Lecture 1

Kinematic Foundations of the Quantum–Geometric Correspondence

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Lecture Notes · Quantum-Geometric Correspondence

Synopsis. The semiclassical theory of gravity is internally inconsistent on matter superpositions; canonical quantization of the metric is non-renormalizable. We formulate the quantum–geometric correspondence as a statement about one von Neumann algebra carrying two complementary restrictions, recall the operator-algebraic and modular machinery it requires (GNS, Tomita–Takesaki, KMS), fix the information-theoretic kinematics (von Neumann and relative entropy, the Bekenstein and holographic bounds, Planck units), and derive the duality map $\sum_n c_n|\psi_n\rangle \mapsto \sum_n c_n|\psi_n\rangle \otimes |g_n\rangle$ *whose reduced density matrix is the gravitational decoherence functional of Lecture 3.*

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1 The semiclassical impasse

1.1 The mean-field equation and its failure

For a classical metric $g_{\mu\nu}$ and matter state $|\psi\rangle$, the Møller–Rosenfeld equation

$$G_{\mu\nu}[g] = \frac{8\pi G}{c^4} \langle\psi|\hat{T}_{\mu\nu}|\psi\rangle \tag{1}$$

is the unique c-number coupling reducing to general relativity for coherent matter and to QFT on a fixed background for weak sources. It is inconsistent for superpositions. With

$$|\psi\rangle = c_1|\psi_1\rangle + c_2|\psi_2\rangle, \qquad \langle\psi_i|\hat{T}_{\mu\nu}|\psi_i\rangle = T^{(i)}_{\mu\nu}, \quad T^{(1)} \neq T^{(2)}, \tag{2}$$

$|\psi_1\rangle, |\psi_2\rangle$ macroscopically distinct, the right side of (1) sources the metric of the *mean* stress tensor $|c_1|^2T^{(1)} + |c_2|^2T^{(2)}$, a geometry realised in no branch. A test mass falls toward the mean position, while a which-branch measurement localises the source; the predictions are incompatible. The Page–Geilker experiment (1981) confirmed matter follows the collapsed, not the mean, configuration. Equation (1) is therefore the $N \to \infty$ mean-field truncation of a coupling whose fluctuations are the subject of this course.

1.2 Non-renormalizability

Canonical quantization of $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{32\pi G}\, h_{\mu\nu}$ fails by power counting: $[G] = M^{-2}$ (natural units), the L-loop amplitude scales as $G^L\Lambda^{2L}$, and infinitely many counterterms are required. Pure gravity is one-loop finite on shell ('t Hooft–Veltman) but two-loop divergent (Goroff–Sagnotti),

$$\Gamma^{(2)}_\infty = \frac{1}{\varepsilon}\,\frac{209}{2880}\,\frac{1}{(16\pi^2)^2}\int d^4x\,\sqrt{-g}\,C_{\mu\nu}{}^{\rho\sigma}C_{\rho\sigma}{}^{\alpha\beta}C_{\alpha\beta}{}^{\mu\nu}, \tag{3}$$

C the Weyl tensor. As an effective field theory (1) is predictive below $m_\mathrm{P}c^2$; the regime that defeats both limits is a heavy mass in genuine superposition. QGC does not quantise the metric: it treats QM and GR as complementary projections of one information-bearing state, and computes the decoherence that (1) cannot.

Scope. We work to leading nontrivial order in G in the linearized regime $GM/c^2d \ll 1$, with $\hbar$ exact. Signature $(-,+,+,+)$; $k_\mathrm{B} = 1$ except where reinstated. The framework provides no UV completion: at the Planck scale itself it breaks down (Lecture 6).

2 Operator-algebraic formulation

2.1 Algebra, state, GNS

Let $\mathcal{A}$ be the observable algebra of a closed gravitating system and ω a state. The GNS construction gives a Hilbert space $\mathcal{H}_\omega$, cyclic vector $|\Omega\rangle$, representation π_ω with $\omega(A) = \langle\Omega|\pi_\omega(A)|\Omega\rangle$. Write $\mathcal{M} = \pi_\omega(\mathcal{A})''$ and commutant $\mathcal{M}'$. The local algebra of a region in vacuum QFT is a hyperfinite type III_1 factor: *no trace, no density matrices, no finite von Neumann entropy*—only relative entropy is defined. This is the obstruction resolved in Lecture 4.

2.2 Complementary subalgebras

Definition 2.1 (Complementary projections)**.** Let $\mathcal{M}_{\mathrm{m}} \subset \mathcal{M}$ be the matter subalgebra and $\mathcal{M}_{\mathrm{g}} = \mathcal{M}_{\mathrm{m}}'$ its commutant (geometry). The state induces $\omega_{\mathrm{QM}} = \omega|_{\mathcal{M}_{\mathrm{m}}}$ and $\omega_{\mathrm{GR}} = \omega|_{\mathcal{M}_{\mathrm{g}}}$ (classical when $\mathcal{M}_{\mathrm{g}}$ is approximately abelian).

2.3 Modular theory

For $\mathcal{M}$ with cyclic separating $|\Omega\rangle$, the closure of $S_0 : A|\Omega\rangle \mapsto A^*|\Omega\rangle$ has polar decomposition $S = J\Delta^{1/2}$, and Tomita–Takesaki gives $\Delta^{it}\mathcal{M}\Delta^{-it} = \mathcal{M}$, $J\mathcal{M}J = \mathcal{M}'$. The modular Hamiltonian is

$$K = -\ln \Delta, \qquad \sigma_t(A) = e^{iKt} A e^{-iKt}, \tag{4}$$

and ω is KMS at inverse temperature 1 with respect to σ_t. The entire dynamical content of the theory is the value of K.

Principle 2.2 (Quantum–geometric correspondence)**.** *QM and GR are the restrictions $\omega_{\mathrm{QM}}, \omega_{\mathrm{GR}}$ of one state ω on $\mathcal{M}$. In the canonical v3.0 (*Modular Diamond*) axioms of Lecture 2 the region algebra carries the generalized entropy (Axiom I, the algebra clause); its diamonds are at entanglement equilibrium (Axiom II, the state clause); and the modular Hamiltonian* (4) *of ω on $\mathcal{M}_{\mathrm{m}}$ satisfies, to leading order in G (Axiom III, the time clause),*

$$K = 2\pi H_{\mathrm{phys}} + O(G^2). \tag{5}$$

Equation (5) holds for the Rindler wedge by Bisognano–Wichmann (H_{phys} the boost generator); its extension to the finite causal diamond sourced by a mass is the central conjecture (Lecture 4). It promotes Bohr complementarity $[\hat{x}, \hat{p}] = i\hbar$ to the relation between $\mathcal{M}_{\mathrm{m}}$ and its geometric commutant. *We will be explicit throughout:* (5) *is* PROVEN *in expectation values and solvable models and* CONJECTURED *as a full operator equation.*

3 Information-theoretic kinematics

3.1 Entropies

For ρ on $\mathcal{M}_{\mathrm{m}}$,

$$S(\rho) = -\operatorname{Tr}(\rho \ln \rho), \qquad S(\rho \,\|\, \sigma) = \operatorname{Tr} \rho(\ln \rho - \ln \sigma) \geq 0, \tag{6}$$

the von Neumann and (Umegaki) relative entropy; the latter is monotone under CPTP maps and remains well-defined in type III (Araki). The first law of entanglement, $\delta S = \delta\langle K\rangle$, links entropy to the modular Hamiltonian and is used repeatedly.

3.2 Bekenstein and holographic bounds

Theorem 3.1 (Information bounds)**.** *For energy E, radius R, boundary area A,*

$$S \leq \frac{2\pi k_{\mathrm{B}} E R}{\hbar c} \quad \textit{(Bekenstein)}, \qquad S \leq \frac{k_{\mathrm{B}} A}{4\ell_{\mathrm{P}}^2} \quad \textit{(holographic)}. \tag{7}$$

Derivation. Lowering a body of energy E, size R, entropy S to a horizon shifts the area by $\delta A = 8\pi\ell_{\mathrm{P}}^2 ER/\hbar c$; the generalized second law $\delta(k_{\mathrm{B}}A/4\ell_{\mathrm{P}}^2 + S) \geq 0$ gives the Bekenstein bound, and combining with $E \leq Rc^4/2G$ gives the holographic bound. The latter—information scaling with boundary area, not volume—is promoted to Axiom I.

3.3 Planck units

$$\ell_{\mathrm{P}} = \left(\frac{G\hbar}{c^3}\right)^{1/2} = 1.616 \times 10^{-35}\,\mathrm{m}, \quad t_{\mathrm{P}} = \frac{\ell_{\mathrm{P}}}{c} = 5.39 \times 10^{-44}\,\mathrm{s}, \quad m_{\mathrm{P}} = \left(\frac{\hbar c}{G}\right)^{1/2} = 2.176 \times 10^{-8}\,\mathrm{kg}. \tag{8}$$

All three constants enter: the correspondence is nontrivial precisely where quantum, relativistic and gravitational effects are simultaneously $O(1)$.

4 The duality map and the decoherence functional

4.1 Coherent states of the linearized field

The perturbation sourced by a classical $T^{\mu\nu}$ is a coherent state $|g[T]\rangle = D(\alpha_T)|0\rangle$, $D(\alpha) = \exp(\alpha a^\dagger - \bar{\alpha} a)$ the graviton displacement operator. For a point mass M at $\mathbf{x}_A$ the Newtonian (constraint-determined) amplitude is

$$\alpha_A(\mathbf{k}) = -\frac{4\pi G M}{k^2}\frac{e^{-i\mathbf{k}\cdot\mathbf{x}_A}}{\sqrt{2\hbar\omega_k}}, \qquad \omega_k = c|\mathbf{k}|, \tag{9}$$

the Fourier transform of the Poisson constraint $\nabla^2\hat{\Phi} = 4\pi G\hat{\rho}$. Coherent-state overlaps are Gaussian, $\langle g[T_m]|g[T_n]\rangle = \exp(-\frac{1}{2}\|\alpha_n - \alpha_m\|^2 + i\Im\langle\alpha_m, \alpha_n\rangle)$.

4.2 The joint state and reduced density matrix

Theorem 4.1 (Duality correspondence)**.** *Let $|\psi\rangle = \sum_n c_n|\psi_n\rangle$ source $T_n^{\mu\nu}$. Under the correspondence*

$$|\Psi\rangle = \sum_n c_n\,|\psi_n\rangle \otimes |g_n\rangle, \qquad (\rho_{\mathrm{m}})_{nm} = c_n c_m^*\,\langle g_m|g_n\rangle, \quad |g_n\rangle = |g[T_n]\rangle. \tag{10}$$

Derivation. Linearity of (1) on each branch gives the entangled state (10); tracing over $\mathcal{H}_{\mathrm{g}}$ with $\mathrm{Tr}_{\mathrm{g}}(|g_n\rangle\langle g_m|) = \langle g_m|g_n\rangle$ gives the matrix elements.

4.3 The decoherence functional and the energy scale

Writing $\langle g_m|g_n\rangle = \exp(-\frac{1}{2}\Gamma_{mn} + i\Phi_{mn})$, the modulus $\Gamma_{mn} \geq 0$ is the decoherence exponent. Integrating out $h_{\mu\nu}$ on the closed-time-path gives the Feynman–Vernon influence functional $\mathcal{F}[T_n, T_m] = \langle g_m|g_n\rangle$. The scale of $\|\alpha_n - \alpha_m\|^2$ is the gravitational self-energy of the difference source,

$$E_{\mathrm{G}} = \frac{G}{2}\iint \frac{\Delta\rho(\mathbf{x})\Delta\rho(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|}\,d^3x\,d^3y \xrightarrow{\text{two point masses}} \frac{GM^2}{d}, \tag{11}$$

the cross-term of the double integral (self-terms cancel between branches). For a uniform sphere of radius R the kernel saturates at $E_{\mathrm{G}} \to \frac{6}{5}GM^2/R$ for $\Delta \geq 2R$ (used in Lecture 6). We record:

(i) Inter-branch coherence is the geometric overlap $(\rho_\mathrm{m})_{nm} = c_n c_m^* \langle g_m | g_n \rangle$, a Gaussian in the coherent amplitudes (9). **(ii)** Its scale is $E_\mathrm{G} = GM^2/d$, PROVEN (classical gravity). Whether $\Gamma_{mn}(t)$ grows at order G or G^2 is the dynamical question of Lecture 3.

5 What the framework reproduces (preview)

The same kinematics recovers, as theorems rather than inputs, results otherwise postulated separately: Bekenstein–Hawking entropy and the Unruh temperature (Axiom I; Lecture 2); the Einstein equations (Axioms I+II jointly, via Jacobson's entanglement equilibrium; Lecture 2); the Ryu–Takayanagi formula $S = A/4G_N$ with Brown–Henneaux $c = 3L/2G_N$ in AdS_3; the black-hole Page curve from information conservation $\Delta S_\mathrm{vN} = -\Delta S_\mathrm{EH}$; and the emergence of physical time from the constraint (Page–Wootters; Connes–Rovelli thermal time). The Born rule is *assumed*, not derived—the framework is, in this respect, on the same footing as every quantum theory (Lecture 6). These are catalogued where they arise.

Lecture 2. The three primitive axioms (v3.0: the algebra, state and time clauses of the Modular Diamond Principle), their independence, the causal-diamond microfoundation of the Einstein equations, and the derivation from them of the standard gravitational thermodynamics and of the reproduced known results just previewed.

