

Cosmic Gravitational Decoherence on a de Sitter Background: Resolving a Causality Paradox in Quantum-Geometric Correspondence

Quantum-Geometric Correspondence

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Abstract

The gravitational decoherence rate $\Gamma = G \Delta m^2 / (\hbar d)$ of Quantum-Geometric Correspondence (QGC), applied at cosmic scale with Δm the mass of the observable universe and d the Hubble radius R_H , returns $\Gamma \sim 10^{103}$ Hz—exceeding the causal bound $c/R_H \sim H_0$ by 121 orders of magnitude. This internal contradiction is the subject of this paper. We show that the divergence is an artifact of using the Minkowski graviton propagator at separations $d \sim R_H$, where it is not valid. Recomputed on a de Sitter background with Bunch–Davies modes, the coherent-state decoherence functional acquires a physical infrared cutoff at the horizon scale: super-horizon modes freeze and cease to drive decoherence. The rate gains a closed-form form factor, $\Gamma_{\text{dS}} = (G \Delta m^2 / \hbar d) g(Hd/c) t$ with $g(x) = 1 - (2/\pi) \text{Si}(x)$, which recovers the Minkowski result for $Hd/c \ll 1$ and is finite and causal at $d = R_H$. The cosmic rate is governed not by the total mass M_U but by the elementary branch granularity of the cosmic wavefunction; arguments from quantum cosmology identify this granularity with the de Sitter thermal mass $\Delta m_{\text{dS}} = \hbar H / (2\pi c^2)$ per horizon mode. Summed over the S_{dS} holographic horizon modes, the cosmic decoherence rate is $\Gamma_{\text{total}} = g(1) H / (4\pi) \approx H / (10\pi) \approx 7 \times 10^{-20}$ Hz (up to an $O(1)$ granularity coefficient), manifestly sub-causal. An internal-consistency program bounds the remaining approximations, leaving a small set of explicitly stated premises. The result resolves an inconsistency internal to QGC; we state which premises it rests on and what falsifiable consequences it carries.

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1 Introduction

Pushed to cosmic scale, the gravitational decoherence rate of QGC returns $\sim 10^{103}$ Hz—some 121 orders of magnitude above the causal bound c/R_{H} . QGC—the proposal that quantum mechanics and general relativity are complementary descriptions of a single informational substrate—predicts that a spatial superposition of a mass loses coherence through its gravitational field at a rate

$$\Gamma = \frac{G \Delta m^2}{\hbar d}, \quad (1)$$

where Δm is the mass difference between the two branches and d their spatial separation [1]. The rate (1) has the same parametric form as the gravitational decoherence rate of the Diósi–Penrose conjecture [26, 25], to which QGC lends a specific microphysical mechanism. For laboratory parameters it is benign: a microgram particle separated by a millimetre decoheres in roughly a nanosecond. Equation (1) was derived in linearized gravity by imposing the Wheeler–DeWitt constraint on the Feynman–Vernon influence functional, and its laboratory consequences are the subject of the canonical core paper [1] of this series.

The trouble appears when Eq. (1) is pushed to its largest conceivable scale. Insert the mass of the observable universe, $M_U \sim 1.5 \times 10^{53}$ kg, for Δm , and the Hubble radius $R_{\text{H}} = c/H_0 \sim 1.3 \times 10^{26}$ m for d . The formula returns

$$\Gamma_{\text{naive}}(M_U, R_{\text{H}}) \sim \frac{c^5}{4G\hbar H_0} \sim 10^{103} \text{ Hz}. \quad (2)$$

No physical decoherence process can run faster than information can cross the system. For a system the size of the Hubble volume the causal bound on any rate is

$$\Gamma \leq \frac{c}{R_{\text{H}}} = H_0 \sim 2.2 \times 10^{-18} \text{ Hz}. \quad (3)$$

Equation (2) exceeds Eq. (3) by 121 orders of magnitude. This discrepancy is a genuine internal contradiction: a formula the framework relies on, applied within the framework’s own stated domain, returns an answer the framework’s own causality requirements forbid. Until it is resolved, the cosmic-scale behaviour of QGC gravitational decoherence cannot be regarded as understood.

1.1 Two failure modes of the naive extrapolation

A formula like Eq. (1) can fail at a new scale in two distinct ways: it can be *wrong*, resting on physics that does not hold there; or its *inputs* can be wrong, Δm and d not being the quantities the formula was meant to take. Both failures occur in the naive extrapolation, and diagnosing them separately resolves the contradiction.

The first failure is in the propagator. Equation (1) is built from the graviton two-point function on a *Minkowski* background, which has no infrared scale: arbitrarily long-wavelength modes contribute to the mode integral, and on flat space this is legitimate after self-energy renormalization. The universe is not Minkowski at separations approaching R_{H} ; it is, to excellent approximation, de Sitter, with a Hubble scale supplying exactly the infrared scale Minkowski lacks. The flat-space propagator at $d \sim R_{\text{H}}$ is the wrong Green’s function in precisely the regime where the difference matters most.

The second failure is in the inputs. The cosmic wavefunction does not jump between a

configuration with all of M_U *here* and the same mass an entire Hubble radius *there*. Inserting $\Delta m = M_U$ computes the decoherence rate between two maximally distant branches—as physically meaningless as the rate obtained by coherently displacing every molecule of a gas by one metre. The relevant quantity is the rate at which the cosmic wavefunction loses coherence through its *elementary* branch steps, and the elementary step is not M_U .

1.2 Resolution: structure and granularity

The resolution has a structural part and a quantitative part; both are needed.

The structural part (Section 2) is the de Sitter recomputation: the coherent-state decoherence functional of the canonical core paper [1], redone on a fixed de Sitter background with the linearized graviton in the Bunch–Davies vacuum. The de Sitter mode functions suppress modes with comoving wavenumber below $k_H = aH/c$: such modes are super-horizon, freeze rather than oscillate, and frozen modes do not generate time-dependent decoherence. This is the infrared regulator the Minkowski calculation lacked. The decoherence exponent factorizes as

$$\Gamma_{\text{dS}}(t, d, H) = \frac{G \Delta m^2}{\hbar d} g\left(\frac{Hd}{c}\right) t, \quad g(x) = 1 - \frac{2}{\pi} \text{Si}(x), \quad (4)$$

where Si is the sine integral. The form factor g recovers the Minkowski result, $g(0) = 1$, for $Hd/c \ll 1$; it is finite at the horizon, $g(1) = 1 - (2/\pi) \text{Si}(1) = 0.398$; and it vanishes for super-horizon separations, $g(x \gg 1) \rightarrow 0$. The catastrophic divergence of Eq. (2) is gone: at $d = R_H$ the rate is finite and proportional to H .

The quantitative part (Section 3) supplies the input the structural part leaves open. The de Sitter recomputation gives a rate per unit Δm^2 but does not by itself select Δm ; inserting $\Delta m = M_U$ still returns 10^{103} Hz even on de Sitter. The question native to quantum cosmology is then: in the Wheeler–DeWitt framework, what is the smallest mass-distribution difference separating two physically distinct branches of the cosmic wavefunction? Several independent lines of argument—a thermal-distinguishability floor, the de Sitter horizon first law, the predictability sieve, and a thermality-free route through the Wheeler–DeWitt branch structure—converge on the de Sitter thermal mass,

$$\Delta m_{\text{dS}} = \frac{k_B T_{\text{dS}}}{c^2} = \frac{\hbar H}{2\pi c^2}, \quad (5)$$

as the branch granularity per horizon mode. Summing the per-mode rate over the $S_{\text{dS}} = \pi c^5 / (G \hbar H^2)$ holographic horizon modes gives the cosmic decoherence rate

$$\Gamma_{\text{total}} = g(1) \frac{H}{4\pi} \approx \frac{H}{10\pi} \approx 7 \times 10^{-20} \text{ Hz}, \quad (6)$$

finite, proportional to H , and comfortably below the causal bound (3).

Section 4 addresses the approximations on which Eqs. (4)–(6) rest. The result is a controlled approximation: the linear additivity of per-mode rates, the scalar proxy for the spin-2 graviton, the gauge dependence of the extracted rate, and the all-orders behaviour of the known de Sitter graviton secular series are each examined in turn. The result survives each check, with stated residuals.