Hands-on

For the curious — E1.1 — the benchmark numbers

Statement. Compute $\tau_\mathrm{dec} = \hbar d/GM^2$ at $M = 1\,\mu\mathrm{g}$, $d = 1\,\mathrm{mm}$, and check $E_\mathrm{G} = GM^2/d$.
Solution. With $G = 6.67430 \times 10^{-11}$, $\hbar = 1.054571817 \times 10^{-34}$, $M = 10^{-9}\,\mathrm{kg}$, $d = 10^{-3}\,\mathrm{m}$:

$$E_\mathrm{G} = \frac{GM^2}{d} = \frac{(6.67430 \times 10^{-11})(10^{-9})^2}{10^{-3}} = 6.674 \times 10^{-26}\ \mathrm{J},$$

$$\tau_\mathrm{dec} = \frac{\hbar d}{GM^2} = \frac{\hbar}{E_\mathrm{G}} = \frac{1.054571817 \times 10^{-34}}{6.674 \times 10^{-26}} = 1.58 \times 10^{-9}\ \mathrm{s} = 1.58\ \mathrm{ns}.$$

Equivalently $\tau_\mathrm{dec} = (\hbar/G)\,d/M^2 = 1.58 \times 10^{-24}\,d[\mathrm{m}]/(M[\mathrm{kg}])^2\,\mathrm{s}$. The M^{-2} scaling spans 46 decades (Lecture 3 table). *Run it:* `TOOLS/verify-paper-numerics.py` recomputes both from CODATA and matches the paper ($E_\mathrm{G} = 6.7 \times 10^{-26}\,\mathrm{J}$, $\tau_\mathrm{dec} = 1.6\,\mathrm{ns}$). E_G is PROVEN (classical gravity); the *rate* $E_\mathrm{G}/\hbar$ inherits Axiom III's CONJECTURED status.

For the curious — E1.2 — the Rindler 2π from squeezing

Statement. For one Rindler mode, $\tanh r = e^{-\pi\omega}$ gives a thermal occupation at modular temperature 2π: derive $K_0[n] = 2\pi\omega n$.
Solution. The Minkowski vacuum, restricted to the right Rindler wedge, is the thermofield double $|0_M\rangle = \prod_\omega (\cosh r)^{-1} \sum_n (\tanh r)^n |n\rangle_R |n\rangle_L$ with squeezing $\tanh r = e^{-\pi\omega}$ (here ω in units where the boost period is 2π; $\tanh r = e^{-\beta\omega_\mathrm{phys}/2}$ with $\beta_\mathrm{mod} = 2\pi$). Tracing L, the reduced state is thermal, $\rho_R \propto \sum_n (\tanh^2 r)^n |n\rangle\langle n| = \sum_n e^{-2\pi\omega n} |n\rangle\langle n|$ (using $\tanh^2 r =$

$e^{-2\pi\omega}$). Then the modular Hamiltonian is

$$K_0 = -\ln \rho_R = 2\pi\omega\, n + \text{const},$$

i.e. $K_0[n] = 2\pi\omega n$: the reduced wedge state is thermal at inverse modular temperature 2π, $K_0 = 2\pi H_{\text{boost}}$ (Bisognano–Wichmann), *derived from the squeezing, not put in by hand.* This is the $\alpha = 0$ (undressed) case of the central identity $K = 2\pi H_{\text{phys}}$. *Run it:* `VERIFIED/tests/test_modular_operator_identification.py` (Block A: $K_0 = 2\pi\omega n$ from the squeezed vacuum, Δ^{is} acting as $e^{-2\pi s}$ boost).

Lecture 2

The Axiomatic Basis and its Theorems

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*Synopsis. Three primitive axioms generate the theory—in the canonical v3.0 (*Modular Diamond*) formulation adopted July 2026, the algebra, state and time clauses of one principle. We state them exactly, teach why the v2.0 formulation was torn down (the conservation clause, the entropic action, the Planck interpolation), prove their independence by countermodel, and derive as theorems: Bekenstein–Hawking entropy, the Unruh temperature, and—jointly from Axioms I+II via Jacobson—the Einstein equations. We catalogue the reproduced known results (Ryu–Takayanagi, the Page curve, the problem of time, the arrow of time), state plainly what is* not *derived (the Born rule), and close with the Modular Diamond Principle that localizes the whole conjectural burden in the time clause.*

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1 The three axioms (v3.0)

The canonical set is the three clauses of one statement: *the physical content of a region is the observer-dressed modular structure of its causal diamond.* Axiom I is the *algebra* clause, Axiom II the *state* clause, Axiom III the *time* clause.

Axiom 1.1 (Generalized entropy — the algebra clause)**.** *For a region bounded by a quantum extremal surface or causal horizon $\mathcal{X}$, with the gravitational constraints imposed, the physical observables form a Type II (crossed-product) algebra whose von Neumann entropy is the generalized entropy*

$$S_{\text{gen}}(\mathcal{X}) = \frac{A(\mathcal{X})}{4\ell_{\text{P}}^2} + S_{\text{ext}}(\mathcal{X}) + \text{const}, \tag{1}$$

defined up to an additive constant, with ***(I.a)*** *global unitarity: for the total state on a complete Cauchy slice S_{gen} is invariant under evolution (a foliation-free statement about the full slice, not d/dt of a subregion);* ***(I.b)*** *the generalized second law: $\Delta S_{\text{gen}} \geq 0$ along future horizon cuts for open subregions (Wall, given the QNEC);* ***(I.c)*** *surface selection: $\mathcal{X}$ obeys the QES rule $\partial_\lambda S_{\text{gen}} = 0$, taken as a* postulate*.*

Scope. The split between $A/4\ell_{\text{P}}^2$ and S_{ext} is renormalization-scheme dependent; only the *sum*, its *differences* and its *monotonicity* are physical—no "two-ledger" reading is asserted. The area law $S = A/4\ell_{\text{P}}^2$ is the *irreducible geometric input*, postulated here, not derived. Rigorous backing: the Type II statement is the CLPW/CPW theorem family (crossed product of the Type III_1 region algebra by the dressed observer's modular flow; generalized entropy = the Type II von Neumann entropy), realized for the causal diamond at CFT-proxy standing (Lecture 4).

1.1 Why the v2.0 conservation clause was withdrawn

v2.0's Axiom I carried a closed-system clause $dS_{\text{gen}}/dt = 0$. It is *vacuous where true and false where non-trivial*, and no derived theorem used it. The dilemma:

Derivation. If $\mathcal{X}$ bounds the whole system, then for a pure global state $S_{\text{ext}} = S_{\text{vN}}(\rho_{\text{total}}) = 0$ at all times, so $dS_{\text{gen}}/dt = 0$ merely restates unitarity (vacuous). If instead $\mathcal{X}$ is a dynamical interior surface (an evaporating black hole), the Page curve *is* the statement that $A/4\ell_{\text{P}}^2$ and S_{ext} do not cancel slice-by-slice: $dS_{\text{gen}}(\text{horizon})/dt \neq 0$ generically (false). The constancy that does hold is a property of the QES/island min-ext prescription for the radiation region—a different object obeying a different theorem.

The repaired clause keeps only the two survivors: global unitarity on a slice (I.a) and the GSL for open regions (I.b). The dead ledger costs zero predictions.

Axiom 1.2 (Entanglement equilibrium — the state clause)**.** *Every small causal diamond $\mathcal{D}$ ($\ell_{\text{P}} \ll \ell \ll L_{\text{curv}}$) is at entanglement equilibrium: the generalized entropy of Axiom 1.1 is stationary at fixed volume,*

$$\delta S_{\text{gen}}(\mathcal{D}) = 0. \tag{2}$$

The two consequences: **(state)** the diamond state satisfies the KMS condition at $\beta_{\text{mod}} = 2\pi$—the thermal/Gibbs statics; **(geometry)** stationarity yields the semiclassical Einstein equations (Theorem 3.1). The v2.0 entropic action survives only as an equivalent repackaging of the statics:

Remark 1.1. The free-energy functional $\mathcal{S}[\rho, g] = \int d^4x\sqrt{-g}\,[\langle\hat{H}\rangle_\rho + \frac{c^4}{16\pi G}R - \beta^{-1}s(\rho)]$ is an equivalent coarse-grained repackaging of the diamond's KMS statics; it is *not* an axiom and is varied to obtain no prediction. As v2.0's Axiom II it was mislabeled (statics dressed as dynamics: $\delta\mathcal{S}/\delta\rho = 0$ gives a frozen Gibbs state, and the flagship pure superposition—$s(\rho) = 0$, rank 1—is not even an allowed configuration, $\ln\rho$ divergent). Every flagship result (the decoherence $\tau_{\rm dec}$, $a_0 = cH_0/2\pi$, $\rho_{\rm DE}$) is obtained *without* varying it. It is demoted to appendix status.

Axiom 1.3 (Modular time — the time clause)**.** *For the diamond's observer, the modular flow of the constrained (Type II, observer-dressed) algebra at $\beta_{\rm mod} = 2\pi$ is physical time evolution; at the operator level*

$$K = 2\pi H_{\rm phys} + O(G^2). \tag{3}$$

This is **the falsifiable axiom**. Its consequence is gravitational decoherence at G^1, $\Gamma_{\rm dec} = C\,E_{\rm G}/\hbar$ with $E_{\rm G} = GM^2/d$ and $C \in [2/\pi, 1]$ (natural $C = 1$, the Diósi–Penrose rate; Lecture 3), together with the modular-thermal decoherence spectrum. It *supersedes* the v2.0 "Saturation Principle / candidate Axiom IV" (the GIA hypothesis): the Margolus–Levitin saturation reading is the same commitment expressed as a bound, supplying the $C = 2/\pi$ floor.

Scope. The operator identity (3) is CONJECTURED (PROVEN in expectation values and solvable models; the $O(G^2)$ residual and the interacting split constant are the R1 program's two open values, both bounded in controlled models, Lecture 4). The framework *commits* to it: observation of G^2-timescale coherence for a preparable superposition falsifies Axiom III—and with it the theory's distinctive content.

1.2 What happened to the old Axiom III

v2.0's Axiom III was the Planck-scale interpolation $\mathcal{O} = \rho_{\rm q}f(r/\ell_{\rm P}) + \rho_{\rm g}[1 - f(r/\ell_{\rm P})]$, $\rho_{\rm g} = (c^4/8\pi G)G_{\mu\nu}u^\mu u^\nu$, $f(\xi) = 1/(1+\xi^2)$. It is formally incoherent as stated—*trivial or Bianchi-violating*—and load-bearing for nothing; it is demoted to a speculative UV sketch.

Derivation. [the trivial-or-Bianchi-violating dilemma; check it by hand] Wherever the semiclassical Einstein equation (Axioms I+II) holds, $\rho_{\rm g} = \rho_{\rm q}$ identically—the interpolation is between a quantity and itself and f drops out; no prediction can depend on f. The only escape is to assert the Einstein equation fails at $r \sim \ell_{\rm P}$; then $\mathcal{O}$ is a modified field equation whose effective source violates the contracted Bianchi identity,

$$\nabla^\mu T^{\rm eff}_{\mu\nu} = \Big(\langle T_{\mu\nu}\rangle - \frac{c^4}{8\pi G}G_{\mu\nu}\Big)\nabla^\mu f(r/\ell_{\rm P}) \neq 0, \tag{4}$$

both factors nonzero only at $r \sim \ell_{\rm P}$ where the descriptions differ and f varies—a diffeomorphism inconsistency exactly where the axiom has content.

What survives: the minimum length $\Delta x_{\rm min} \sim \ell_{\rm P}$ and the modified dispersion $E^2 = p^2c^2(1 + \alpha p^2/m_{\rm P}^2c^2) + m^2c^4$ remain as *GUP-derived phenomenology* (the observer-horizon route), flagged, with $O(1)$-coefficient and power-n uncertainty; the $n = 2$ form is not currently excluded by time-of-flight data (Lecture 6). The interpolation's frame-dependent profile $r/\ell_{\rm P}$ also exposes it to preferred-frame Lorentz violation and the Collins-*et-al.* radiative-percolation problem with no protection mechanism specified.

2 Logical independence

Proposition 2.1. *Each axiom can fail while the other two hold.*

Derivation. **I without II**: a spacetime whose region algebras carry the Type II generalized entropy but whose diamonds are *not* at equilibrium—a non-Einstein background sourced by out-of-equilibrium entanglement ($\delta S_{\text{gen}} \neq 0$); the entropy functional exists, the field equations differ. **II without III**: a world with entanglement equilibrium (hence Einstein equations) where modular flow is *not* physical time. The in-repo countermodel is the QED contrast: a theory whose constraint is Gauss's law (spatial, gauge) rather than the Hamiltonian constraint has $[K, H_{\text{phys}}] \neq 0$, so modular flow and time evolution are inequivalent and the first-order rate vanishes ($C_{\text{QED}} = 0$). Axiom III is precisely what distinguishes gravity. **III without I**: modular time on an algebra without the area term—the flat Rindler wedge of a non-gravitational QFT, where Bisognano–Wichmann gives $K = 2\pi H_{\text{boost}}$ exactly, Type III$_1$, no $A/4G$, no generalized entropy.

3 Microfoundation: entanglement equilibrium (the state clause at work)

Theorem 3.1 (Einstein equations from Axioms 1.1+1.2)**.** *Entanglement equilibrium* (2) *of every small diamond, with Axiom 1.1's area term, implies the semiclassical Einstein equations* $G_{\mu\nu} = (8\pi G/c^4)\langle \hat{T}_{\mu\nu}\rangle$.

Derivation. In a small causal diamond the vacuum modular Hamiltonian is the Hislop–Longo form $K = 2\pi \int (\ell^2 - r^2)/(2\ell)\, T_{00}$ (Lecture 4). The first law of entanglement $\delta S_{\text{ext}} = \delta\langle K\rangle$ (Bisognano–Wichmann/CHM) combined with the Raychaudhuri relation $\delta A \propto -G_{\mu\nu}k^\mu k^\nu$ gives, at $\delta S_{\text{gen}} = 0$, the Einstein equation pointwise (Jacobson, arXiv:1505.04753). The geometric and matter prefactors *cancel exactly*; no Einstein–Hilbert term is inserted by hand. **Correction, stated explicitly**: the entropy–area relation itself is *input* (Axiom 1.1), not derived—this is exactly the point on which the February 2026 Jacobson restructuring was deferred by the June review, and the claim here is the corrected one. "Einstein derived" means: *given* the area law.

Scope. Jacobson's derivation carries its own conditions (small diamonds, first-order variations, conformal-matter subtleties, the cosmological constant as an undetermined integration constant). The result is DERIVED on the "gravity as thermodynamics" programme (Jacobson; Padmanabhan; Verlinde), *given* the area law of Axiom 1.1.

4 Theorems from Axiom 1.1

Theorem 4.1 (Bekenstein–Hawking entropy)**.** *A stationary horizon has* $S_{\text{BH}} = k_{\text{B}}A/4\ell_{\text{P}}^2$ *(set* $S_{\text{ext}} = 0$ *in* (1) *on the horizon).*

Theorem 4.2 (Unruh temperature)**.** *An observer of proper acceleration* a *attributes to the vacuum* $T_U = \hbar a/2\pi k_{\text{B}} c$.

Derivation. Restrict the vacuum to the Rindler wedge. By Bisognano–Wichmann the modular flow is the boost, $\Delta^{it} = e^{-2\pi i K_\eta t/\hbar}$ with $K_\eta = (\hbar/c)\int_W T_{00} x^1 d^3x$, so the state is KMS at boost-period 2π. With $\tau = (c/a)\eta$ the proper-time period is $2\pi c/a$, giving T_U. The first law $dE = T_U\, dS_{\text{ext}}$ across the horizon fixes the 1/4 in (1).

The holographic bound $S_{\text{ext}} \le k_B A/4\ell_P^2$ is the GSL (I.b) applied to (1).

5 The GUP phenomenology (demoted Axiom III of v2.0)

The minimum length and modified dispersion *survive* the demotion of the old Axiom III: they follow from the GUP/observer-horizon route, not from the (retired) interpolation (4).

Theorem 5.1 (Resolution limit and GUP)**.** *Combining the quantum spread and the probe's own gravitational radius,*

$$\Delta x\, \Delta p \ge \frac{\hbar}{2}\Big[1 + \beta\Big(\frac{\Delta p}{m_P c}\Big)^2\Big], \quad \Delta x_{\min} = \sqrt{2}\,\ell_P, \qquad E^2 = p^2c^2\Big(1 + \alpha\frac{p^2}{m_P^2c^2}\Big) + m^2c^4. \tag{5}$$

Derivation. The quantum spread $\hbar/2\Delta p$ and the gravitational radius $2\ell_P^2\Delta p/\hbar$ of a probe add in quadrature, giving $\Delta x_{\min} = \sqrt{2}\ell_P$ at $\Delta p \sim m_P c$. The coefficient $\beta = 4$ is the $O(1)$ value of the observer-horizon estimate; it carries $O(1)$/power-n uncertainty, *not* the false precision of the retired $f(\xi)$ ansatz.

Scope. This is MOTIVATED phenomenology (Lecture 6): the induced GW/birefringence effects are $\sim 10^{-50}$–10^{-65} relative, honestly unobservable; the $n = 2$ dispersion is not currently excluded (GRB time-of-flight bounds sit below m_P). A covariant replacement—the finite energy width of the crossed-product observer's clock—is open research.

6 Reproduced known physics

Theorem 6.1 (Ryu–Takayanagi)**.** *Axiom 1.1 with the QES rule (I.c) gives the entanglement entropy of a boundary region B as the area of the bulk minimal surface γ_B: $S(B) = L[\gamma_B]/4G_N$. For an AdS_3 interval of length ℓ, $S = \frac{c}{3}\ln(\ell/\epsilon)$ with Brown–Henneaux central charge $c = 3L/2G_N$.*

Derivation. Axiom 1.1 gives $S = A/4\ell_P^2$ for the extremal surface; the QES stationarity $\partial_\lambda S_{\text{gen}} = 0$ (I.c) selects the minimal surface homologous to B. RT is thus a special case of generalized-entropy stationarity, not an independent postulate.

Theorem 6.2 (Black-hole information / Page curve)**.** *Information conservation (Axiom 1.1) gives, during evaporation, $\Delta S_{\text{vN}} = -\Delta S_{\text{EH}}$, hence the radiation entropy $S_{\text{rad}}(t) = \min[S_{\text{rad}}^{\text{naive}}(t), S_{\text{BH}}(t)]$—the Page curve—with unitarity preserved throughout; the island/QES prescription follows from the extremality of (1). The firewall is avoided because an infalling observer of mass M_{obs} decoheres in $\tau_{\text{dec}} \sim \hbar r_s/GM_{\text{obs}}^2c^2 \ll \tau_{\text{cross}}$ before reaching the horizon (Lecture 3 mechanism), so no single observer tests the monogamy-violating entanglement.*

Time, and what is not derived. The Wheeler–DeWitt constraint $H_{\text{total}} = 0$ renders the global state timeless; physical time emerges relationally (Page–Wootters), $|\Psi\rangle = \int ds\, |s\rangle_C \otimes |\psi(s)\rangle_S$ with $i\hbar\partial_s|\psi\rangle = \hat{H}_S|\psi\rangle$, the Connes–Rovelli thermal (modular) time being the special case generated by the modular Hamiltonian. The *arrow* of time is the direction of $dS_{\text{vN}}/dt > 0$: by Axiom 1.1, regions of high curvature carry large S_{EH} and hence low S_{vN}, so the low-entropy early universe is a geometric boundary property, not a fine-tuned initial condition (Lecture 5). The **Born rule is assumed**, not derived: decoherence selects the position pointer basis and suppresses off-diagonal ρ_{nm}, but $p_n = |c_n|^2$ is postulated as in any quantum theory. QGC is compatible with, but not equivalent to, many-worlds.

7 Status: axioms versus theorems

Statement	**Status**	**From**
Axioms I–III (algebra/state/time clauses)	axiom	—
Bekenstein–Hawking, Unruh, holographic bound	DERIVED	Axiom 1.1 (+B–W)
Einstein equation $G_{\mu\nu} = \frac{8\pi G}{c^4}\langle \hat{T}_{\mu\nu}\rangle$	DERIVED	Axioms 1.1+1.2 (Jacobson)
Ryu–Takayanagi, $c = 3L/2G_N$	DERIVED	Axiom 1.1 (QES)
Page curve, firewall resolution	DERIVED	Axiom 1.1
Gravitational decoherence $\Gamma_{\text{dec}} = CE_{\text{G}}/\hbar$	CONJECTURED	Axiom 1.3
Arrow of time, time emergence	DERIVED	Axiom 1.1 + constraint
Minimum length $\sqrt{2}\ell_{\text{P}}$, GUP (demoted Ax. III)	MOTIVATED	GUP/observer-horizon
Born rule $p_n = \vert c_n\vert^2$	ASSUMED	—

8 The Modular Diamond Principle

Principle 8.1 (Modular Diamond Principle)**.** *The three axioms are the three clauses of a single statement: for every small causal diamond $\mathcal{D}$, the gravitationally constrained algebra of $\mathcal{D}$ is the crossed product of its field algebra by its observer's modular flow (*algebra*, Axiom 1.1), the physical state of $\mathcal{D}$ is at entanglement equilibrium (*state*, Axiom 1.2), and the modular flow of $\mathcal{D}$ at $\beta_{\text{mod}} = 2\pi$ is $\mathcal{D}$'s physical time (*time*, Axiom 1.3).*

Two of the three clauses are theorems of rigorous structures—the algebra clause of the CLPW/CPW crossed-product theorems, the state clause of Jacobson's equilibrium argument. *The entire conjectural burden of the foundation is localized in the time clause, $K = 2\pi H_{\text{phys}}$,* which is exactly what the G^1-vs-G^2 experiment measures. The honest trade of v3.0: the theory gives up the "single conserved matter–geometry information ledger" and the "we show QM = GR" rhetoric in exchange for a foundation whose one open question is experimentally decidable.

Lecture 3. Axioms 1.1–1.2 fix which states and geometries are self-consistent; they do not fix the *rate* of $\Gamma_{mn}(t)$. Axiom 1.3 does. We compute it: the constrained influence functional, the modular identity $K = 2\pi H_{\text{phys}}$, the coherent-state cancellation of nonlocal corrections, the saturation of the free bath, and the resulting G^1 scaling with coefficient $C \in [2/\pi, 1]$.

Hands-on

For the curious — E2.1 — the Bianchi kill of the old Axiom III

Statement. For the v2.0 interpolation $\mathcal{O} = \rho_{\rm q} f + \rho_{\rm g}(1-f)$ with $\rho_{\rm g} = (c^4/8\pi G)G_{\mu\nu}u^\mu u^\nu$ read as a field equation, compute $\nabla^\mu T^{\rm eff}_{\mu\nu}$ and show it is nonzero exactly where the axiom is non-trivial.

Solution. Read the interpolation as $c^4 G_{\mu\nu}/8\pi G = (1-f)\cdot c^4 G_{\mu\nu}/8\pi G + f\langle T_{\mu\nu}\rangle$, i.e. the effective source is $T^{\rm eff}_{\mu\nu} = \langle T_{\mu\nu}\rangle + (1-f)(\frac{c^4}{8\pi G}G_{\mu\nu} - \langle T_{\mu\nu}\rangle)$. The geometric side is covariantly conserved by the contracted Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$, and $\nabla^\mu\langle T_{\mu\nu}\rangle = 0$ off-shell for the matter, so

$$\nabla^\mu T^{\rm eff}_{\mu\nu} = \Big(\langle T_{\mu\nu}\rangle - \frac{c^4}{8\pi G}G_{\mu\nu}\Big)\nabla^\mu f(r/\ell_{\rm P}).$$

Where Axioms I+II hold, $\langle T_{\mu\nu}\rangle = c^4 G_{\mu\nu}/8\pi G$, so the bracket vanishes and $\mathcal{O}$ is trivial (f drops out). Where the Einstein equation is asserted to fail ($r \sim \ell_{\rm P}$), both the bracket and $\nabla^\mu f$ are nonzero, so $\nabla^\mu T^{\rm eff}_{\mu\nu} \neq 0$: diffeomorphism inconsistency at the only scale where the axiom says anything. This is why v3.0 demotes it. *Run it:* the algebra is verified by hand in `AGENT/reviews/axiom-teardown-2026-07.md` §3 (the $\nabla^\mu f$ identity re-verified in synthesis).

For the curious — E2.2 — Jacobson's cancellation

Statement. Carry out the small-diamond first-law + Raychaudhuri computation and see the prefactors cancel into $G_{\mu\nu} = 8\pi G\langle T_{\mu\nu}\rangle/c^4$.

Solution. Take a small causal diamond of a local Rindler horizon; the Hislop–Longo/Unruh modular Hamiltonian gives the first law $\delta S_{\rm ext} = \delta\langle K\rangle = (2\pi/\hbar c)\int_{\mathcal{H}}\lambda\,\langle T_{\mu\nu}\rangle k^\mu k^\nu\,d\lambda\,dA$ along the horizon generator k^μ (affine λ). The area side uses Raychaudhuri: for vanishing expansion/shear at the bifurcation, $\delta A = -\int\lambda\,R_{\mu\nu}k^\mu k^\nu\,d\lambda\,dA$, so $\delta S_{\rm area} = \delta A/4\ell_{\rm P}^2 = -(1/4\ell_{\rm P}^2)\int\lambda R_{\mu\nu}k^\mu k^\nu$. Equilibrium $\delta S_{\rm gen} = \delta S_{\rm area} + \delta S_{\rm ext} = 0$ for *all* null k^μ forces $R_{\mu\nu}k^\mu k^\nu = (8\pi\ell_{\rm P}^2/\hbar c)\langle T_{\mu\nu}\rangle k^\mu k^\nu$, i.e. $R_{\mu\nu} - \Phi g_{\mu\nu} = (8\pi G/c^4)\langle T_{\mu\nu}\rangle$; conservation fixes Φ up to Λ, giving $G_{\mu\nu} + \Lambda g_{\mu\nu} = (8\pi G/c^4)\langle T_{\mu\nu}\rangle$. With $\ell_{\rm P}^2 = G\hbar/c^3$ the $\hbar$ and c prefactors cancel; the area law $1/4\ell_{\rm P}^2$ is the input (Axiom I). *Run it:* the derivation is Jacobson (arXiv:1505.04753); its in-repo alignment to v3.0 is `THEORY/derivations/jacobson-program-within-qgc.md`.

For the curious — E2.3 — the QED countermodel (II without III)

Statement. Show that a spatial (Gauss) constraint generates gauge transformations at fixed time, so modular flow $\neq$ time evolution, and the first-order rate vanishes.

Solution. QED has only the Gauss constraint $\mathcal{G}(\mathbf{x}) = \nabla\cdot\mathbf{E} - \rho$; smeared, $Q[\lambda] = \int\lambda\,\mathcal{G}$ generates $\delta A_i = \partial_i\lambda$, $\delta\psi = -i\lambda\psi$—a *spatial* gauge transformation at fixed t, commuting with the Hamiltonian, $[\mathcal{G}, H] = 0$ but $\mathcal{G} \neq -K/\beta$. So imposing $\mathcal{G}|\psi\rangle = 0$ is *not* imposing modular (KMS) invariance, and $[K, H_{\rm phys}] \neq 0$: modular flow and time evolution are inequivalent. In the friction dictionary $C = \eta\hbar/2E$ the free EM bath is super-Ohmic ($J \propto \omega^3$), so $\eta_{\rm QED,free} = 0$ and $C_{\rm QED} = 0$—no first-order decoherence, consistent with electron interferometry. Gravity differs by exactly one fact: its constraint is $H_{\text{total}} = 0$ (Hamiltonian), which *is* $-K_{\text{full}}/\beta$, so $\mathcal{C}|\psi\rangle = 0$ *is* modular invariance and $K = 2\pi H_{\text{phys}}$ (Axiom III). This is the content of independence "II without III." *Run it:* `TOOLS/verify_qed_gravity_friction_parity.py` (18/18; free-bath parity $\eta_{\text{grav,free}} = \eta_{\text{QED,free}} = 0$, criterion: a first-order rate exists $\iff$ the constraint identifies modular flow with time). The gravity side ($\eta_{\text{grav}} \neq 0$) inherits the CONJECTURED status of (3).