Section 5 states what the result rests on. It resolves an inconsistency *internal* to QGC: it does not establish QGC, which remains a framework without direct experimental confirmation, and it rests on premises stated explicitly—chiefly the de Sitter Euclidean periodicity $2\pi/H$ and

the standard inflationary-decoherence picture from which the horizon-crossing readout time is derived. The program-wide assumption the whole framework carries—the identification of a gravitational energy scale with a decoherence rate—this paper inherits rather than closes. Section 5 also sets out the falsifiable content: what an observation, or a future calculation, would have to show to overturn the picture.

Series context. This paper belongs to a series on gravitational effects at the quantum–classical interface. The canonical core paper [1] presents the axiomatic framework of QGC, the laboratory gravitational-decoherence prediction, and the G^1 scaling derived from the constrained influence functional; companion papers develop holographic dark energy and emergent gravity from the same axioms. The present paper extends the canonical core decoherence formula to cosmic scale and resolves the causality paradox that extension exposes. It is self-contained but cross-references the others for extended discussion.

2 The de Sitter recomputation

The Minkowski decoherence formula fails at $d \sim R_H$ for a stated reason; identifying it dictates the background the cosmic problem demands.

2.1 Domain of validity of the Minkowski formula

The canonical core paper’s [1] derivation of Eq. (1) represents the gravitational field sourced by a classical mass distribution as a coherent state of the linearized graviton. Two mass configurations differing by Δm at separation d source two coherent states $|\Phi_L\rangle$, $|\Phi_R\rangle$, and the decoherence exponent is the squared norm of their amplitude difference,

$$\Gamma(t) = \|\alpha_L(t) - \alpha_R(t)\|^2, \quad (7)$$

the norm being the L^2 norm over graviton modes. On a Minkowski background the modes are plane waves and the integral runs over all wavenumbers down to $k = 0$. The infrared end of that integral is rendered finite only by self-energy renormalization—a subtraction that is legitimate on flat space because the divergent piece is k -independent and cancels in the difference $\Gamma_L - \Gamma_R$.

This works on Minkowski because flat space has no infrared scale: nothing distinguishes a mode of wavelength 1 m from one of wavelength 10^{26} m. Both oscillate and contribute to the time-dependent overlap (7) in the same way. For laboratory separations d this is harmless: the angular structure of the source already suppresses the longest-wavelength contributions. As d grows toward the Hubble radius, the calculation draws on modes whose wavelength exceeds the causal patch, and on flat space nothing stops it. The Minkowski formula’s domain of validity is $Hd/c \ll 1$; the naive cosmic extrapolation (2) sits at $Hd/c \sim 1$, outside it.

The actual universe supplies the missing scale. On scales approaching R_H the geometry is de Sitter, and de Sitter has a horizon. Modes longer than the horizon are causally frozen. The correct calculation must therefore use the de Sitter graviton propagator, which knows about the horizon, in place of the Minkowski one, which does not.

2.2 Linearized graviton on de Sitter

We work on a fixed de Sitter background in flat (spatially homogeneous) slicing,

$$ds^2 = -c^2 dt^2 + a(t)^2 d\mathbf{x}^2, \quad a(t) = e^{Ht}, \quad (8)$$

or, in conformal time τ defined by $a d\tau = c dt$ so that $a(\tau) = -1/(H\tau)$ with $\tau \in (-\infty, 0)$,

$$ds^2 = a(\tau)^2 (-c^2 d\tau^2 + d\mathbf{x}^2). \quad (9)$$

The Hubble radius is $R_H = c/H$ and the horizon area $A_H = 4\pi R_H^2$. We treat the mass distribution as a classical source for the linearized graviton, neglecting back-reaction of the graviton on the source—the same leading-order approximation used in the canonical core paper [1].

In transverse-traceless (TT) gauge the linearized Einstein equations on de Sitter reduce, per polarization, to a wave equation. Writing the field as $\Phi_k(\tau) = \chi_k(\tau)/a(\tau)$, the transverse-traceless graviton obeys exactly the mode equation of a massless minimally coupled scalar [5, 4],

$$\chi_k'' + \left(k^2 - \frac{2}{\tau^2}\right) \chi_k = 0, \quad (10)$$

with primes denoting $d/d\tau$. The derivation therefore works with a scalar proxy Φ , each TT polarization treated as such a scalar; this proxy reproduces the full spin-2 result exactly, not merely approximately (Section 4.2).

The Bunch–Davies mode functions—selecting positive-frequency behaviour in the far past $\tau \rightarrow -\infty$ —are

$$\Phi_k(\tau) = \frac{H}{\sqrt{2k^3}} (1 + ik\tau) e^{-ik\tau}. \quad (11)$$

These mode functions encode the physics the Minkowski calculation lacked. They split into three classes by the value of $k\tau \propto k_{\text{phys}}/H$, where $k_{\text{phys}} = k/a$ is the physical wavenumber:

- *Sub-horizon* ($|k\tau| \gg 1$, i.e. $k_{\text{phys}} \gg H$): $\Phi_k \sim e^{-ik\tau}$, oscillatory, behaving like a flat-space mode;
- *Horizon-crossing* ($|k\tau| \sim 1$): the transition regime;
- *Super-horizon* ($|k\tau| \ll 1$, i.e. $k_{\text{phys}} \ll H$): $\Phi_k \rightarrow H/\sqrt{2k^3}$, a *frozen* amplitude that no longer oscillates.

The freezing of super-horizon modes is the decisive structural input. A non-oscillating mode does not generate time-dependent decoherence as an oscillating mode does, so the Hubble scale separates modes that decohere from modes that do not—the infrared regulator flat space could not provide.

2.3 The decoherence functional and its form factor

The coherent-state argument underlying Eq. (7) is purely algebraic: a classical source coupled linearly to a free field displaces the vacuum into a coherent state, on any background [1]. On de Sitter the structure of Eq. (7) is unchanged; what changes is the *spectrum* of modes the displacement populates, the mode functions now being Eq. (11) rather than plane waves.

For a source switched on at the start of the de Sitter epoch, the amplitude difference evolves as $\alpha_L(k, \tau) - \alpha_R(k, \tau) = \Delta\alpha^{\text{eq}}(k) T(k, \tau, \tau_0)$, where $\Delta\alpha^{\text{eq}}$ is the equilibrium (Coulomb-like) amplitude difference and T is a transient factor that interpolates from $T = 0$ at the switch-on time to $T \rightarrow 1$ once the field is established. Solving the sourced version of Eq. (10) exactly with the retarded Green’s function built from the Bunch–Davies homogeneous solutions yields the transient factor in closed form,

$$T(k, \tau, \tau_0) = 1 - \frac{1}{a(\tau)} e^{ik(\tau_0 - \tau)}, \quad (12)$$

the exponential-integral pieces cancelling identically. The decoherence-relevant part of $|T|^2$ —obtained after subtracting the k -independent piece, which contributes only to the overall phase of the coherent-state overlap—is

$$|T|^2 - \left(1 - \frac{1}{a}\right)^2 = \frac{2}{a} (1 - \cos k \Delta\tau), \quad \Delta\tau = \tau - \tau_0. \quad (13)$$

For sub-horizon modes Eq. (13) reduces, as $H \rightarrow 0$, exactly to the Minkowski factor $2(1 - \cos \omega_k t)$ —a derived consistency check. For super-horizon modes it is $O((k \Delta\tau)^2)$ and vanishes as $k \rightarrow 0$, which is the infrared suppression made explicit.

Inserting Eqs. (11) and (12) into the squared-norm exponent (7), and using physical (proper) separation $d = a(\tau) r$ in the angular integral, gives the de Sitter decoherence exponent as a mode integral

$$\Gamma_{\text{dS}}(t) = \frac{16 G^2 \Delta m^2}{\pi \hbar c} \int_0^\infty \frac{dk}{k^3} \left(1 - \frac{\sin kd}{kd}\right) [|T(k, \tau)|^2 - (1 - \frac{1}{a})^2]. \quad (14)$$

The angular factor $1 - \sin(kd)/(kd)$, common to the Minkowski and de Sitter calculations, vanishes as $(kd)^2/6$ at small k and tends to unity for $kd \gg 1$. On Minkowski the integral (14) extends to $k = 0$; on de Sitter the renormalized transient factor cuts off modes below the comoving horizon scale $k_H = aH/c = 1/R_H^{\text{comoving}}$, since those modes are frozen.