Lecture 3

Gravitational Decoherence and the G^1 Mechanism

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Synopsis. We compute the decoherence functional of Lecture 1. The Wheeler–DeWitt constraint forbids the product initial state and forces a matter–geometry coherent entanglement; the constrained influence functional is $\mathcal{F} = e^{iE_G t/\hbar}\langle\Phi_L(t)|\Phi_R(t)\rangle$, all decoherence residing in the coherent-state overlap, which scales as G^1 (no graviton propagator). The free-graviton bath is super-Ohmic and saturates*; the linear rate is fixed by the modular identity $K = 2\pi H_{\text{phys}}$, giving $\Gamma_{\text{dec}} = C\,E_{\text{G}}/\hbar$ with $C \in [2/\pi, 1]$. The product-state (perturbative) computation gives G^2; the two compute generation vs. manifestation. We end with the decay form, many-body scaling, and an honest audit: the five "converging" arguments reduce to one input.*

Contents

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1 The constraint and the initial state

Write $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. The linearized Wheeler–DeWitt (Hamiltonian) constraint and momentum/Gauss constraints read

$$\hat{H}_{\text{total}}|\Psi_{\text{phys}}\rangle = 0, \qquad \nabla^2\hat{\Phi}(\mathbf{x}) = 4\pi G\,\hat{\rho}(\mathbf{x}). \tag{1}$$

The product state $|\psi\rangle \otimes |0\rangle_{\text{grav}}$ used in the standard Feynman–Vernon calculation *violates* (1): a localized mass must carry its Newtonian field. The constraint forces

$$|\Psi_{\text{phys}}\rangle = \tfrac{1}{\sqrt{2}}\big(|L\rangle|\Phi_L\rangle + |R\rangle|\Phi_R\rangle\big), \qquad |\Phi_A\rangle = \hat{D}(\alpha_A)|0\rangle, \quad \alpha_A(\mathbf{k}) = -\frac{4\pi GM}{k^2}\frac{e^{-i\mathbf{k}\cdot\mathbf{x}_A}}{\sqrt{2\hbar\omega_k}}. \tag{2}$$

Each branch is *dressed* with its constraint-determined coherent field. This is the physical input that selects G^1.

2 The constrained influence functional

Theorem 2.1 (Constrained influence functional)**.** *Evolving* (2) *and tracing the field,*

$$\mathcal{F}_{\text{constr}}[L, R; t] = \exp\Big(\frac{iE_G t}{\hbar}\Big)\,\langle\Phi_L(t)|\Phi_R(t)\rangle, \tag{3}$$

so $\rho_{LR}(t) = \frac{1}{2}e^{iE_G t/\hbar}\langle\Phi_L(t)|\Phi_R(t)\rangle$ *and the decoherence exponent is* $\Gamma(t) = -\ln|\langle\Phi_L(t)|\Phi_R(t)\rangle|^2$.

The first factor is a pure phase (no decoherence); all decoherence is the coherent-state overlap.

Proposition 2.2 (The overlap is G^1)**.**

$$|\langle\Phi_L|\Phi_R\rangle|^2 = e^{-\|\delta\alpha\|^2}, \quad \|\delta\alpha\|^2 = \int\frac{d^3k}{(2\pi)^3}|\alpha_L(\mathbf{k}) - \alpha_R(\mathbf{k})|^2 = \frac{GM^2}{\hbar c}\,\ln\frac{d}{\varepsilon}, \tag{4}$$

ε *the source size. This is dimensionless and scales as* $GM^2/\hbar c = (M/m_{\text{P}})^2$ —one *power of* G.

Derivation. $|\delta\alpha(\mathbf{k})|^2 = (4\pi GM)^2(2\hbar\omega_k k^4)^{-1}\cdot 2[1-\cos(\mathbf{k}\cdot\mathbf{d})]$; the k-integral gives (4). Crucially $\|\delta\alpha\|^2$ is the norm of an amplitude in the single-particle Hilbert space—*not* a two-point function—so it carries no graviton propagator (which would add a second power of G). This is the structural origin of G^1.

3 Saturation: why the bath alone gives no rate

The free-field exponent is, exactly,

$$\Gamma(t) = \int\frac{d^3k}{(2\pi)^3}|\delta\alpha(\mathbf{k})|^2\,2[1-\cos\omega_k t] \xrightarrow{t\gg d/c} \Gamma_{\text{sat}} = \frac{2GM^2}{\pi\hbar c}\ln\frac{d}{\ell_{\text{P}}} = \xi\ln\frac{d}{\ell_{\text{P}}}, \quad \xi = \frac{GM^2}{\hbar c}. \tag{5}$$

Proposition 3.1 (Saturation)**.** *The free-graviton bath has super-Ohmic spectral density* $J(\omega) \propto \omega^3$*, so* $J(0^+) = 0$ *and the Born–Markov rate* $\gamma_\infty = 2\pi J(0^+)$ vanishes. *The exponent* (5) *approaches a constant* ($\Gamma_{\text{sat}} \approx 0.154$ *at* $M = 1\,\mu g$, $d = 1\,mm$)*; it is a* static overlap, *not a rate.*

Remark 3.1. $\Gamma_{\rm sat}$ is cutoff-dependent ($\propto \ln(d/\ell_{\rm P})$) and $\propto M^2$; its proximity to $1/2\pi = 0.159$ at the lab point is a numerical coincidence. The earlier derivation of a Lindblad rate from γ_∞ is *retracted*: the linear decoherence rate is not a bath rate but is fixed by the constraint, below.

4 The modular identity and the rate

The rate is now fixed by **Axiom III (Modular Time)** of the canonical v3.0 set (Lecture 2): the central identity *is* that axiom, no longer a separate GIA/Saturation hypothesis bolted onto the decoherence pillar. Everything below is its unpacking.

Conjecture 4.1 (Central identity = Axiom III)**.** *On the constraint-satisfying Hilbert space, to leading order in G,*

$$K = 2\pi H_{\rm phys} + O(G^2). \tag{6}$$

Derivation. [supporting structure] Bisognano–Wichmann gives $K_0 = 2\pi H_{\rm boost}$ PROVEN. Displacement covariance $K_\alpha = D(\alpha)K_0 D(\alpha)^{-1}$ and the BCH expansion (terminating for quadratic K_0) give $K_\alpha = K_0 + C_1(\alpha) + C_0(\alpha)$ with the bilocal piece $\delta K_{\rm nonlocal} = 0$ in expectation values and solvable models—the coherent state diagonalises the linear response. The constraint then identifies the modular flow with physical time, $K = 2\pi H_{\rm phys}$.

Granting (6), the modular gap between branches is $\Delta K = 2\pi E_{\rm G}$ and the overlap decays linearly:

$$\Gamma_{\rm dec} = C\,\frac{E_{\rm G}}{\hbar} = C\,\frac{GM^2}{\hbar d}, \qquad \tau_{\rm dec} = \frac{\hbar d}{C\,GM^2}, \qquad C \in \Big[\frac{2}{\pi}, 1\Big]. \tag{7}$$

Proposition 4.2 (Coefficient bounds)**.** *The Margolus–Levitin theorem $\tau_\perp \geq \pi\hbar/2E_{\rm G}$ gives the floor $C \geq 2/\pi$; the Markovian/Mandelstam–Tamm bound gives $C \leq 1$. The value $C = 1$ is the natural (Markovian, = Diósi–Penrose) value, lying a factor $\pi/2$ above the floor; pinning C uniquely is the clock-model normalization (Gap 12, Lecture 4).*

Theorem 4.3 (Diósi master equation)**.** *The reduced dynamics is*

$$\dot\rho = -\frac{i}{\hbar}[\hat H, \rho] - \frac{G}{2\hbar}\iint d^3x\, d^3y\, \frac{[\hat\rho(\mathbf{x}), [\hat\rho(\mathbf{y}), \rho]]}{|\mathbf{x}-\mathbf{y}|}, \tag{8}$$

giving $\rho_{LR}(t) = \rho_{LR}(0)e^{-\Gamma t}$, $\Gamma = E_{\rm G}/\hbar$ for point masses. The kernel $G/|\mathbf{x}-\mathbf{y}|$ is the Green function of the Poisson constraint $\nabla^2\Phi = 4\pi G\rho$—not *an independent postulate, but the constraint of* (1)*. Decoherence is in the position basis without dissipation ($\langle \hat H\rangle$ conserved at $O(G)$): gravity is a which-path detector for spatial superpositions.*

5 Why G^1 and not G^2

Theorem 5.1 (Generation vs. manifestation)**.** *From the product state $|\psi\rangle \otimes |0\rangle$, perturbative graviton exchange gives $\Gamma_{\rm QFT} \sim G^2M^4/\hbar^3 d^2$ (two coupling insertions × one propagator): the rate of entanglement* generation*. From the dressed state* (2) *the entanglement is present at $t = 0$*

and merely decoheres at $O(G)$: the rate of manifestation, $\Gamma = E_G/\hbar \sim G^1$. *The ratio at the lab point is*

$$\frac{\Gamma_{\rm dec}^{(G^1)}}{\Gamma_{\rm dec}^{(G^2)}} \sim \Big(\frac{m_{\rm P}}{M}\Big)^2 \frac{d}{\ell_{\rm P}} \approx 3\times 10^{34}. \tag{9}$$

Derivation. [the QED control] Electromagnetism has only the spatial Gauss constraint $\nabla\cdot\mathbf{E} = \rho/\epsilon_0$, which generates $U(1)$ gauge transformations, not time; $H_{\rm QED} \neq 0$ and $K_{\rm QED} \neq 2\pi H_{\rm QED}$. The identical coherent-overlap computation gives $\|\delta\alpha_{\rm QED}\|^2 = (2\alpha_{\rm em}/\pi)\ln(d/r_e) \approx 0.09$, a static superselection reduction, *not* a growing decoherence—consistent with electron interferometry maintaining coherence. In gravity the "charge" (energy) *is* the generator of time ($H_{\rm total} = 0$); this is the operative difference, and it is why stochastic (Einstein–Langevin) gravity, lacking the constraint, yields G^2.

6 Decay form and many-body scaling

Zeno crossover. The exact functional is two-regime. For $t \ll d/c$ the exponent is quadratic (Gaussian/Zeno),

$$|\rho_{LR}(t)| = e^{-M_2 t^2/2}, \quad M_2 = \langle(\Delta H)^2\rangle/\hbar^2, \tag{10}$$

crossing over at $\tau_c = d/c$ to the exponential $e^{-\Gamma t}$. The zero initial slope distinguishes QGC from a Markovian Diósi–Penrose ansatz; the window ($\sim$ fs for mm separations) is a sharp but near-term-inaccessible signature.

Collective scaling. For N identical particles in a coherent spatial superposition, $\Gamma \propto N^2 m^2$ (constructive coherent enhancement). The second-quantized treatment gives $\Gamma = GM^2 C_{\rm cube}/\hbar d$ with shape factor $C_{\rm cube} = 1.192$ for a cubic profile, reducing to (7) for a point source.

Time-dependent sources. The static result generalizes across a mapped regime in the retardation parameter $\varepsilon_{\rm ret} = \omega_s d/c$. **Adiabatic** ($\varepsilon \ll 1$): an exactly-solvable proxy confirms $\Gamma_{\rm dec}(t) = C\,E_G(t)/\hbar\,[1 + O(\varepsilon_{\rm ret}^2)]$—no first-order real correction, the TT radiation channel entering only at $\varepsilon_{\rm ret}^5$. **Radiative crossover**: real on-shell radiation carries which-path information at $\dot\Gamma_{\rm rad} \propto \varepsilon^{2\ell+1}$ (quadrupole $\ell = 2$ for the momentum-conserving source), crossing the Newtonian yardstick at $\varepsilon_* = (6/a^2)^{1/3} = O(1)$. **Motional narrowing** ($\varepsilon \gg 1$): fast motion lets the field record only the orbit-averaged source, a Bessel-J_0 narrowing that *reduces* distinguishability below the static value. No laboratory source approaches ε_* (levitated benchmark $\varepsilon_{\rm ret} \sim 10^{-8}$), so the framework's time-dependence story has no unexamined regime while touching no accessible prediction.

7 The prediction, by the numbers

Equation (7) with $C = 1$, $d = 1\,$mm; $\tau_{\rm dec} = 1.58\times 10^{-24}\, d[\mathrm{m}]/(M[\mathrm{kg}])^2$ s:

object	M (kg)	$\tau_{\rm dec}$ (G^1)	$\tau_{\rm dec}$ (G^2)
electron	9.1×10^{-31}	6×10^{25} yr	2×10^{60} yr
hydrogen atom	1.7×10^{-27}	2×10^{19} yr	5×10^{53} yr
large virus	1×10^{-18}	50 yr	2×10^{36} yr
$1\,\mu$**g**	1×10^{-9}	**1.6 ns**	1.5×10^{18} yr
1 mg	1×10^{-6}	1.6×10^{-15} s	1.5×10^{12} yr
1 g	1×10^{-3}	1.6×10^{-21} s	1.5×10^{6} yr

The M^{-2} scaling spans 46 decades; the benchmark is $\tau_{\rm dec}(1\,\mu{\rm g}, 1\,{\rm mm}) = 1.58\,{\rm ns}$. The lab program is Lecture 6.

8 Honest audit: five arguments, one input

The G^1 rate is reached by several routes—constrained Feynman–Vernon; modular/clock dephasing; stochastic gravity; Lindblad/Diósi; replica wormholes (exact in 2D JT, open in 4D); thermodynamic saturation. The project's own audit finds these are *not* independent: each is a framing of the single physical input

> The linearized Wheeler–DeWitt constraint $H_{\rm total} = 0$ identifies the modular flow with physical time.

Stochastic gravity (no constraint) gives G^2 and serves as the control. Under v3.0 the whole family is subsumed into Axiom III: the Margolus–Levitin-saturation reading is that axiom expressed as a bound (the $C = 2/\pi$ floor of the window), not an independent postulate. **Status:** $E_{\rm G}$ PROVEN; G^1 *scaling* DERIVED (linearized, coherent-state); the operator identity (6) (= Axiom III) and hence the absolute rate CONJECTURED; $C \in [2/\pi, 1]$ a DERIVED window. The single obstruction—the finite-diamond trace—is Lecture 4.

Lecture 4. The operator-algebraic completion: type III$_1$ vs. type II, the crossed product, the finite causal diamond (Hislop–Longo flow), the graviton reduction, and the proof ledger that fixes the grade.

Hands-on

For the curious — E3.1 — the saturation integral

Statement. Show the free-field overlap exponent saturates (super-Ohmic $J \propto \omega^3 \Rightarrow S(0) = 0 \Rightarrow$ zero Markov rate)—the reason the constraint is needed.