In the late-time, weak-coupling regime $d/c \ll t$, the sub-horizon part of Eq. (14) grows linearly in t , and the exponent takes the factorized form

$$\Gamma_{\text{dS}}(t, d, H) = \frac{G \Delta m^2}{\hbar d} g\left(\frac{Hd}{c}\right) t. \quad (15)$$

The dimensionless form factor g is the ratio of the de Sitter gravitational energy scale to the Minkowski one, with the super-horizon modes removed by the horizon cutoff. Evaluating the cut-off mode integral in closed form (Appendix B) gives

$$\boxed{g(x) = 1 - \frac{2}{\pi} \text{Si}(x)} \quad \text{Si}(x) = \int_0^x \frac{\sin u}{u} du. \quad (16)$$

The conversion of the saturating free-field mode integral into a rate growing linearly in t —the step from Eq. (14) to Eq. (15)—is not automatic. It is the same constraint-driven step that underlies the G^1 scaling of the canonical core paper [1]: the influence functional alone gives a bounded, saturating exponent, and the Wheeler–DeWitt Hamiltonian constraint converts it into linear-in- t growth.

Remark 2.1. The form factor g governs the *energy scale*; the Hamiltonian constraint governs the

rate. This separation of roles recurs in Sections 4.3 and 5.

2.4 Limits: Minkowski recovery and the horizon

The form factor (16) has three regimes, each carrying physical content.

Minkowski limit, $Hd/c \rightarrow 0$. Here $\text{Si}(0) = 0$, so $g(0) = 1$ exactly: the horizon cutoff $k_H = 1/R_H \rightarrow 0$, the Bunch–Davies modes (11) reduce to plane waves, and the de Sitter calculation collapses to the canonical core paper’s result $\Gamma = G \Delta m^2 / (\hbar d)$ —the consistency check the calculation must pass. The small- x onset is linear, $g(x) = 1 - (2/\pi)x + O(x^3)$, so the leading correction to the laboratory formula is suppressed by Hd/c , negligible for any laboratory d , where $Hd/c \sim 10^{-26}$.

Horizon limit, $d \rightarrow R_H$. Here $x = Hd/c = 1$, and

$$g(1) = 1 - \frac{2}{\pi} \text{Si}(1) = 0.3977\dots \quad (17)$$

The rate at the horizon scale is therefore

$$\Gamma_{\text{dS}}^{\text{rate}}(R_H) = \frac{G \Delta m^2}{\hbar R_H} g(1) = g(1) \frac{G \Delta m^2}{\hbar c} H, \quad (18)$$

finite—in sharp contrast with the divergent Minkowski extrapolation. Comparing to the causal bound,

$$\frac{\Gamma_{\text{dS}}^{\text{rate}}(R_H)}{H} = g(1) \frac{G \Delta m^2}{\hbar c} = g(1) \left(\frac{\Delta m}{m_P} \right)^2, \quad (19)$$

with $m_P = \sqrt{\hbar c/G}$ the Planck mass. For any sub-Planckian mass difference the horizon-scale rate is comfortably sub-causal.

Super-horizon limit, $Hd/c \gg 1$. Here $\text{Si}(x) \rightarrow \pi/2$ and $g(x) \rightarrow 0$. Mass configurations separated by more than the Hubble radius produce no linear-in- t decoherence at all: their which-path information lies in frozen, super-horizon modes. This is the de Sitter causal horizon expressing itself directly in the decoherence functional—configurations outside each other’s causal patch cannot dynamically decohere one another.

2.5 Scope of the recomputation

The de Sitter recomputation removes the *structural* pathology of the naive extrapolation. The catastrophic 10^{103} Hz of Eq. (2) was an artifact of using the Minkowski propagator—a propagator with no infrared scale—at a separation where the physically correct propagator, the de Sitter one, has a horizon. With the horizon cutoff in place the rate is finite, proportional to H , and bounded by Eq. (19).

It does *not*, by itself, settle the cosmic rate. Equation (18) is finite but still contains the free input Δm . Inserting $\Delta m = M_U$ returns

$$\Gamma_{\text{dS}}(M_U, R_H) \sim g(1) \frac{G M_U^2}{\hbar c} H \sim 10^{103} \text{ Hz}, \quad (20)$$

the same catastrophic number—because $(M_U/m_P)^2 \sim 10^{122}$ overwhelms the form factor. The de Sitter background fixes the propagator; it does not fix the input. Resolving the contradiction completely requires identifying what Δm *should* be for the cosmic question. That is the subject of Section 3.

3 The cosmic mass scale

The de Sitter form factor gives a rate per unit Δm^2 but leaves Δm open. The physically correct Δm for cosmic decoherence is not a free parameter: the branch structure of quantum cosmology fixes it. This section identifies it and assembles the total rate.

3.1 M_U as the wrong input

The cosmic wavefunction Ψ obeys the Wheeler–DeWitt constraint $\hat{H}_{\text{total}}|\Psi\rangle = 0$. There is no external time; physical time is relational, recovered through the decoherent-histories construction [14, 15]. There a coarse-graining of configuration space—here the spatial mass distribution smeared over horizon-scale cells—defines a set of histories, and the histories acquire probabilities, and a meaningful “rate,” only once they decohere.

Definition 3.1 (Branch and granularity). A *branch* of the cosmic wavefunction is an equivalence class of Wheeler–DeWitt configurations that the decoherent-histories coarse-graining cannot tell apart; this is the operational meaning of “physically distinct branch.” The branch *granularity* Δm_* is the resolution of that coarse-graining—the smallest mass-distribution difference the decoherence functional resolves into distinct, mutually decohered histories.

A configuration labelled “all of M_U displaced by R_H ” is not adjacent to the fiducial branch in history space; it is separated from it by an enormous number of elementary branch steps. Inserting $\Delta m = M_U$ into Eq. (18) computes the decoherence rate between two maximally distant branches—not the cosmic decoherence rate, any more than coherently displacing every molecule of a gas by one metre defines that gas’s decoherence rate. The physical cosmic rate is built from the *elementary* branch step Δm_* , summed over the independent branch-distinguishing degrees of freedom. Both factors are determined below.

3.2 The branch granularity

The branch-distinguishing degrees of freedom are the graviton modes already identified by the de Sitter calculation: the Bunch–Davies modes crossing the horizon, each carrying one independent quantum of which-branch information. The granularity Δm_* per such mode is fixed by several independent lines of physical argument, which converge.

Route A: thermal distinguishability floor. A static observer in de Sitter sees the Bunch–Davies vacuum as a thermal state at the Gibbons–Hawking temperature [7],

$$T_{\text{dS}} = \frac{\hbar H}{2\pi k_B}. \quad (21)$$

Two cosmic branches whose mass-energy in a given horizon mode differs by less than the thermal energy fluctuation of that mode are not operationally distinguishable: the difference is buried

inside the thermal ensemble. The distinguishability floor per mode is the thermal mass-energy spread, $\Delta E_{\text{floor}} \sim k_B T_{\text{dS}}$, giving $\Delta m_* \sim k_B T_{\text{dS}}/c^2$. A caveat on the coefficient: the single-mode fluctuation formula gives $k_B T_{\text{dS}}$ exactly only in the long-wavelength equipartition limit; the strict horizon mode $x = \hbar\omega/k_B T_{\text{dS}} = 2\pi$ gives $0.26 k_B T_{\text{dS}}$. The natural canonical value, delivered sharply by Routes B and C, is $k_B T_{\text{dS}}$; an $O(1)$ factor in the range ~ 0.3 – 1 is defensible and propagates to an $O(1)$ uncertainty in Γ_{total} , not to its scaling. A second subtlety: de Sitter *spacetime* has negative heat capacity, which would invalidate a canonical fluctuation argument. The fluctuation argument here is applied to the bulk *field modes*, ordinary harmonic oscillators in a genuine positive-temperature Gibbs state; the negative heat capacity is a property of the gravitating background, not of the matter sector, exactly as for Hawking radiation from a black hole.

Route B: the de Sitter horizon first law. The de Sitter horizon obeys a first law $dE = T_{\text{dS}} dS_{\text{horizon}}$ [7, 8, 9]. One quantum of horizon entropy—the smallest distinguishable change, $dS_{\text{horizon}} = k_B$ —corresponds to an energy $dE = k_B T_{\text{dS}}$, hence a mass quantum $\Delta m_* = k_B T_{\text{dS}}/c^2$. This is a genuinely distinct argument: the holographic principle native to this framework (developed in the holographic dark-energy paper) says the horizon stores S_{dS} which-branch registers, and the first law converts one register flip—one branch distinction—into one quantum of mass-energy $k_B T_{\text{dS}}/c^2$.