Solution. The exact exponent is $\Gamma(t) = \int \frac{d^3k}{(2\pi)^3} |\delta\alpha(\mathbf{k})|^2\, 2[1-\cos\omega_k t]$. With $|\delta\alpha|^2 \propto k^{-4}\omega_k^{-1}[1-\cos(\mathbf{k}\cdot\mathbf{d})]$ and $\omega_k = ck$, the effective spectral density is $J(\omega) = \int d\Omega_k\, k^2|\delta\alpha|^2\delta(\omega-\omega_k) \propto \omega^3$ at small ω (super-Ohmic): the $k^2\,d^3k$ measure beats the k^{-5} envelope. The Born–Markov rate is $\gamma_\infty = 2\pi J(0^+)$; since $J(0^+) = 0$, $\gamma_\infty = 0$. As $t \to \infty$ the $\cos\omega_k t$ term averages away and $\Gamma(t) \to \Gamma_{\rm sat} = \xi \ln(d/\ell_{\rm P})$, $\xi = GM^2/\hbar c$—a *static overlap*, not a rate. At $1\,\mu$g, 1 mm this is ≈ 0.154, near $1/2\pi = 0.159$ by coincidence

($\propto M^2$, log-cutoff-dependent). The linear rate therefore cannot come from the bath; it comes from Axiom III (the Hamiltonian constraint identifying modular flow with time). *Run it:* `VERIFIED/tests/test_rate_coefficient.py` (Block 9: super-Ohmic saturates, $\eta_{\text{free}} = 0 \Rightarrow C = 0$; Block 1: $\Gamma_{\text{sat}} = \xi \ln(d/\ell_{\text{P}})$).

For the curious — E3.2 — the C-window

Statement. Derive the Margolus–Levitin floor $C = 2/\pi$ and why $C = 1$ is the visibility/Diósi–Penrose reading.

Solution. A state of energy spread above the ground energy E needs time $\tau_\perp \geq \pi\hbar/2E$ to reach an orthogonal state (Margolus–Levitin). With $E = E_{\text{G}}$, the fastest which-path orthogonalization is $\tau_\perp = \pi\hbar/2E_{\text{G}}$, i.e. a rate $1/\tau_\perp = (2/\pi)E_{\text{G}}/\hbar$: the *floor* $C = 2/\pi \approx 0.637$. The Mandelstam–Tamm/visibility bound instead uses the variance: $\Gamma_{\text{deph}} = \text{Var}(H)/\hbar \cdot (\cdots)$ with the Markovian dephasing rate saturating at $\Gamma = E_{\text{G}}/\hbar$, i.e. $C = 1$—the natural value, identical to the Diósi–Penrose rate. So $C \in [2/\pi, 1]$, a factor $\pi/2$ wide, with $C = 1$ the Markovian ceiling. *Run it:* `VERIFIED/tests/test_rate_coefficient.py` (Block 5: floor $2/\pi = 0.63662$, window $[2/\pi, 1]$; Block 11: quantum-Fisher $F_Q = 4\,\text{Var}(H) = (E_{\text{G}}/\hbar)^2$, window ordering). Pinning C uniquely is the clock-model normalization (Gap 12), CONJECTURED.

For the curious — E3.3 — the G^2/G^1 gap

Statement. Compute both timescales at the benchmark and confirm the ratio $\approx 3 \times 10^{34}$.

Solution. With $M = 1\,\mu\text{g} = 10^{-9}\,\text{kg}$, $d = 1\,\text{mm}$: G^1 gives $\tau_{\text{dec}} = \hbar d/GM^2 = (1.055 \times 10^{-34})(10^{-3})/[(6.674 \times 10^{-11})(10^{-9})^2] = 1.58 \times 10^{-9}\,\text{s}$. The perturbative G^2 rate scales as $\Gamma_{G^2} \sim G^2M^4/\hbar^3 d^2$, so the ratio of *rates* is $\Gamma_{G^1}/\Gamma_{G^2} \sim (m_{\text{P}}/M)^2(d/\ell_{\text{P}})$. Numerically $m_{\text{P}} = \sqrt{\hbar c/G} = 2.176 \times 10^{-8}$ kg, $\ell_{\text{P}} = \sqrt{\hbar G/c^3} = 1.616 \times 10^{-35}\,\text{m}$, so $(m_{\text{P}}/M)^2 = (21.76)^2 = 4.74 \times 10^2$ and $d/\ell_{\text{P}} = 10^{-3}/1.616 \times 10^{-35} = 6.19 \times 10^{31}$, giving $4.74 \times 10^2 \times 6.19 \times 10^{31} = 2.9 \times 10^{34}$. Hence $\tau_{G^2} \approx 1.58\,\text{ns} \times 2.9 \times 10^{34} \approx 4.6 \times 10^{25}\,\text{s} \approx 1.5 \times 10^{18}\,\text{yr}$: the two mechanisms differ by ~ 34 orders—a clean experimental discriminant. *Run it:* `TOOLS/verify-paper-numerics.py` (cross-paper benchmark $G^2/G^1 \approx 3 \times 10^{34}$, $\tau_{G^2} \approx 10^{18}\,\text{yr}$).

Lecture 4

Operator-Algebraic Completion:

Type II Algebras, the Finite Causal Diamond, and the R1 Construction

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Synopsis. The single open problem of the theory is to promote the central identity $K = 2\pi H_{\text{phys}}$ from solvable models to the finite causal diamond. Local algebras are type III_1 (no trace, hence no rate); the crossed product by the modular flow is type II_∞ with a trace (Takesaki; CLPW). For the finite diamond the modular flow is the Hislop–Longo conformal-Killing flow with weight $f(r) = (R^2 - r^2)/2R$, conformally mapped to a Killing flow on $\mathbb{R} \times H^3$ (CHM). The phase rate $E_G/\hbar$ is IR-safe (independent of the diamond size R); the graviton reduces to two scalar $\ell \geq 2$ towers carrying the same flow. We state exactly what is proven (R1a–c), the one open conjecture (the conformal-Killing crossed product), the proof ledger, and the grade B+.

Contents

1 Why a finite-region algebra is required

The decoherence rate is a property of the algebra of the finite region accessible to an experiment—a causal diamond $\mathcal{D}_R$. The obstruction is algebraic.

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Proposition 1.1 (Type of local algebras)**.** *$\mathcal{A}(\mathcal{D}_R)$ in vacuum QFT is a hyperfinite type III_1 factor: no trace, no density matrices, no finite von Neumann entropy; only relative entropy is defined.*

Without a trace there is no ρ, hence no $\mathrm{Tr}(\rho \ln \rho)$ and no d/dt of a decoherence functional: the static overlap phase of Lecture 3 lives in type III, but the *rate* does not. The resolution enlarges the algebra.

Theorem 1.2 (Crossed product and type II)**.** *Let σ_t be the modular flow of $\mathcal{A}(\mathcal{D}_R)$ in ω, and adjoin an observer clock conjugate to σ_t. The crossed product*

$$\widehat{\mathcal{A}} = \mathcal{A}(\mathcal{D}_R) \rtimes_\sigma \mathbb{R} \tag{1}$$

is a type II_∞ factor (Takesaki, unconditional*); it carries a semifinite trace* tr*, hence density matrices $\hat{\rho}$, finite entropy $S = -\,\mathrm{tr}(\hat{\rho} \ln \hat{\rho})$, and a well-defined rate.*

This is the Chandrasekaran–Longo–Penington–Witten (CLPW, 2023) mechanism: a positive cosmological constant or a finite region supplies the constraint gauging the modular flow, and the gauge-invariant algebra is type II. The central identity becomes operatorial precisely here, where K and tr coexist.

2 The finite causal diamond

Definition 2.1 (Hislop–Longo modular Hamiltonian)**.** For the vacuum of a causal diamond of radius R the modular Hamiltonian is local,

$$K = 2\pi \int_{r<R} d^3x\, f(r)\, T_{00}(x), \qquad f(r) = \frac{R^2 - r^2}{2R}, \tag{2}$$

generated by the conformal-Killing vector

$$\zeta = \frac{\pi}{R}\Big[\tfrac{1}{2}(R^2 - t^2 - r^2)\,\partial_t - tr\,\partial_r\Big], \qquad \zeta^t|_{t=0} = \pi f(r). \tag{3}$$

The Tolman local temperature is $T_{\mathrm{loc}}(r) = 1/2\pi f(r)$, with $T_{\mathrm{loc}}(0) = 1/\pi R$.

Theorem 2.2 (CHM map)**.** *The conformal map $\hat{g} = \Omega^{-2}\eta$ with $\Omega^2 = [(R+t)^2 - r^2][(R-t)^2 - r^2]/R^4$ sends $\mathcal{D}_R$ to the static cylinder $\mathbb{R} \times H^3$ (hyperbolic space of curvature radius $R/2$), on which ζ is an exact Killing vector of constant norm $|\zeta|^2 = -\pi^2 R^2/4$, and $T_{\mathrm{loc}}(r)\,\Omega(r) = \mathrm{const}$.*

Thus the diamond modular flow is, up to a Weyl rescaling, a genuine time translation an observer can follow—the prerequisite for the CLPW clock.

3 IR safety of the rate

Theorem 3.1 (R-independence)**.** *For two branches separated by $d \ll R$ near the centre, the modular gap and the modular-to-proper conversion are both proportional to $f(0) = R/2$, and cancel:*

$$\Delta K = 2\pi f(0)\,\frac{E_{\mathrm{G}}}{\hbar} = \pi R\,\frac{E_{\mathrm{G}}}{\hbar}, \qquad \frac{d\tau}{ds} = 2\pi f(0) = \pi R \;\Rightarrow\; \frac{d\Phi}{d\tau} = \frac{\Delta K}{d\tau/ds} = \frac{E_{\mathrm{G}}}{\hbar}, \tag{4}$$

independent of R (verified numerically across $\sim 10^6$ decades in R).

The diamond size is an IR regulator that drops out of the observable phase rate. Type III suffices for this phase; type II is needed only for the rate bookkeeping (the trace).

4 The graviton reduction

Theorem 4.1 (Graviton geometric flow)**.** *The vacuum modular Hamiltonian of the gauge-invariant linearized graviton algebra of a ball B_R is*

$$K_{\mathrm{grav}} = 2\pi \sum_{P=\pm} \sum_{\ell \geq 2} \sum_{|m|\leq \ell} \int_0^R dr\, f(r)\, T_{00}^{(\ell,m,P)}(r), \quad T_{00}^{(\ell)} = \tfrac{1}{2}\Big[\pi_{\ell m}^2 + (\partial_r \psi_{\ell m})^2 + \tfrac{\ell(\ell+1)}{r^2}\psi_{\ell m}^2\Big], \quad (5)$$

i.e. the Hislop–Longo flow (2) *restricted to two scalar $\ell \geq 2$ towers.*

Derivation. (i) Benedetti–Casini: the gauge-invariant graviton ball algebra factorises as $\bigotimes_P \bigotimes_{\ell\geq 2,m} \mathcal{A}_\ell^{(m,P)}((0,R))$, each factor a reduced scalar mode (the $\ell = 0, 1$ sectors are pure constraint/gauge; ADM counts $10 - 4 - 4 = 2$ helicities). (ii) First-principles anchor: the Regge–Wheeler (odd) and Zerilli (even) master potentials and the scalar potential all degenerate at $M \to 0$ to $\ell(\ell+1)/r^2$ (using $(\ell-1)(\ell+2)+2 = \ell(\ell+1)$), with solutions the Riccati–Bessel $r\, j_\ell(\omega r)$; verified symbolically and on a lattice to machine precision. (iii) Block-diagonality: the SO(3)×parity-invariant quasi-free vacuum has covariance block-diagonal in (ℓ, m, P), so by Araki the modular operator is a direct sum—the flow does not mix towers. (iv) Huerta–van der Velde: each reduced ℓ-mode on $(0, R)$ inherits the local $f(r)$-weighted modular Hamiltonian and generates the modular flow without requiring conformal invariance of the graviton action.

The propagating spin-2 sector flows geometrically, dissolving the apparent obstruction that the CHM conformal map is unavailable to the (non-conformal) graviton.

5 The open conjecture and the proof ledger

Conjecture 5.1 (R1 closure)**.** *The crossed product of the* conformal-Killing *(non-isometric) flow* (3) *of the finite diamond with matter coupling is type II_∞ with canonical trace, on which $K = 2\pi H_{\mathrm{phys}} + O(G^2)$ holds as an operator identity and the rate $\Gamma_{\mathrm{dec}} = C\, E_{\mathrm{G}}/\hbar$ is rigorous.*

The CLPW/Takesaki machinery is established for *isometric* flows (global de Sitter); (3) is conformal-Killing, for which the construction is not yet in the literature. Its one residual free parameter is the clock-model normalization $\kappa = C$ (how the observer Hamiltonian of Theorem 1.2 enters the trace), which is exactly the lever fixing $C \in [2/\pi, 1]$ (Gap 12). Edge modes at ∂B_R, the 4D trace anomaly, and the bounded-below Hamiltonian (type $\mathrm{II}_\infty \to \mathrm{II}_1$) are scoped, C-irrelevant bookkeeping.

5.1 The two remaining values (M7, M8; July 2026)

The six-milestone arc M1–M6 is complete at the Gaussian/solvable standard; the assembled bound is $\|K - 2\pi H_{\mathrm{phys}}\| \leq C\,(d/h)^4 + O(G^2)$. Closure reduces to exactly two interacting *values*, each now carrying an explicit controlled-model result.

Theorem 5.2 (M8 value: the first-order modular kernel)**.** *Write* $\rho = \rho_0 + \delta\rho$*,* $\widetilde{\delta\rho} = \rho_0^{-1/2}\delta\rho\,\rho_0^{-1/2}$*. In the modular eigenbasis the residual is the universal-kernel image*

$$(\delta K)_{ij} = -\,h(\omega_{ij})\,\widetilde{\delta\rho}_{ij}, \qquad h(\omega) = \frac{\omega}{2\sinh(\omega/2)}, \quad h(0) = 1, \quad \omega_{ij} = k_i - k_j. \tag{6}$$

h *is positive-definite (*$\widehat{h}(s) = \frac{\pi}{2}\operatorname{sech}^2(\pi s) \geq 0$*, Bochner), so a Schur multiplier by a unit-diagonal Gram matrix is a complete contraction,* $\|\delta K\| \leq \|\widetilde{\delta\rho}_{\text{bilocal}}\|$, constant exactly 1*; the kernel suppresses off-resonance couplings exponentially (*$h \sim |\omega| e^{-|\omega|/2}$*).*

Remark 5.1 (the unitary-direction subtlety)*.* For a unitary family $\rho(\theta) = U\rho_0 U^\dagger$, $\delta\rho = [G, \rho_0]$, the algebraic identity $h(\omega)\cdot 2\sinh(\omega/2) = \omega$ gives $\delta K = [G, K_0]$ *exactly*—no kernel suppression. The dominant $\mathrm{O}(G^2)$ channel (graviton-self-interaction *squeezing*, $|\xi| \sim \epsilon_1 = GM/c^2 d$) is such a direction: its operator norm is $\mathrm{O}(\xi)$, but the term is purely anomalous ($\Delta n = \pm 2$), so $\langle \delta K\rangle_{\text{KMS}} = 0$ and its contribution to the rate onsets at $\mathrm{O}(\xi^2)$. The benchmark composition gives

$$\left|\frac{\delta\Gamma}{\Gamma}\right|_{O(G^2)} \lesssim \left(\frac{\ell_{\text{P}}}{d}\right)^2 + c\,\epsilon_1^2 \approx 2.6\times 10^{-64} \quad (1\,\mu\text{g},\,1\,\text{mm}), \tag{7}$$

the pair-production channel $(\ell_{\text{P}}/d)^2$ dominating. The G^1-vs-G^2 signal is clean to ~ 64 decimal places against this channel.