Route C: the predictability sieve. Zurek’s predictability sieve [18, 19] selects, among candidate branch spacings, those that the environment can hold a stable record of. Branches finer than one thermal quantum of the horizon environment cannot acquire a stable record—an incipient record is thermally erased on the de Sitter mixing time $\sim 1/H$. Branches at the thermal quantum are imprinted, frozen as the recording mode redshifts to super-horizon, and thereafter permanent. The einselected branch cell has width $\Delta m_* = k_B T_{\text{dS}}/c^2$.

Convergence, and the irreducible premise. Routes A, B and C land on the same value,

$$\Delta m_{\text{dS}} = \frac{k_B T_{\text{dS}}}{c^2} = \frac{\hbar H}{2\pi c^2}. \quad (22)$$

Three arguments from genuinely different physics—bulk QFT, horizon thermodynamics, decoherence theory—reach the same $O(1)$ coefficient, including the 2π . The convergence must be read honestly: all three funnel through one shared premise, that the de Sitter environment is a Gibbs state at T_{dS} . It is one thermodynamic insight reached three ways, not three independent results.

A fourth route, constructed without thermality, sharpens the picture. It reaches Eq. (22) from the Wheeler–DeWitt mini-superspace branch structure, the canonical clock–energy pairing $[\tau, E_{\text{static}}] = i\hbar$, and the energy–time uncertainty floor $\Delta E_* = \hbar/\beta_{\text{dS}}$ —using no Gibbs state, no occupation number, no temperature. What this route cannot avoid, and what Routes A–C also rely on, is the strictly weaker, *geometric* fact that Euclidean de Sitter has period

$$\beta_{\text{dS}} = \frac{2\pi}{H}. \quad (23)$$

Euclidean de Sitter is the round four-sphere of radius c/H ; the period (23) is a property of that

manifold. Thermality is *derived* from Eq. (23) via the KMS theorem, not the reverse. The four routes therefore do not share the premise of thermality—three assume it, one does not—but all four share the premise (23), a fact about the classical de Sitter background geometry. The cosmic-decoherence sector rests on that geometric premise, not on the hypothesis that the universe is in a hot thermal state. The granularity (22) is thus physically well-motivated and reduced to the single geometric premise (23); it is not derived with the rigor of a theorem.

3.3 The holographic mode count

The cosmic wavefunction does not have infinitely many branch-labelling degrees of freedom. The holographic principle, native to this framework through the holographic dark-energy paper, caps the information content of the de Sitter causal diamond at the Gibbons–Hawking entropy,

$$S_{\text{dS}} = \frac{A_H}{4\ell_P^2} = \frac{\pi c^5}{G\hbar H^2}, \quad (24)$$

with $\ell_P^2 = G\hbar/c^3$ the Planck area. This is the maximum number of independent which-branch registers the horizon can hold; finer modes would have to be stored on sub-Planckian horizon area, which the holographic bound forbids. The sum over branch-distinguishing modes therefore runs over exactly $N_{\text{modes}} = S_{\text{dS}}$ modes. Equivalently: the de Sitter infrared cutoff $k_H = 1/R_H$ removes super-horizon modes, the Planck scale cuts the ultraviolet end, and the number of independent modes in between is the horizon area in Planck units. The mode-counting statement and the holographic statement are the same statement. For $H = H_0$, Eq. (24) gives $S_{\text{dS}} \sim 2.5 \times 10^{122}$, consistent with standard estimates of the entropy of the observable universe [11].

3.4 Assembling the total rate

The per-mode rate follows from Eq. (18) with $\Delta m = \Delta m_{\text{dS}}$ —each thermal-mass branch quantum is itself a horizon-scale object, $d = R_H$, and so carries the horizon form factor $g(1)$:

$$\Gamma_{\text{per mode}} = g(1) \frac{G \Delta m_{\text{dS}}^2 H}{\hbar c} = g(1) \frac{G \hbar H^3}{4\pi^2 c^5} = g(1) \frac{t_P^2 H^3}{4\pi^2}, \quad (25)$$

using $t_P^2 = G\hbar/c^5$. Numerically, for $H = H_0$, this is $\sim 4 \times 10^{-142}$ Hz per mode—each individual mode decoheres on a timescale enormously longer than the Hubble time, as it must, since a single mode carries only one thermal quantum’s worth of mass difference.

The total cosmic decoherence rate is the sum of the per-mode rates. In the free-field (Gaussian) approximation the Bunch–Davies state factorizes mode by mode and the linearized graviton coupling is mode-diagonal, so the total decoherence exponent is the sum of per-mode exponents,

$$\Gamma_{\text{total}}(t) = \sum_{i=1}^{S_{\text{dS}}} \Gamma_i(t). \quad (26)$$

This is the sum of S_{dS} *independent* two-branch decoherence processes, each with the *same* elementary granularity Δm_{dS} —not a partition of M_U into S_{dS} pieces. The Δm^2 non-linearity in Eq. (1) acts within one mode and does not couple distinct modes. The validity of Eq. (26) beyond strict Gaussian order, and the smallness of the mode–mode coupling correction, are taken up in Section 4.1.

Combining Eqs. (25), (24) and (26),

$$\Gamma_{\text{total}} = S_{\text{dS}} \cdot \Gamma_{\text{per mode}} = \frac{\pi c^5}{G \hbar H^2} \cdot g(1) \frac{G \hbar H^3}{4\pi^2 c^5} = g(1) \frac{H}{4\pi}. \quad (27)$$

The cancellation is exact: G , \hbar and c all drop out, and the result depends only on H and on the form factor $g(1)$. The bare algebraic identity $S_{\text{dS}} \cdot t_P^2 H^3 / (4\pi^2) = H / (4\pi)$ is exact and $g(1)$ -independent—it is the $g(1) = 1$ idealization, the relation between the holographic mode count and the per-mode-rate *formula*. The physical rate carries the form factor:

$$\Gamma_{\text{total}} = g(1) \frac{H}{4\pi} = \left[1 - \frac{2}{\pi} \text{Si}(1)\right] \frac{H}{4\pi} \approx \frac{H}{10\pi} \approx 7 \times 10^{-20} \text{ Hz} \quad (H = H_0). \quad (28)$$

Equivalently $\Gamma_{\text{total}} t_H = g(1) / (4\pi) \approx 0.032$: the observable universe undergoes a fraction $\approx 1/30$ of a cosmic decoherence event per Hubble time.

Several internal checks support Eq. (28). *Dimensions*: $[\Delta m_{\text{dS}}] = [\hbar H / c^2] = \text{kg}$, $[\Gamma_{\text{per mode}}] = [t_P^2 H^3] = \text{s}^{-1}$, $[\Gamma_{\text{total}}] = [H] = \text{s}^{-1}$, with $g(1)$ dimensionless. *Causality*: $\Gamma_{\text{total}} = g(1) H / (4\pi) < H = c / R_H$, sub-causal by the factor $4\pi / g(1) \approx 10\pi$ —with $g(1) < 1$, even further below the bound than the $g(1) = 1$ idealization. *Minkowski limit*: as $H \rightarrow 0$, $\Delta m_{\text{dS}} \rightarrow 0$ and $S_{\text{dS}} \rightarrow \infty$, but the product $\Gamma_{\text{total}} = g(1) H / (4\pi) \rightarrow 0$ —no cosmic decoherence in flat space, as required, since flat space has no horizon and no branch granularity. *Laboratory consistency*: for a laboratory superposition the relevant Δm is the actual mass difference, vastly larger than $\Delta m_{\text{dS}} \sim 10^{-70} \text{ kg}$; the cosmic granularity is a floor, irrelevant when an experiment resolves branches far coarser than it, so the laboratory formula (1) with $g(0) = 1$ is recovered without conflict.

The numerical values in Eq. (28) follow from these expressions and the constants of Appendix A.

3.5 The contradiction resolved

The contradiction was that the naive cosmic rate $\sim 10^{103} \text{ Hz}$ violates causality by 121 orders of magnitude. Its resolution has the two parts this section and the previous one supply:

- *Structural* (Section 2): the Minkowski formula is the wrong propagator at $d \sim R_H$. The de Sitter propagator with its horizon cutoff $k_H = 1 / R_H$ gives a finite rate, proportional to Δm^2 and to H .
- *Granularity* (this section): the cosmic Δm is not M_U . It is the de Sitter thermal mass $\Delta m_{\text{dS}} = \hbar H / (2\pi c^2)$ per horizon mode. The S_{dS} horizon modes decohere independently and their rates add, giving $\Gamma_{\text{total}} = g(1) H / (4\pi)$.

The 10^{103} Hz number came from inserting $\Delta m = M_U$, which the quantum-cosmology analysis identifies as the wrong physical input—a maximally distant branch jump, not a branch granularity. With the propagator corrected and the granularity identified, the cosmic decoherence rate is finite, proportional to H , and manifestly sub-causal. The internal contradiction is removed.

4 Internal consistency

The result of Section 3 is a controlled approximation: linearized gravity, the Bunch–Davies Gaussian vacuum, a scalar proxy for the graviton, and a particular extraction of the rate from

the saturating mode integral. Each approximation is examined here for whether it could affect the cosmic rate (28). None of the four checks changes the rate; each leaves a stated residual, collected in Table 1.

4.1 Linear additivity of per-mode rates

Equation (26)—the statement that the S_{dS} horizon modes decohere into independent environments whose decoherence exponents add—is exact at strict Gaussian order. Mode–mode coupling enters only through graviton self-interaction, which is $O(G^2)$, parametrically subleading to the $O(G)$ leading coupling, exactly as for the G^1 scaling itself.

That subleading correction was computed rather than merely power-counted. An explicit leg-by-leg counting of powers of G in the connected two-vertex cut diagram finds the per-pair correction to be $O(\epsilon_{\text{dS}})$, with $\epsilon_{\text{dS}} = G\hbar H^2/(2\pi c^5) = 1/(2S_{\text{dS}})$ the de Sitter loop-counting parameter—one power of ϵ_{dS} , not the ϵ_{dS}^2 a naive estimate would suggest, because ϵ_{dS} is itself $O(G)$. One power is not, on its own, enough to defeat the combinatorial S_{dS}^2 pair count: a coherent same-sign sum over all pairs would give a correction of order $S_{\text{dS}} \epsilon_{\text{dS}} = O(1)$, and linear additivity would fail.

What saves additivity is a sign. The per-pair matrix element carries a factor $\cos\theta_{ij}$, so the signed pair sum is a structure-factor sum, generically incoherent. Writing $\chi = \langle e^{i\psi} \rangle$ for the single-mode characteristic function of the combined configuration-plus-mode-label phase, the correction is

$$\frac{\delta\Gamma_{\text{total}}}{\Gamma_{\text{total}}} \sim \frac{c_v}{4} \max(|\chi|^2, 1/S_{\text{dS}}), \quad (29)$$

with $c_v \approx 0.18$ a computed $O(1)$ coefficient. Additivity holds, $\delta\Gamma_{\text{total}}/\Gamma_{\text{total}} \ll 1$, iff $|\chi| \ll 1$. The de Sitter mode spectrum fixes $|\chi|$: the holographic horizon modes occupy a band in comoving wavenumber of width $\sim k_H S_{\text{dS}}^{1/3}$, the mode-label phase is the exact Bunch–Davies phase at the readout time, and the resulting closed-form characteristic function is $|\chi| \sim 3 S_{\text{dS}}^{-1/3} \sim 5 \times 10^{-41}$.

Proposition 4.1 (Linear additivity). *The per-mode decoherence exponents add up to a residual $\delta\Gamma_{\text{total}}/\Gamma_{\text{total}} \sim (c_v/4) |\chi|^2 \lesssim 10^{-81}$, negligible by a margin no plausible sharpening could erode.*

The readout time entering $|\chi|$ is fixed, not assumed. Reading out a mode’s which-path phase below horizon crossing is not a valid decoherent-histories coarse-graining: a sub-horizon Bunch–Davies mode oscillates and its phase time-averages away (the same convergence factor that suppresses sub-horizon contributions to the form factor), and the which-path record is registered only when the mode freezes and classicalizes at horizon crossing—the squeezing mechanism of inflationary decoherence [2]. Operationally, the crossed-product observer of Section 4.3 reads a mode when it crosses the observer’s horizon. The readout condition $|\tau_*| \gtrsim 1/k_H$ —carried through the earlier stages of this program as an explicit premise—is therefore derived within the standard inflationary-decoherence picture, and linear additivity holds unconditionally within it.

4.2 The spin-2 graviton

The derivation used a scalar proxy: each transverse-traceless graviton polarization treated as a massless minimally coupled scalar. Redoing the key computations with the full spin-2 graviton $h_{\mu\nu}^{\text{TT}}$, its two polarizations, and the appropriate tensor harmonics on de Sitter [6] confirms the proxy. The transverse-traceless graviton on de Sitter obeys, per polarization, *exactly* the

scalar-proxy mode equation (10) [5]. Consequently the form factor $g(x) = 1 - (2/\pi) \text{Si}(x)$ and its horizon value $g(1) = 0.398$ carry a spin-2 rescaling factor of exactly 1.000. The only quantitative refinement is the per-pair coefficient of Section 4.1, rescaled by the polarization factor $8/15$ from $c_v = 0.34$ (scalar proxy) to $c_v = 0.18$ —a change in a safe direction that does not affect the power of ϵ_{dS} . The cosmic rate (28) is unchanged. The scalar proxy is not an approximation here; for the mode structure that governs the form factor it is exact.

4.3 Gauge-invariant rate extraction

A decoherence “rate” computed on a cosmological background risks depending on the choice of time slicing. A foliation-dependent extraction bears this out: the proper-time and conformal-time observers give different $O(1)$ coefficients, a spread $g(1) \in [0.21, 0.43]$.

This is resolved by extracting the rate from an intrinsic, foliation-free clock. The de Sitter static-patch crossed-product algebra of observables [20, 21] is of Type II_1 , with a finite, observer-independent trace; its modular flow is the static-patch Killing flow. Reading the decoherence rate off the modular-flow derivative of the modular relative entropy makes no reference to any cosmological time slicing. The gauge-invariant rate so obtained is

$$\Gamma_{\text{inv}} = g(1) \frac{H}{4\pi}, \quad g(1) = 1 - \frac{2}{\pi} \text{Si}(1) = 0.398, \quad (30)$$

the energy-scale value of the form factor, pinned exactly. The earlier spread $[0.21, 0.43]$ is identified as an artifact of frame-dependent phase-averaging, which the modular extraction bypasses, and is eliminated.

One element of this extraction is not a gauge ambiguity but an irreducible input: the rate is per unit comoving proper time with the surface-gravity normalization $\kappa = H$ —which is again the de Sitter Euclidean period $2\pi/H$, the same geometric premise (23) that the granularity rests on. de Sitter homogeneity makes all comoving observers equivalent, so this is not a residual ambiguity; it is a framework input, and it is the *same* input already on the table.

4.4 All-orders secular control

The graviton on de Sitter has an infrared sector with secular growth: graviton self-interaction produces loop corrections growing as $\ln a$ at each order in perturbation theory [22, 23]. This must not invalidate the linearized calculation on the timescale of interest.

The first point is structural: the secular factor $\ln a$ multiplies *only* the $O(G^2)$ mode–mode coupling correction of Section 4.1, never the leading additive sum (27), which is built from the Gaussian Bunch–Davies state and the mode-diagonal linearized coupling and carries no self-interaction loop. The leading cosmic rate is untouched by secular growth at any order.

The second point concerns the timescale. The cosmic wavefunction decoheres at the *sum* rate over S_{dS} modes, which reaches $O(1)$ after a few Hubble times, $t_{\text{dec}} \sim 4\pi/H$, even though any single mode would take $\sim 10^{123}$ Hubble times in isolation. The per-mode time is a counterfactual that never enters, since all S_{dS} channels decohere in parallel. The secular factor is therefore evaluated at $t \sim t_{\text{dec}}$, where $\ln a = O(1)$.

This timescale argument can be promoted to an all-orders bound. The leading-log tower $L(x) = 1 + \sum_n c_{n,n} x^n$ has secular argument $x = \kappa^2 H^2 \ln a$ with $\kappa^2 H^2 \sim \epsilon_{\text{dS}}$; its $O(1)$ coefficients give a geometric bound $|L(x) - 1| \leq C|x|/(1 - |x|)$. On the cosmic-decoherence timescale the

argument is $x_{\text{dec}} \sim 5 \times 10^{-122}$, so the all-orders secular correction to the $O(G^2)$ additivity term is bounded far below unity, with many tens of orders of margin; the crossover at which it could reach $O(1)$ lies more than a hundred orders of magnitude beyond t_{dec} . This bound uses only the established leading-log structure and the gauge-invariant readout time of Section 4.3—no resummation is needed. A Starobinsky/dynamical-renormalization-group resummation strengthens it further, indicating the leading-log tower sums to a bounded equilibrium-type function rather than a divergent series.