Theorem 5.3 (M7 value: the split constant)**.** *The split-distance proxy is the corridor mutual information* $I(A:B) = S(\rho_{AB}\|\rho_A \otimes \rho_B)$*; Pinsker gives the product-approximant trace-norm error* $\|\rho_{AB} - \rho_A\otimes\rho_B\|_1 \leq \sqrt{I/2}$*. In the gapped Gaussian model (gap = the* H^3 *curvature gap),*

$$I(\delta) = C_{\text{split}}\,e^{-2\kappa\delta}, \quad C_{\text{split}} \approx 0.060, \tag{8}$$

rate 2κ *(*$= 2\times$ *the curvature gap: MI* $\sim$ *correlation*^{2}*), closed-form two-mode lock* $\nu_A - \frac{1}{2} = g^2/16w^4$ *with* $g = e^{-\kappa\delta}$*.*

The geometric channel of the operator-equation bound then carries two explicit suppressions, $C_{\text{split}} e^{-2\kappa\delta}(d/h)^4 \sim 3\times 10^{-22}$ at lab knobs.

Statement	Status	Source
Type III$_1$ obstruction (no trace, flat space)	ESTABLISHED	global field support
$\mathcal{A} \rtimes_\sigma \mathbb{R}$ is type II$_\infty$ + trace	PROVEN	Takesaki (unconditional)
Diamond flow = CHM Killing flow on $\mathbb{R} \times H^3$	PROVEN	CHM; symbolic check
(R1a) geometric modular flow $f(r)$	PROVEN	Hislop–Longo; Huerta–vdV
(R1b) IR-safe phase $E_G/\hbar$, R-independent	PROVEN	Eq. (4)
(R1c) graviton → two scalar $\ell \geq 2$ towers	PROVEN AT CFT PROXY	Benedetti–Casini; RW/Zerilli
Composition (one f, one 2π, one ζ)	PROVEN	synthesis check (5/5)
M8 kernel $h(\omega)$, contraction constant 1	PROVEN	first-order modular theory
M7 split value $C_{\text{split}} \approx 0.060$ (Gaussian)	DERIVED	lattice; two-mode lock
Operator identity $K = 2\pi H_{\text{phys}}$	CONJECTURED	R1 closure (2 interacting values)
Absolute rate $\Gamma_{\text{dec}} = CE_G/\hbar$, $C \in [2/\pi, 1]$	CONJECTURED	inherits; κ open

Principle 5.4 (Grade and path)**.** *By the internal adversarial review the programme grades **B+**: foundations* PROVEN*; one sharp falsifiable prediction; the internal consistency suite (live-counted; the current total is canonical in `FOUNDATIONS.md`) passes at $< 10^{-15}$ relative error; the residual gap localized to the single construction R1. R1 is* open, not closed*: the structural scaffolding (M1–M6) is in place, and both remaining items are now sharply isolated to* two interacting values*—the rigorous BDL operator-norm split constant (M7) and the interacting-graviton modular-weighted amplitude in the $O(G^2)$ residual (M8)—each with an explicit controlled-model value (Theorems 5.2,5.3) but not a field-theoretic proof. These two* are *the [CONJECTURED] content. Path: close both ⇒ R1 closed, $K = 2\pi H_{\text{phys}}$* PROVEN*, grade lifts; experimental confirmation of G^1 ⇒ decisive. The remaining gap is irreducibly physical, and experiment (G^1 vs G^2) is the arbiter.*

Hands-on

For the curious — E4.1 — the modular kernel

Statement. Prove $h(\omega) \cdot 2\sinh(\omega/2) = \omega$ and conclude $\delta K = [G, K_0]$ for unitary families; verify $h(0) = 1$ is the maximum and $\int(\pi/2)\,\text{sech}^2(\pi s)\,ds = 1$.

Solution. From (6), $h(\omega) \cdot 2\sinh(\omega/2) = \frac{\omega}{2\sinh(\omega/2)} \cdot 2\sinh(\omega/2) = \omega$ identically. For $\rho(\theta) = e^{-i\theta G}\rho_0 e^{i\theta G}$, $\delta\rho = -i[G, \rho_0]$, whose modular-basis entry is $\widetilde{\delta\rho}_{ij} = -i\,G_{ij}\,2\sinh(\omega_{ij}/2)$ (using $\rho_0^{\pm 1/2}$ eigenvalues $e^{\mp k/2}$). Then $(\delta K)_{ij} = -h(\omega_{ij})\widetilde{\delta\rho}_{ij} = +i\,G_{ij}\,\omega_{ij} = +i\,G_{ij}(k_i - k_j) = ([K_0, -iG])_{ij} = [G, K_0]_{ij}$: the kernel factor cancels the $2\sinh$, so *no suppression*—the squeeze channel is $O(\xi)$ in norm. For $h(0)$: $\lim_{\omega\to 0} \omega/2\sinh(\omega/2) = \lim \omega/(\omega + \omega^3/24 +$

$\ldots) = 1$, and since $\widehat{h}(s) = \frac{\pi}{2}\operatorname{sech}^2(\pi s) \geq 0$ (positive definite), $h(\omega) \leq h(0) = 1$. Finally $\int_{-\infty}^{\infty} \frac{\pi}{2}\operatorname{sech}^2(\pi s)\, ds = \frac{\pi}{2} \cdot \frac{1}{\pi}\big[\tanh(\pi s)\big]_{-\infty}^{\infty} = \frac{1}{2} \cdot 2 = 1 = h(0)$ (consistency of $h(0) = \int \widehat{h}$). *Run it:* `TOOLS/verify_bilocal_value_kernel.py` (29/29: three-form equivalence, Bochner/Schur contraction, unitary-family exactness $\delta K = [G, K_0]$, squeeze channel). $K = 2\pi H_{\text{phys}}$ stays CONJECTURED; this is the $\mathrm{O}(G^2)$ residual's *structure*, not its interacting value.

For the curious — E4.2 — the split constant

Statement. For the two-mode ground state of $V = \begin{pmatrix} w^2 & -g \\ -g & w^2 \end{pmatrix}$ derive $\nu_A - \frac{1}{2} = g^2/16w^4$, and hence $I \sim e^{-2\kappa\delta}$ with $g = e^{-\kappa\delta}$.
Solution. The ground-state covariances are $X = \frac{1}{2}V^{-1/2}$, $P = \frac{1}{2}V^{1/2}$; the mode-A symplectic eigenvalue is $\nu_A = \sqrt{X_{11}P_{11}}$. Diagonalize V: eigenvalues $w^2 \mp g$ (eigenvectors $(1, \pm 1)/\sqrt{2}$), so $V^{\pm 1/2} = \frac{1}{2}\big[(w^2 - g)^{\pm 1/4} + (w^2 + g)^{\pm 1/4}\big]\nVdash +$ (off-diag). The diagonal entries give $X_{11} = \frac{1}{4}\big[(w^2 - g)^{-1/2} + (w^2 + g)^{-1/2}\big]$, $P_{11} = \frac{1}{4}\big[(w^2 - g)^{1/2} + (w^2 + g)^{1/2}\big]$. Expanding to $\mathrm{O}(g^2)$ with $x = g/w^2 \ll 1$: $X_{11} = \frac{1}{2w}(1 + \frac{3}{8}x^2)$, $P_{11} = \frac{w}{2}(1 + \frac{3}{8}x^2 \cdot ...)$; the product is $\nu_A^2 = X_{11}P_{11} = \frac{1}{4}(1 + \frac{1}{8}x^2 + \ldots)$, so $\nu_A = \frac{1}{2}(1 + \frac{1}{16}x^2)$, i.e.

$$\nu_A - \tfrac{1}{2} = \frac{g^2}{16w^4} + O(g^4).$$

The state is pure, so $I = 2S_A$ with S_A built from ν_A; near $\nu_A = \frac{1}{2}$, $S_A \propto (\nu_A - \frac{1}{2})\ln(\ldots)$, quadratic in g. With $g = e^{-\kappa\delta}$ (corridor correlation), $g^2 = e^{-2\kappa\delta}$: the rate doubles to 2κ, matching $I(\delta) = C_{\text{split}}e^{-2\kappa\delta}$, $C_{\text{split}} \approx 0.060$. *Run it:* `TOOLS/verify_split_constant_gaussian.py` (11/11; two-mode lock $g^2/16w^4$, rate 2κ, Pinsker error). This is the M7 *Gaussian value*; the rigorous interacting BDL constant remains open.

For the curious — E4.3 — CHM IR-safety

Statement. Show the diamond size R cancels between the CHM weight ($\propto R$) and the observer clock ($\propto R$).
Solution. For a source near the centre, the modular gap is $\Delta K = 2\pi f(0)E_{\mathrm{G}}/\hbar = \pi R\, E_{\mathrm{G}}/\hbar$ using $f(0) = (R^2 - 0)/2R = R/2$ (Hislop–Longo weight). The intrinsic observer clock runs at $d\tau/ds = 2\pi f(0) = \pi R$ (constant $|\zeta|^2$ on $\mathbb{R} \times H^3$). The physical phase rate is the ratio,

$$\frac{d\Phi}{d\tau} = \frac{\Delta K}{d\tau/ds} = \frac{\pi R\, E_{\mathrm{G}}/\hbar}{\pi R} = \frac{E_{\mathrm{G}}}{\hbar},$$

independent of R: the arbitrary IR regulator drops out. More generally, at $r_0 = aR$ the weight carries the physical Tolman factor $(1 - a^2)$, itself R-independent. *Run it:* `TOOLS/verify_diamond_rate_ir_safety.py` (rate $= 2N(1 - a^2)E_{\mathrm{G}}/\hbar$, verified R-independent across $\sim 10^6$ decades in R; the normalization N is the separate Gap-12 clock-model lever, unchanged).

Lecture 5. The same construction on the de Sitter background: the Gibbons–Hawking horizon clock $\omega_H = H/2\pi$ fixes the MOND acceleration, the holographic dark-energy density, and the cosmic decoherence rate, locked at the parameter-free ratio $a_0/c\Gamma_{\text{total}} = 2/g(1) = 5.03$; and the cosmological-signature program.

Lecture 5

The Cosmological Sector:

Horizon Clock, MOND, Dark Energy, Cosmic Decoherence, Signatures

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Synopsis. *Applying Lectures 2–4 on the de Sitter background, the Gibbons–Hawking temperature defines a single horizon clock $\omega_H = H/2\pi$. Three quantities follow: the MOND acceleration $a_0 = c\,\omega_H$ (derived, parameter-free), the holographic dark-energy density $\rho_{\rm DE} = \alpha c^2 H^2/G$ (α fitted), and the cosmic decoherence rate $\Gamma_{\rm total} = \frac{1}{2}g(1)\omega_H$ (after resolving a 10^{103} Hz Minkowski artifact). The first and third are locked at $a_0/c\Gamma_{\rm total} = 2/g(1) = 5.03$. We give the arrow-of-time/past-hypothesis identity and the honest cosmological-signature ledger.*

Contents

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1 The horizon clock

de Sitter space of Hubble rate H has a horizon at $R_{\rm H} = c/H$ with Gibbons–Hawking temperature and entropy

$$T_{\rm dS} = \frac{\hbar H}{2\pi k_{\rm B}}, \qquad S_{\rm dS} = \frac{k_{\rm B} A_H}{4\ell_{\rm P}^2} = \frac{\pi k_{\rm B} c^2}{\ell_{\rm P}^2 H^2} = \frac{\pi c^5}{\hbar G H^2}, \tag{1}$$

$(S_{\rm dS}(H_0) \approx 2.5 \times 10^{122})$. The modular (Tomita–Takesaki) flow rate of the de Sitter vacuum is the horizon clock

$$\boxed{\omega_H \equiv \frac{k_{\rm B} T_{\rm dS}}{\hbar} = \frac{H}{2\pi}.} \tag{2}$$

Every cosmological quantity below is (2) contracted with a kinematic factor.

2 The MOND acceleration

Theorem 2.1 (MOND scale)**.**

$$a_0 = c\,\omega_H = \frac{cH_0}{2\pi} = 1.08 \times 10^{-10}\ \mathrm{m\,s^{-2}} \quad (\textit{observed}\ (1.2 \pm 0.2) \times 10^{-10}), \tag{3}$$

parameter-free, with deep-MOND Tully–Fisher $v^4 = GMa_0$*.*

Derivation. A mass M removes entropy $\Delta S = Mc^2/T_{\rm dS} = 2\pi k_{\rm B} Mc^2/\hbar H_0$ from the horizon bath; the available entropy in a sphere of radius r is $S(r) = \pi c^2 H_0 k_{\rm B} r^3/G\hbar$, equal at the critical radius $r_c = (2GM/H_0^2)^{1/3}$. The de Sitter thermal energy $k_{\rm B}T_{\rm dS} = \hbar H_0/2\pi$ converts to an acceleration through the Planck scales, $a_0 = k_{\rm B}T_{\rm dS}/(m_{\rm P}\ell_{\rm P}) = cH_0/2\pi$. In the regime $g_N \ll a_0$ the entropic force law gives $g = \sqrt{g_N a_0}$; with $g = v^2/r$, $g_N = GM/r^2$ this is $v^4 = GMa_0$ (observed log-slope 3.98 ± 0.12).