Two residuals survive: the full spin-2 all-orders graviton resummation, and the complete subleading-log resummation. Both are open problems of the quantum-field-theory-on-de-Sitter literature, not of QGC; and the timescale bound, which does not resum the series, shows that neither can affect the cosmic rate—on $t \sim t_{\text{dec}}$ the secular argument is $x \sim 10^{-121}$ regardless of how the literature resolves them.

4.5 Status after the consistency checks

Table 1 collects the four checks. Each was passed; the cosmic rate $\Gamma_{\text{total}} = g(1)H/(4\pi)$ is unchanged by all of them. What remains is a small, explicitly named set of premises—the de Sitter Euclidean period (23), the standard inflationary-decoherence picture behind the horizon-crossing readout time, the holographic finiteness of the mode count, the decoherent-histories definition of relational time, and the framework-wide energy-to-rate identification—and it is to these that the discussion turns.

Table 1. *Outcome of the internal-consistency checks on the cosmic decoherence rate $\Gamma_{\text{total}} = g(1)H/(4\pi)$.*

Approximation	Finding	Residual
Linear additivity (4.1)	Mode–mode coupling is $O(G^2)$; the signed pair sum is incoherent; $\delta\Gamma_{\text{total}}/\Gamma_{\text{total}}$ bounded with many tens of orders of margin	Readout time derived at horizon crossing (standard classicalization); additivity unconditional within that picture
Scalar proxy (4.2)	TT graviton obeys exactly the scalar mode equation; $g(1)$ rescaling is 1.000	None; proxy is exact for the form factor
Rate gauge (4.3)	Type II_1 modular flow gives a foliation-free rate; $g(1) = 0.398$ pinned	The normalization $\kappa = H$, i.e. the period $2\pi/H$ — the same geometric premise
Secular growth (4.4)	Dresses only the $O(G^2)$ term; bounded with many tens of orders of margin on t_{dec}	Full spin-2 / subleading-log resummation — literature problems, cannot affect the rate

5 Discussion

This section states what the result is, what it rests on, and what would overturn it.

5.1 What is established

The structural claim is firm. The catastrophic cosmic rate of Eq. (2) is an artifact of using the Minkowski graviton propagator at separations $d \sim R_{\text{H}}$, where that propagator is not the

correct Green’s function. This is not a matter of interpretation: the Minkowski propagator has no infrared scale, the de Sitter propagator has a horizon, and at $d \sim R_H$ the difference is the whole effect. Recomputing the coherent-state decoherence functional on a de Sitter background produces the form factor $g(x) = 1 - (2/\pi) \text{Si}(x)$, which is finite for all x , recovers the canonical core paper’s Minkowski result as $x \rightarrow 0$, and vanishes for super-horizon separations. The divergence is gone, and gone for a stated structural reason.

The quantitative claim is more qualified. Given the de Sitter form factor, the holographic mode count (24), the branch granularity (22), and linear additivity, the cosmic rate is $\Gamma_{\text{total}} = g(1) H/(4\pi) \approx 7 \times 10^{-20}$ Hz by arithmetic. The form factor $g(1) = 0.398$ is computed in closed form and pinned gauge-invariantly. The granularity Δm_{dS} is reduced—through four converging routes—to the single geometric premise that Euclidean de Sitter has period $2\pi/H$. Linear additivity holds with the readout time derived at horizon crossing (Section 4.1). The internal-consistency program of Section 4 closes the scalar-proxy and secular-growth approximations outright. What remains is a residual $O(1)$ uncertainty in the granularity coefficient (the Route-A mode-weighting ambiguity, a factor in ~ 0.3 – 1) and the named premises below.

The verbs are chosen precisely. The structural cure of the contradiction *is shown*. The de Sitter form factor *is derived* from the coherent-state functional. The granularity Δm_{dS} *is reduced* to a geometric premise—well-motivated and convergent, not proven with the rigor of a theorem. The total rate $\Gamma_{\text{total}} = g(1) H/(4\pi)$ *follows* from those inputs. The cosmic rate is not proven; what is established is the removal of an internal inconsistency, replacing a 121-order causality violation with a finite, sub-causal, premise-labelled result.

5.2 The residuals, stated openly

Three classes of residual survive, marking where the result is soft.

The readout time. Linear additivity (Section 4.1) holds because the single-mode characteristic function $|\chi| \sim 5 \times 10^{-41}$ is small at the horizon-crossing readout time. That readout time is derived, not assumed: a sub-horizon mode’s which-path phase time-averages away, the record is registered only when the mode freezes at horizon crossing [2], and the crossed-product observer of Section 4.3 supplies the operational reading. The derivation stands within the standard inflationary-decoherence picture; a reader who rejects that picture reinstates the readout time as a premise, and a readout forced below horizon crossing would give $|\chi| = O(1)$ and break additivity.

The energy-to-rate identification. The whole QGC decoherence program, beginning with the canonical core paper [1], rests on the identification of a gravitational energy scale with a decoherence rate: the influence functional supplies a saturating energy scale, and a Hamiltonian constraint converts it into a linearly growing rate, $\Gamma = E_G/\hbar$ rather than the bare modular gap frequency or its square. The de Sitter calculation inherits this identification; it does not establish it. This is the central G^1 assumption of the framework, not specific to de Sitter or to cosmic scale, and the deepest open question on which the present result—like the canonical core paper [1]—depends. The present paper does not close it.

Literature-level open problems. The all-orders secular control of Section 4.4 leaves two residuals—the full spin-2 all-orders graviton resummation and the complete subleading-log resummation—that are open problems of quantum field theory on de Sitter space, not of QGC. The timescale bound shows that neither can affect the cosmic rate, because on the relevant timescale the secular argument is $\sim 10^{-121}$ regardless of how those problems are resolved. They are residuals of the surrounding literature that the present result is insulated from, not residuals of the present result.

The largest caveat is not a residual of this calculation but its setting: QGC is a framework without direct experimental confirmation. This paper resolves a contradiction *internal* to that framework. It strengthens QGC by removing a self-inconsistency; it does not, and cannot, establish QGC. The cosmic decoherence rate $\Gamma_{\text{total}} = g(1)H/(4\pi)$ is a prediction *of* QGC, conditional on QGC.

5.3 Relation to prior work

Decoherence in quantum cosmology has a long history. The recognition that the universe’s wavefunction acquires classical behaviour through the decoherence of its coarse-grained histories goes back to Zeh [12], Kiefer [13], and Halliwell [14, 16], and the decoherent-histories formalism we use to define a cosmic branch is that of Gell-Mann and Hartle [15]. Barvinsky, Kamenshchik and Kiefer [17] computed decoherence of the cosmological wavefunction by its field environment. The present paper does not reopen the foundations of that program; it adopts the decoherent-histories definition of a branch and asks a sharper, quantitative question—the branch *granularity*—that the QGC decoherence formula makes well posed.

The granularity itself draws on the Gibbons–Hawking thermodynamics of the de Sitter horizon [7], on the thermodynamic reading of the Einstein equation [8, 9], and on Zurek’s einselection [18, 19]; the holographic mode count is the Gibbons–Hawking entropy used as a bound on which-branch information [10]. The gauge-invariant rate extraction uses the de Sitter algebra of observables of Chandrasekaran, Longo, Penington and Witten [20] and the crossed-product construction of Witten [21]. The graviton infrared sector and its secular growth are the subject of the Tsamis–Woodard program [22, 23, 24]. A separate graviton channel—bremsstrahlung emission into an ambient graviton bath, treated by a quantum Boltzmann equation [27]—contributes at G^2 in thermal equilibrium and is suppressed by the bath occupation number, remaining far below $\Gamma_{\text{total}} = g(1)H/(4\pi)$ at the graviton occupations relevant here.

Most directly related is the recent local calculation of the decoherence of quantum superpositions in de Sitter spacetime by Li [3], which independently obtains a de Sitter horizon form factor with a pinned prefactor for the same physical setting. Our $g(x) = 1 - (2/\pi)\text{Si}(x)$ and its $g(1) = 0.398$ are consistent with that line of work; the present paper’s distinct contribution is the use of the form factor to remove the QGC cosmic-rate causality paradox and to fix the branch granularity, rather than the form factor in isolation.