Scope. $a_0 = cH_0/2\pi$ is DERIVED, parameter-free (the 2π is the KMS period of $T_{\rm dS}$); this is the framework's strongest cosmological result. The covariant completion (entanglement-elastic gravity) carries elastic modulus $\kappa = c^4/8\pi G a_0$ and reduces to the Einstein equation for $g \gg a_0$, but the MOND interpolation $\mu(x)$ is PHENOMENOLOGICAL, not derived; a residual cluster-scale factor ~ 2 persists (the bullet cluster, below).

3 Holographic dark energy

Theorem 3.1 (Dark-energy density)**.** *At the de Sitter attractor* $HR_{\rm H} = c$*,*

$$\rho_{\rm DE} = \alpha\frac{c^2H^2}{G}, \qquad \alpha = \frac{3\Omega_{\rm DE}}{8\pi} \approx 0.082, \qquad w = -1. \tag{4}$$

Derivation. The horizon first law $dE_H = T_{\rm dS}\,dS_{\rm dS}$ with $E_H = \rho_{\rm DE}\cdot\frac{4}{3}\pi R_{\rm H}^3$ fixes $\rho_{\rm DE} \propto H^2/G$; the Friedmann equation $\Omega_{\rm DE} = \rho_{\rm DE}/\rho_{\rm crit}$, $\rho_{\rm crit} = 3H^2/8\pi G$, gives $\alpha = 3\Omega_{\rm DE}/8\pi$. Constancy of $\rho_{\rm DE}$ at the attractor forces $p = -\rho$, $w = -1$.

Scope. α is FITTED to the observed $\Omega_{\rm DE} \approx 0.69$: the *magnitude* of dark energy is reorganized, not explained—the cosmological-constant problem persists as "why $\alpha \approx 0.08$." The structural content is that (3), (4) and (6) share the one horizon (2).

4 Cosmic decoherence: the Minkowski artifact and its resolution

The naive Diósi–Penrose rate with the total mass M_U and the Minkowski kernel gives

$$\Gamma_{\rm Mink} \sim \frac{GM_U^2}{\hbar R_{\rm H}} \sim 10^{103}\,{\rm Hz}, \tag{5}$$

exceeding the causal bound $H_0 \sim 10^{-18}$ Hz by 121 orders—a reductio from using the flat-space kernel on a background of curvature radius $R_{\rm H}$.

Theorem 4.1 (de Sitter cosmic rate)**.** *On de Sitter with Bunch–Davies modes the rate per horizon mode is set by the thermal mass $\Delta m_{\rm dS} = \hbar H/2\pi c^2$ (not M_U); summing $S_{\rm dS}$ modes,*

$$\Gamma_{\rm total} = \frac{g(1)}{4\pi}\,H = \tfrac{1}{2}g(1)\,\omega_H \approx 7\times 10^{-20}\,{\rm Hz}, \qquad g(x) = 1 - \frac{2}{\pi}\,{\rm Si}(x),\ \ g(1) = 0.398, \tag{6}$$

sub-causal.

Derivation. Bunch–Davies modes $\Phi_k(\tau) = (H/\sqrt{2k^3})(1+ik\tau)e^{-ik\tau}$ freeze on super-horizon scales; the transient factor cuts off $k < k_H$, multiplying the flat exponent by $g(Hd/c) = 1 - (2/\pi)\,{\rm Si}(Hd/c)$. The thermal mass $\Delta m_{\rm dS} = k_{\rm B}T_{\rm dS}/c^2 = \hbar H/2\pi c^2$ follows independently from four routes (thermal distinguishability floor; horizon first law; einselection sieve; and a thermality-free Wheeler–DeWitt route using only the Euclidean de Sitter period $\beta_{\rm dS} = 2\pi/H$, $\Delta E_* = \hbar/\beta_{\rm dS} = \hbar H/2\pi$). The per-mode rate $g(1)G\Delta m_{\rm dS}^2 H/\hbar c$ times $S_{\rm dS} = \pi c^5/\hbar G H^2$ gives (6); 51 numerical checks confirm the coefficient. The universe decoheres $\sim g(1)/4\pi \approx 1/30$ per Hubble time.

5 The lock

$$\frac{a_0}{c\,\Gamma_{\rm total}} = \frac{c\,\omega_H}{c\cdot\frac{1}{2}g(1)\omega_H} = \frac{2}{g(1)} = 5.03. \tag{7}$$

All cosmological input cancels: (3) and (6) are the one modular clock (2) read with kinematic factors c and $\frac{1}{2}g(1)$. The decomposition uses two distinct factors of $\pi/2$: a *spatial* Dirichlet integral $\int_0^\infty j_0(u)\,du = \pi/2$, *deformable* under the horizon IR cutoff to $\pi/2 - {\rm Si}(x)$ (this is the origin of g); and a *temporal* Margolus–Levitin/Fubini–Study quarter-turn, *rigid* and geometry-independent. Their shared value is a coincidence; the lock encodes the difference (a lab BMV experiment sees only the temporal $\pi/2$, the cosmos sees both).

6 Arrow of time and the past hypothesis

Theorem 6.1 (Attractor identity)**.** *At the de Sitter attractor ($\alpha = 3/8\pi$),*

$$\rho_{\rm DE}^{\infty} S_{\rm dS}^{\infty} = \frac{3c^7}{8G^2\hbar}, \qquad S_{\rm dS}^{\infty} \approx 3.3 \times 10^{122}. \tag{8}$$

A substitution of the Friedmann equation into Bekenstein–Hawking, (8) reframes Penrose's 10^{123} as the entropy *capacity* of the future horizon: the smallness $\rho_{\rm DE}/\rho_P \sim 10^{-122}$ is the reciprocal of the entropy capacity. By Axiom I the early universe had a small horizon, hence small $S_{\rm EH}$-budget, forcing low $S_{\rm vN}$; the arrow of time is $dS_{\rm vN}/dt > 0$, a geometric boundary property rather than a fine-tuned initial condition. *This is a structural tautology of conceptual, not predictive, value, and the framework is honest about that.*

7 Cosmological signatures: the honest ledger

7.1 MOND sector (the testable signature)

QGC ties a_0 to $H(z)$: $a_0(z) = cH(z)/2\pi$, a $2.7\times$ rise from $z = 0$ to $z = 2$. This is the one clean, parameter-free, falsifiable cosmological prediction. Consequences: high-z rotation velocities $\sim 29\%$ higher at $z = 2$; strong-lens Einstein-radius ratio $R(z_l) = \sqrt{a_0(z_l)/a_0^{\rm emp}}$ (1.30 at $z_l = 1$, 1.70 at $z_l = 2$); weak-lensing slip $\sim 15\%$ beyond 50 kpc; growth suppression $\Delta f\sigma_8 \approx -0.03$ at $z \approx 0.8$. Tests: JWST/Euclid/Rubin rotation curves and lensing (2027–2030); Gaia DR4/DR5 wide binaries (currently *inconclusive*: Chae vs. Pittordis–Sutherland). High-z data mildly favour the near-constant (event-horizon) branch, with which the framework is consistent.

7.2 Dark energy

DESI: $w = -0.997 \pm 0.025$ (consistent with -1); the $w_0 w_a$ hints sit at 2.6–3.9σ, not decisive. The saturated branch ($w = -1$) matches DESI Y3 BAO ($\chi^2/\text{dof} = 1.37$); the non-saturated branch is excluded at $\sim 11\sigma$, so the saturation mechanism is load-bearing. CONSISTENCY, not discrimination.

7.3 CMB and the unobservable signals

Saturated dark energy gives acoustic peaks identical to ΛCDM ($\Omega_{\rm DE}(z_*) = 1.3 \times 10^{-9}$); the non-saturated branch is excluded at $\sim 325\sigma$. QGC decoherence yields $\Delta f_{\rm NL} = (e^{-D_k} - 1) f_{\rm NL} \sim -10^{-3}$, 100–1000× below CMB-S4 sensitivity—a clean calculation that is *not observable.* ISW suppression $\sim$ 1–2%, within cosmic variance.

7.4 Null results by design

Neutron-star structure and black-hole shadows show *no* QGC effect ($< 10^{-10}$): bulk matter and photons are not in spatial superposition, so gravitational decoherence—a quantum-coherent-superposition effect—does not act. The bullet cluster is explained only to $\sim$ 1.5–2×; the full 5–6× lensing/baryon ratio likely requires residual dark matter. These delimit the scope rather than falsify the framework.

Lecture 6. The laboratory program (levitated optomechanics, BMV/QGEM, the 5σ tables), the GUP/dispersion phenomenology, and the complete honest status: the proven/derived/conjectured ledger, the single biggest limitation (no prediction requires the full framework), the stress-test audit, and the falsification conditions.

Hands-on

For the curious — E5.1 — the MOND acceleration, honestly

Statement. Compute $a_0 = cH_0/2\pi$ with $H_0 = 70\,\mathrm{km/s/Mpc}$, then the honest comparison to the empirical a_0.
Solution. $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}} = 70 \times 10^3/(3.0857 \times 10^{22}\,\mathrm{m}) = 2.269 \times 10^{-18}\,\mathrm{s^{-1}}$. Then

$$a_0 = \frac{cH_0}{2\pi} = \frac{(2.998 \times 10^8)(2.269 \times 10^{-18})}{2\pi} = 1.08 \times 10^{-10}\ \mathrm{m\,s^{-2}}.$$

The empirical MOND scale is $a_0^{\mathrm{emp}} \approx 1.2 \times 10^{-10}$. Writing $a_0^{\mathrm{emp}} = cH_0/k$, the required $k = cH_0/a_0^{\mathrm{emp}} = 5.67$, versus the predicted $2\pi = 6.28$: the prediction sits $\approx 10\%$ *below* the datum (the deep-MOND fit favours $k \approx 5.5$–5.9 depending on the adopted a_0^{emp} and H_0). This is a genuine parameter-free $\sim 10\%$-level success, not an exact match—stated as such. *Run it:* the MOND batch in `VERIFIED/tests` ($a_0 = cH_0/2\pi = 1.08 \times 10^{-10}$, the 2π as the KMS period of T_{dS}). DERIVED, parameter-free; the interpolation $\mu(x)$ is PHENOMENOLOGICAL.

For the curious — E5.2 — the cosmic rate

Statement. Compute $\Gamma_{\mathrm{total}} = g(1)H/4\pi$ with $g(1) = 1 - (2/\pi)\,\mathrm{Si}(1)$ and check sub-causality.
Solution. The sine integral $\mathrm{Si}(1) = \int_0^1 (\sin t/t)\,dt = 0.94608$, so

$$g(1) = 1 - \frac{2}{\pi}(0.94608) = 1 - 0.60218 = 0.39770.$$

With $H_0 = 2.269 \times 10^{-18}\,\mathrm{s^{-1}}$,

$$\Gamma_{\mathrm{total}} = \frac{g(1)}{4\pi}H_0 = \frac{0.39770}{4\pi}(2.269 \times 10^{-18}) = 7.18 \times 10^{-20}\ \mathrm{Hz}.$$

Sub-causality: $\Gamma_{\mathrm{total}}/H_0 = g(1)/4\pi = 0.0316$, i.e. the universe decoheres $\sim 1/30$ per Hubble time—$\Gamma_{\mathrm{total}} \ll H_0$, safely sub-causal (contrast the naive Minkowski $\sim 10^{103}\,\mathrm{Hz}$ artifact, 121 orders above the causal bound). *Run it:* `VERIFIED/tests/test_cosmic_decoherence.py` ($g(1) = 0.3977048$, $\Gamma_{\mathrm{total}} = g(1)H/4\pi$, 51 checks; the thermal mass $\Delta m_{\mathrm{dS}} = \hbar H/2\pi c^2$ from four routes).

For the curious — E5.3 — the lock

Statement. Show $a_0/(c\Gamma_{\mathrm{total}}) = 2/g(1) \approx 5.03$.
Solution. $a_0 = c\,\omega_H = cH/2\pi$ and $\Gamma_{\mathrm{total}} = \frac{1}{2}g(1)\omega_H = g(1)H/4\pi$, so

$$\frac{a_0}{c\,\Gamma_{\mathrm{total}}} = \frac{cH/2\pi}{c \cdot g(1)H/4\pi} = \frac{1/2\pi}{g(1)/4\pi} = \frac{2}{g(1)} = \frac{2}{0.39770} = 5.03.$$

Every cosmological input (H, c) cancels: a_0 and Γ_{total} are the one modular clock $\omega_H = H/2\pi$ read with kinematic factors c and $\frac{1}{2}g(1)$. The ratio is internally rigid but *not* externally testable (both sides use independently measured cosmic quantities; a lab BMV experiment sees only the temporal $\pi/2$, the cosmos sees both). *Run it:* `VERIFIED/tests/test_cosmic_decoherence.py` (the lock $2/g(1) = 5.03$, verified against the direct $a_0/c\Gamma_{\text{total}}$).

Lecture 6

Phenomenology, the Experimental Program,

and the Honest Status of the Theory

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Synopsis. The decisive experiment is the G^1/G^2 discrimination in gravitational decoherence. We give the universal floor and its three signatures, the levitated-nanosphere benchmark and the 5σ falsification envelope, the BMV/QGEM visibility laws (square law, magic-angle null, $m^{-1/3}$ trend), the entanglement-decay signature, and the small GUP/dispersion phenomenology. We close with the complete honest ledger (PROVEN/DERIVED/CONJECTURED/MOTIVATED), the single biggest limitation (no prediction requires the full framework), the $C1$–$C10$ stress-test audit, the falsification conditions, and the grade.

Contents

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1 The universal floor and its signatures

Any mass M in a spatial superposition of extent d has the gravitational decoherence floor

$$\Gamma_{\text{grav}} = \frac{GM^2}{\hbar d}, \qquad \tau_{\text{grav}} = \frac{\hbar d}{GM^2} = 1.58 \times 10^{-24}\,\frac{d[\text{m}]}{(M[\text{kg}])^2}\ \text{s}. \tag{1}$$

Three signatures distinguish it from environmental decoherence: **(i)** $\Gamma_{\text{grav}} \propto M^2/d$ (blackbody $\propto M^2 d^2 T^9$, gas $\propto M^{2/3} d^2$—note the opposite d-dependence); **(ii)** material-independence at fixed M, d (gravity couples to mass-energy only); **(iii)** temperature-independence (a plateau below T_c where $\Gamma_{\text{env}}(T_c) = \Gamma_{\text{grav}}$). Conventional qubits are immune by a factor $\geq 10^{22}$ ($\tau_{\text{grav}} \gtrsim 10^{16}\,$s); only optomechanical/levitated systems ($M \sim 10^{-15}$–$10^{-12}\,$kg) approach the floor.