What is new here is not any one of those ingredients but their assembly into the resolution of a specific, quantitative contradiction. The contribution is to show that the QGC laboratory decoherence formula, taken to cosmic scale, produces a causality paradox; that the paradox is a propagator artifact; and that the corrected calculation, fed the branch granularity that quantum cosmology supplies, returns a finite sub-causal rate $\Gamma_{\text{total}} = g(1)H/(4\pi)$.

5.4 Falsifiability

A resolution of an internal inconsistency should still expose itself to refutation. There are three distinct ways the picture of this paper could be shown wrong.

By a calculation. The result makes a sharp internal claim: the cosmic decoherence rate of QGC is $\Gamma_{\text{total}} = g(1) H/(4\pi)$, with $g(1) = 1 - (2/\pi) \text{Si}(1)$ a closed-form number. The chain leading to it—the de Sitter form factor, the per-mode rate (25), the holographic mode count, linear additivity—is explicit and reproducible. A computation showing that the de Sitter coherent-state functional does *not* yield $g(x) = 1 - (2/\pi) \text{Si}(x)$, or that the signed mode-mode pair sum is in fact coherent (so that additivity fails and $\delta\Gamma_{\text{total}}/\Gamma_{\text{total}} = O(1)$), or that the branch granularity is parametrically different from $\hbar H/(2\pi c^2)$, would overturn the result. The most exposed link is the readout time: a demonstration that the decoherent-histories readout is forced below horizon crossing—against the standard classicalization picture on which its derivation rests—would make $|\chi| = O(1)$ and break linear additivity. This is the sharpest concrete target for an adversarial calculation.

By the laboratory. The cosmic result and the laboratory prediction of the canonical core paper [1] are not independent: the de Sitter form factor reduces to the laboratory formula (1) as $Hd/c \rightarrow 0$, with $g(0) = 1$ exact. The laboratory G^1 rate is testable, with current technology striving toward the required regime; observation of G^2 scaling rather than G^1 would falsify the constrained-influence-functional mechanism of the canonical core paper [1]. Because the cosmic calculation uses the *same* mechanism—the same constraint-extracted rate, the same coherent-state overlap, only on a de Sitter rather than a Minkowski background—a laboratory falsification of G^1 would also remove the basis of the cosmic result. The cosmic and laboratory predictions stand or fall together on the mechanism, even though the cosmic rate is far too slow to observe directly.

By cosmology. The predicted rate $\Gamma_{\text{total}} = g(1) H/(4\pi) \approx H/(10\pi)$ is too slow to act as a direct observational signal: a fraction $\sim 1/30$ of a decoherence event per Hubble time is not something a measurement resolves. Its content is instead a sharp internal statement—the universe decoheres at a definite fraction of the Hubble rate, with the coefficient fixed by $g(1)$ and the geometry. The result also makes a structural prediction that is in principle distinguishing: cosmic gravitational decoherence is set by the de Sitter scale H and not by the total mass M_U ; configurations separated by more than a Hubble radius do not dynamically decohere one another. A demonstration that cosmic-scale superpositions decohere at a rate controlled by M_U rather than H , or across super-horizon separations, would contradict the picture. We do not overstate this: there is at present no experiment that probes cosmic-scale gravitational decoherence, and we claim none.

The result’s primary exposure is to calculation. It is a falsifiable claim about what QGC predicts, with an explicit derivation chain and one explicitly marked soft link; it shares the laboratory program’s exposure through the common G^1 mechanism; and it removes a contradiction that, left standing, would have been reason to doubt the framework.

6 Conclusion

The gravitational decoherence formula $\Gamma = G \Delta m^2 / (\hbar d)$ of Quantum-Geometric Correspondence, applied without examination at cosmic scale, returns a rate $\sim 10^{103}$ Hz—121 orders of magnitude above the causal bound c/R_H . This was a genuine internal contradiction of the framework. This paper has resolved it.

The resolution is in two parts. The divergence is a propagator artifact: the formula is built on the Minkowski graviton two-point function, which has no infrared scale, and at separations approaching the Hubble radius that is the wrong Green’s function. Recomputed on a de Sitter background, the coherent-state decoherence functional acquires a closed-form form factor $g(x) = 1 - (2/\pi) \text{Si}(x)$, which recovers the laboratory result for $Hd/c \ll 1$, is finite at the horizon, and vanishes for super-horizon separations—the de Sitter horizon supplying the infrared cutoff that flat space lacked. Separately, the cosmic decoherence rate is not governed by the total mass M_U but by the elementary branch granularity of the cosmic wavefunction, identified by several converging arguments with the de Sitter thermal mass $\Delta m_{\text{dS}} = \hbar H / (2\pi c^2)$ per horizon mode. Summed over the S_{dS} holographic horizon modes, the cosmic rate is

$$\Gamma_{\text{total}} = g(1) \frac{H}{4\pi} \approx \frac{H}{10\pi} \approx 7 \times 10^{-20} \text{ Hz} \quad (\text{up to an } O(1) \text{ granularity coefficient}), \quad (31)$$

finite, proportional to H , and comfortably sub-causal.

An internal-consistency program established that this rate is unchanged by the remaining approximations: the scalar proxy is exact for the graviton mode structure that governs the form factor; the rate is gauge-invariant under the de Sitter modular flow; linear additivity of per-mode rates holds with many tens of orders of margin; and the known de Sitter graviton secular growth dresses only an $O(G^2)$ correction and is bounded far below any level that could matter. The result rests on a small, named set of premises—chiefly the geometric de Sitter Euclidean period $2\pi/H$, the standard inflationary-decoherence picture from which the horizon-crossing readout time is derived, and the framework-wide identification of a gravitational energy scale with a decoherence rate—and we have stated each one openly.

The result strengthens QGC by removing a self-inconsistency; it does not establish the framework, which awaits experimental test. Its sharpest exposure is to calculation: the derivation chain is explicit and reproducible, its softest link the horizon-crossing readout time, now derived within the standard classicalization picture rather than assumed. Through the common G^1 mechanism it also shares the falsifiability of the laboratory program of the canonical core paper [1]. What the paper provides is a finite, premise-labelled, causally consistent answer to a question the framework had previously left as an open contradiction: the universe decoheres gravitationally, into its own horizon, at a definite fraction of the Hubble rate.

Appendices

A Conventions and notation

This appendix collects the conventions used throughout the paper.

We work in SI units. The fundamental constants are Newton's constant $G = 6.674 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$, the reduced Planck constant $\hbar = 1.055 \times 10^{-34} \text{ J}\cdot\text{s}$, and the speed of light $c = 2.998 \times 10^8 \text{ m/s}$. From these we form the Planck length $\ell_P = \sqrt{G\hbar/c^3} = 1.616 \times 10^{-35} \text{ m}$, the Planck time $t_P = \ell_P/c$ so that $t_P^2 = G\hbar/c^5$, and the Planck mass $m_P = \sqrt{\hbar c/G} = 2.176 \times 10^{-8} \text{ kg}$.

The cosmological inputs are the Hubble rate H , taken at its present value $H_0 \approx 2.2 \times 10^{-18} \text{ s}^{-1}$ for numerical estimates; the Hubble radius $R_H = c/H$; the Hubble time $t_H = 1/H$; and the horizon area $A_H = 4\pi R_H^2$. The de Sitter (Gibbons–Hawking) temperature is $T_{\text{dS}} = \hbar H/(2\pi k_B)$, and the de Sitter entropy is $S_{\text{dS}} = A_H/(4\ell_P^2) = \pi c^5/(G\hbar H^2)$.

We adopt the metric signature $(-, +, +, +)$. The de Sitter background is written in flat slicing as $ds^2 = -c^2 dt^2 + a(t)^2 d\mathbf{x}^2$ with $a(t) = e^{Ht}$, or in conformal time τ (defined by $a d\tau = c dt$) as $ds^2 = a(\tau)^2(-c^2 d\tau^2 + d\mathbf{x}^2)$ with $a(\tau) = -1/(H\tau)$ and $\tau \in (-\infty, 0)$. The linearized graviton is a perturbation $h_{\mu\nu}$ of this background; in transverse-traceless gauge each polarization is treated, in the scalar proxy, as a massless minimally coupled scalar Φ .

The primary quantities of the decoherence calculation are the branch mass difference Δm , the proper separation $d = a(\tau) |\mathbf{x}_L - \mathbf{x}_R|$, the comoving wavenumber k , the physical wavenumber $k_{\text{phys}} = k/a$, and the comoving horizon wavenumber $k_H = aH/c$. The dimensionless de Sitter parameter is $x = Hd/c = d/R_H$. The decoherence exponent $\Gamma(t)$ enters the off-diagonal density-matrix element as $\rho_{LR}(t) = \rho_{LR}(0) e^{-\Gamma(t)}$; the decoherence *rate* is $\Gamma(t)/t$ in the linear-growth regime.

The sine integral is $\text{Si}(x) = \int_0^x (\sin u/u) du$, with $\text{Si}(0) = 0$ and $\text{Si}(\infty) = \pi/2$. The de Sitter form factor is $g(x) = 1 - (2/\pi) \text{Si}(x)$, with $g(0) = 1$, $g(1) = 0.3977\dots$, and $g(x \rightarrow \infty) = 0$.

B The de Sitter form factor

This appendix derives the closed form (16) of the de Sitter form factor $g(x)$ and records the limiting behaviour used in the main text.

B.1 The form factor as an energy-scale ratio

The free-field de Sitter decoherence functional grows logarithmically, not linearly, in time; the linear-in- t decoherence *rate* is set by the gravitational energy scale E_G through the constraint-extraction mechanism—the influence functional supplies the energy scale, the Hamiltonian constraint supplies the rate. The form factor $g(Hd/c)$ of the factorized law (15) is accordingly the ratio of the de Sitter gravitational energy scale at separation d to its Minkowski counterpart.

The d -dependent part of the gravitational interaction energy of the two mass configurations is built from the Coulomb propagator $1/k^2$; its dependence on the separation is carried by the radial integral

$$E_G^{\text{Mink}} \propto \int_0^\infty dk \frac{\sin kd}{kd} = \frac{\pi}{2d}. \quad (32)$$

On de Sitter, modes with comoving wavenumber $k < k_H = 1/R_H$ are super-horizon: their transient factor is frozen and they do not contribute to the time-dependent energy scale. The de

Sitter energy scale is the same integral carrying the horizon infrared cutoff,

$$E_G^{\text{dS}}(x) \propto \int_{k_H}^{\infty} dk \frac{\sin kd}{kd} = \frac{1}{d} \left(\frac{\pi}{2} - \text{Si}(x) \right), \quad x = k_H d = \frac{Hd}{c}, \quad (33)$$

where the evaluation uses $\int_x^{\infty} u^{-1} \sin u \, du = \pi/2 - \text{Si}(x)$.

B.2 The closed form

The form factor is the ratio of (33) to (32):

$$g(x) = \frac{E_G^{\text{dS}}(x)}{E_G^{\text{Mink}}} = \frac{\pi/2 - \text{Si}(x)}{\pi/2} = 1 - \frac{2}{\pi} \text{Si}(x). \quad (34)$$

No free constant enters: the Minkowski normalization $g(0) = 1$ is automatic, since as $H \rightarrow 0$ the cutoff $k_H \rightarrow 0$ and (33) reduces identically to (32).

B.3 Limits

From Eq. (34) and the standard values of the sine integral:

- $\text{Si}(0) = 0$, so $g(0) = 1$. This is the Minkowski limit; the de Sitter calculation reproduces the canonical core paper's result $\Gamma = G \Delta m^2 / (\hbar d)$ exactly.
- Small- x expansion: $\text{Si}(x) = x - x^3/18 + O(x^5)$, hence

$$g(x) = 1 - \frac{2}{\pi} x + \frac{1}{9\pi} x^3 + O(x^5). \quad (35)$$

The leading correction to the laboratory formula is linear in Hd/c and, for any laboratory separation where $Hd/c \sim 10^{-26}$, entirely negligible.

- At the horizon, $\text{Si}(1) = 0.946083\dots$, so

$$g(1) = 1 - \frac{2}{\pi} (0.946083\dots) = 0.39768\dots, \quad (36)$$

the value used throughout as $g(1) \approx 0.398$.

- $\text{Si}(\infty) = \pi/2$, so $g(x \rightarrow \infty) = 1 - (2/\pi)(\pi/2) = 0$. Super-horizon separations produce no linear-in- t decoherence.

The function g decreases monotonically from 1 to 0 as x runs from 0 to ∞ , since $g'(x) = -(2/\pi) \sin(x)/x$ is negative for $0 < x < \pi$ and the subsequent oscillations of $\sin(x)/x$ are too small to reverse the trend; g is positive throughout because $\text{Si}(x) < \pi/2$ for all finite x .

C The holographic mode count and the cancellation

This appendix records the algebra behind the cancellation in Eq. (27) and the numerical estimates quoted in the main text.

C.1 The exact cancellation

The per-mode rate (25) is, with $t_P^2 = G\hbar/c^5$,

$$\Gamma_{\text{per mode}} = g(1) \frac{G \Delta m_{\text{dS}}^2 H}{\hbar c}, \quad \Delta m_{\text{dS}} = \frac{\hbar H}{2\pi c^2}. \quad (37)$$

Substituting the granularity,

$$\Gamma_{\text{per mode}} = g(1) \frac{G H}{\hbar c} \cdot \frac{\hbar^2 H^2}{4\pi^2 c^4} = g(1) \frac{G \hbar H^3}{4\pi^2 c^5} = g(1) \frac{t_P^2 H^3}{4\pi^2}. \quad (38)$$

The holographic mode count (24) is

$$S_{\text{dS}} = \frac{A_H}{4\ell_P^2} = \frac{4\pi(c/H)^2}{4(G\hbar/c^3)} = \frac{\pi c^5}{G \hbar H^2}. \quad (39)$$

The product is

$$\Gamma_{\text{total}} = S_{\text{dS}} \cdot \Gamma_{\text{per mode}} = \frac{\pi c^5}{G \hbar H^2} \cdot g(1) \frac{G \hbar H^3}{4\pi^2 c^5} = g(1) \frac{H}{4\pi}. \quad (40)$$

Every factor of G , \hbar and c cancels. The combination $S_{\text{dS}} \cdot t_P^2 H^3 / (4\pi^2) = H / (4\pi)$ is an exact algebraic identity, independent of the form factor; it is the $g(1) = 1$ idealization. The physical rate carries $g(1)$, giving $\Gamma_{\text{total}} = g(1) H / (4\pi)$.

This cancellation is not an accident of bookkeeping. It expresses the structural fact that the holographic mode count and the per-mode rate are built from the same horizon: the number of which-branch registers and the rate at which each is written are both set by the de Sitter scale, so their product depends only on H .

C.2 Numerical values

At the present epoch, $H = H_0 \approx 2.20 \times 10^{-18} \text{ s}^{-1}$:

$$T_{\text{dS}} = \frac{\hbar H_0}{2\pi k_B} \approx 2.7 \times 10^{-30} \text{ K}, \quad (41)$$

$$\Delta m_{\text{dS}} = \frac{\hbar H_0}{2\pi c^2} \approx 4 \times 10^{-70} \text{ kg}, \quad (42)$$

$$S_{\text{dS}} = \frac{\pi c^5}{G \hbar H_0^2} \approx 2.5 \times 10^{122}, \quad (43)$$

$$\Gamma_{\text{per mode}} = g(1) \frac{t_P^2 H_0^3}{4\pi^2} \approx 4 \times 10^{-142} \text{ Hz}, \quad (44)$$

$$\Gamma_{\text{total}} = g(1) \frac{H_0}{4\pi} \approx 7 \times 10^{-20} \text{ Hz}. \quad (45)$$

The $g(1) = 1$ idealization would give $H_0 / (4\pi) \approx 1.75 \times 10^{-19} \text{ Hz}$; the physical rate is smaller by the factor $g(1) \approx 0.398$. The causal bound is $c/R_H = H_0 \approx 2.2 \times 10^{-18} \text{ Hz}$, so $\Gamma_{\text{total}}/H_0 = g(1)/(4\pi) \approx 0.032$: the cosmic rate is sub-causal by a factor of roughly 30. The mode count $S_{\text{dS}} \approx 2.5 \times 10^{122}$ agrees with standard estimates of the entropy of the observable universe [11].

All values above follow from the displayed expressions and the constants of Appendix A.

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