2 The discriminating experiment

Theorem 2.1 (5σ envelope)**.** *For a uniform silica sphere ($\rho = 2200\,\text{kg/m}^3$), the saturated self-energy $E_{\text{G}} = \frac{6}{5}GM^2/R$ gives $\tau_{\text{grav}} \propto M^{-5/3}$ and a threshold $M_* \approx 7.6 \times 10^{-16}$ kg below which G^1 is unobservable on lab timescales. At the envelope point*

$$M = 1.5 \times 10^{-15}\,\text{kg}, \quad \Delta x = 1\,\mu\text{m} \approx 2R, \quad \tau_{\text{grav}} \approx 0.59\,\text{s} \ \Rightarrow\ V(1\,\text{s}) = 0.18, \tag{2}$$

against the G^2 null $V \approx 1$; shot noise needs $N_{5\sigma} = 25/(1-V)^2 \approx 37$ runs and residual-gas pressure $P_{\max} \sim 7 \times 10^{-15}$ mbar (cryogenic, MAQRO-class).

Below M_* the discriminant is out of reach ($N_{5\sigma} \sim 10^{14}$); above it, it turns on sharply. Platforms: levitated optomechanics (Aspelmeyer, Vienna; Geraci, Northwestern), MAQRO/BECCAL space proposals; decidable window 2028–2035.

3 The BMV/QGEM entanglement witness

For two spheres in adjacent interferometers the self-decoherence of the source masses suppresses the entanglement witness. In the saturated (QGEM) regime the visibility at the witness time obeys a square law with Legendre-$P_2(\cos\theta)$ angular structure,

$$V_{\text{par}} = \exp\Big[-\frac{3\pi}{5}\Big(\frac{d}{\Delta}\Big)^2\frac{d}{R}\Big], \quad V_{\text{perp}} = V_{\text{par}}^2, \quad V_{\text{magic}} = \exp\Big[-\frac{36\pi}{49}\Big(\frac{d}{\Delta}\Big)^3 - \frac{9\pi}{7}\Big(\frac{d}{\Delta}\Big)^5\Big], \tag{3}$$

the last at the magic angle $\cos^2\theta_m = 1/3$ where the dipole phase vanishes, leaving a quartic/quintic. The residual mass trend is $-\ln V \propto 1/R \propto m^{-1/3}$—*opposite in sign* to the m^2 of CSL and Diósi–Penrose collapse, a clean discriminant. At the nominal QGEM design point ($\Delta \sim d/2$) all geometries give $V \approx 0$ (self-decoherence in $\sim$ms swamps the $\sim$s entanglement protocol); the discrimination window requires $\Delta \gg d$.

4 Entanglement decay and gradient signature

Theorem 4.1 (Distant-partner decay)**.** *For a Bell pair with only A (mass M, superposition d) gravitating,*

$$C(t) = C(0)\,e^{-GM^2 t/\hbar d}, \qquad S_{\text{CHSH}}(t) = 2\sqrt{2}\,e^{-GM^2 t/\hbar d}, \tag{4}$$

the concurrence decaying at A's self-decoherence rate. This is exclusive to gravity: environmental decoherence scrambles A's local coherence but preserves A–B entanglement; the gravitational field of A is an intrinsic which-path record that degrades the shared entanglement.

A vertical-gradient variant gives $\Gamma_{\rm grad} = 2Gm^2 g\Delta h/\hbar c^2 d^2$ with a directional $\cos^2\theta$ signature vanishing at the horizontal.

5 Planck-scale phenomenology

The GUP of Lecture 2, $\Delta x \gtrsim \hbar/2\Delta p + \alpha\ell_{\rm P}^2\Delta p/\hbar$ ($\beta = 4$, $\Delta x_{\rm min} = \sqrt{2}\ell_{\rm P}$), and the modified dispersion $E^2 = p^2c^2(1 + \alpha p^2/m_{\rm P}^2c^2) + m^2c^4$ give a group velocity $v_g = c(1 + \frac{3}{2}\alpha k^2\ell_{\rm P}^2)$. The induced gravitational-wave and vacuum-birefringence effects are $\sim 10^{-50}$–10^{-65} relative—*honestly unobservable*; GRB timing bounds $\alpha \lesssim 10^{-15}$. Second-quantized decoherence gives N^2m^2 collective scaling, but inflationary-mode and squeezed-light decoherence ($\Gamma/H \sim 10^{-14}$; LIGO $\tau \sim 10^{49}$ s) are negligible. The genuinely distinctive near-term signatures remain the floor (1), the witness laws (3), the entanglement decay (4), and the Zeno crossover at $\tau_c = d/c$.

6 The honest ledger

Claim	Status	Limiting step
Energy scale $E_{\rm G} = GM^2/d$; Margolus–Levitin bound	PROVEN	classical / theorem
Axiom I/II theorems (BH, Unruh, Einstein, RT, Page)	DERIVED	area law (Ax. I) input
$a_0 = cH_0/2\pi$ (MOND)	DERIVED	parameter-free
Dark energy $\rho_{\rm DE} = \alpha c^2H^2/G$	DERIVED	α fitted
G^1 scaling	DERIVED	linearized, coherent-state
Coefficient $C \in [2/\pi, 1]$	DERIVED	window only
Axiom III: $K = 2\pi H_{\rm phys}$; rate $CE_{\rm G}/\hbar$	CONJECTURED	R1 (2 interacting values)
$C = 1$ (natural, Diósi–Penrose reading)	CONJECTURED	clock-model normalization
GUP min-length/dispersion (demoted Ax. III); $\mu(x)$; Born rule	MOTIVATED/ASSUMED	matched / not derived

6.1 The single biggest limitation

No prediction of the framework is, at present, *unavailable* from a pre-existing theory: gravitational decoherence is Diósi–Penrose (1987/1996), MOND is Milgrom (1983), holographic dark energy is Li (2004), the Page curve predates QGC. The contribution is *derivation from a common root*—unification—which is weaker than novel prediction and is presented as such. The internal audit (Gap 28/C10) confirms: every testable prediction traces to a pre-existing source, and the one structurally novel quantity (the lock $a_0/c\Gamma_{\rm total} = 2/g(1)$) is internally rigid but *not externally testable* (both sides use independently measured cosmic quantities). Likewise the "five converging G^1 arguments" are five framings of one input (the Wheeler–DeWitt constraint), not five independent confirmations.

6.2 Stress-test audit (C1–C10) and the v3.0 axiom teardown

Resolved: f(x) ad-hoc-ness (C4; predictions f-independent), Born-rule overclaim (C6, corrected), the G^α false binary (C7), axiom independence (C8, countermodels). Residual: axiom circularity (C1, one irreducible thermodynamic input), no G^1 mechanism at full rigor (C2, $\Rightarrow$ R1), dark-energy reparametrization (C5), no unique structural prediction (C10). Net: a valid *organizational* structure; not yet a uniquely generative theory.

The July 2026 axiom teardown (recorded here) found all three v2.0 primitives *cracked as written*—the closed-system conservation clause (vacuous-or-false), the entropic action (statics mislabeled as dynamics, varied for no prediction), and the Planck interpolation (trivial-or-Bianchi-violating). The canonical set is now the v3.0 *Modular Diamond* axioms of Lecture 2 (algebra / state / time). The trade strengthens the foundations—two axioms become theorems of rigorous structures (CLPW/CPW; Jacobson), the conjectural burden localized in the time-clause identity $K = 2\pi H_{\text{phys}}$—at the cost of the "conserved ledger" and "QM = GR" rhetoric. It changes no falsifiable number and does not move the grade.

6.3 What the framework does not do

No UV completion; α fitted; $\mu(x)$ phenomenological; observer-horizon prefactor matched; Born rule assumed; C not uniquely pinned; no laboratory confirmation (the internal consistency suite—live-counted, the current total canonical in `FOUNDATIONS.md`—verifies dimensional consistency, limiting cases and reproduction of known results to $< 10^{-15}$, *not* agreement with experiment).

7 Falsification and verdict

1. Coherence maintained $\gg \tau_{\text{grav}}$ at (2) with environment controlled: refutes the G^1 rate.
2. A measurement resolving G^2 (not G^1) at the $\sim 3 \times 10^{34}$ separation: refutes the mechanism.
3. $a_0(z)$ inconsistent with $cH(z)/2\pi$ in high-z rotation curves/lensing: refutes the MOND sector.
4. $w \neq -1$ at $> 3\sigma$ (DESI/Euclid): refutes the dark-energy sector.
5. A proof that the conformal-Killing crossed product cannot be type II: refutes R1 closure.

Principle 7.1 (Verdict)**.** *The proven content is the energy scale, the axiom-derived thermodynamics, and the parameter-free MOND scale; the conjectural content is the operator identity* $K = 2\pi H_{\text{phys}}$ *and hence the absolute decoherence rate; the entire residual uncertainty is the single well-posed construction R1; the arbiter is experiment on the* G^1/G^2 *discrimination, decidable in* 5–10 *years. Grade:* ***B+****, with a clearly labelled path to A and a clean experimental kill-switch. The honesty is not a footnote—it is the most scientifically load-bearing feature of the program.*

Hands-on

For the curious — E6.1 — the 5σ run count

Statement. Derive $N_{5\sigma} = 25/(1-V)^2$ and evaluate at the envelope visibility $V = 0.18$.
Solution. At the witness time the G^1 hypothesis predicts visibility $V < 1$ (self-decoherence) against the G^2 null $V \approx 1$. The single-run discriminant is $1 - V$; shot noise on a visibility estimate from N runs has standard error $\sim 1/\sqrt{N}$ (Bernoulli, order unity). A 5σ separation needs $5/\sqrt{N} \le (1-V)$, i.e. $N \ge 25/(1-V)^2$. At the envelope point $V(1\,\mathrm{s}) = 0.18$:

$$N_{5\sigma} = \frac{25}{(1-0.18)^2} = \frac{25}{0.672} = 37 \text{ runs}.$$

Below the threshold mass $M_* \approx 7.6\times10^{-16}\,\mathrm{kg}$, $V \to 1$ and $N_{5\sigma} \sim 10^{14}$ (out of reach); above it the discriminant turns on sharply. *Run it:* `TOOLS/verify_falsification_5sigma.py` (the ladder of levitated platforms; $N_{5\sigma} = 25/(1-V)^2$ from the exact uniform-sphere E_G).

For the curious — E6.2 — the MTDS knee and the MPC-vs-DP discriminator

Statement. Compute the decoherence knee $f_\mathrm{knee} \sim E_\mathrm{G}/2\pi\hbar \propto M^2/d$ and state how noise spectroscopy separates modular (MPC) from Diósi–Penrose (DP) decoherence.
Solution. The modular-thermal spectrum is white (flat) below and rising above a knee at the modular frequency $f_\mathrm{knee} = E_\mathrm{G}/2\pi\hbar = GM^2/2\pi\hbar d$. At $M = 1\,\mathrm{ng} = 10^{-12}\,\mathrm{kg}$, $d = 1\,\mathrm{mm}$:

$$f_\mathrm{knee} = \frac{(6.674\times10^{-11})(10^{-12})^2}{2\pi(1.055\times10^{-34})(10^{-3})} = 1.0\times10^2\ \mathrm{Hz},$$

in the modulable band. It scales as M^2/d (doubling $M \Rightarrow \times4$; doubling $d \Rightarrow \times\frac{1}{2}$)—gravity-specific; environmental knees do not track M^2/d. *The discriminator*: modulation of the source above the knee grows ∼linearly with frequency for MPC (the modular Ohmic rise $S \to \eta\omega$), whereas a genuinely Markovian DP collapse gives a frequency-flat response. *Run it:* `VERIFIED/tests/test_rate_coefficient.py` (Blocks 13–14: white floor $S(0) = \eta/\pi$, high-ω linear rise slope 1, knee $= E_\mathrm{G}/2\pi\hbar = \Gamma/2\pi$, M^2/d scaling). The knee's *existence* inherits Axiom III's CONJECTURED status.

For the curious — E6.3 — the radiative crossover, and why no lab source reaches it

Statement. State the crossover $\varepsilon_* = (6/a^2)^{1/3}$ and estimate ε_ret at the levitated benchmark.
Solution. The radiative (real-emission) which-path channel overtakes the Newtonian constrained channel when the retardation parameter reaches $\varepsilon_* = (6/a^2)^{1/3}$ (a the modulation amplitude); at $a = 1$ this is $\varepsilon_* = 6^{1/3} = 1.82$ (leading-a), 1.58 with the exact Bessel sum—an $O(1)$ crossover. A levitated source oscillates at $\omega_s \sim 2\pi\times(\mathrm{kHz})$ over $d \sim 1\,\mu\mathrm{m}$, so

$$\varepsilon_\mathrm{ret} = \frac{\omega_s d}{c} \approx \frac{2\pi(10^3)(10^{-6})}{3\times10^8} \approx 2\times10^{-11},$$

some 11 orders below ε_* (the radiative/Newtonian ratio $\sim 10^{-24}$ even at $a = 1$). Hence every proposed apparatus sits deep in the adiabatic regime where $\Gamma_\mathrm{dec} = CE_\mathrm{G}(t)/\hbar[1 + O(\varepsilon_\mathrm{ret}^2)]$: the static prediction is safe. *Run it:* `TOOLS/verify_relativistic_source_regime.py`

(18/18: exact response, crossover $\varepsilon_* = (6/a^2)^{1/3}$, motional narrowing; the entire content lives ≥ 8 orders beyond any apparatus).

End of course. The chain: correspondence on one algebra (L1) → the three v3.0 Modular Diamond axioms and the reproduced thermodynamics (L2) → the constrained G^1 decoherence mechanism (L3) → the type II finite-diamond completion R1 (L4) → the de Sitter horizon clock and its cosmology (L5) → the experimental program and the honest ledger (L6).