Emergent Gravity from Entanglement Equilibrium: MOND Phenomenology and Covariant Extension

Quantum-Geometric Correspondence

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Abstract

The MOND acceleration scale $a_0 = cH_0/(2\pi) \approx 1.08 \times 10^{-10}$ m/s^2 emerges from de Sitter horizon thermodynamics with no adjustable parameters, matching observed values within 10%. The de Sitter cosmological horizon establishes a thermal bath at the Gibbons–Hawking temperature $T_{\text{dS}} = \hbar H_0/(2\pi k_B)$, generating extensive (volume-law) entropy in addition to the standard area-law contribution. We postulate that when local entanglement equilibrium—the stationarity of generalized entropy under allowed variations—fails at low accelerations, the volume entropy becomes dynamically relevant and produces MOND-like corrections; this failure of local equilibrium is a physical hypothesis, not a derived result. The same construction gives the Tully–Fisher relation $v^4 = GMa_0$ and a clean regime separation: general relativity remains exact when $\epsilon = g_N/a_0 \gg 1$ (solar system, pulsars) and corrections appear only when $\epsilon \lesssim 1$ (galaxy outskirts), with the volume entropy saturating the Bousso covariant entropy bound.

We also sketch Entanglement-Elastic Gravity (EEG), a candidate covariant completion in which an elastic displacement field ψ encoding entanglement strain sources a modified Einstein equation. We present EEG as a schematic proposal rather than a finished theory: the quadratic action used here is a linearized caricature, and reproducing the deep-MOND limit requires a nonlinear (Bekenstein–Milgrom/AQUAL-type) kinetic functional, which we leave to future work. What the framework fixes independent of that completion are the acceleration scale a_0 and the elastic modulus $\kappa = c^4/(8\pi G a_0)$. Beyond the rotation curves and Tully–Fisher relation it reproduces, the framework points to a weak-lensing slip $|\Phi - \Psi|/|\Phi| \sim 15\%$ beyond 50 kpc and a growth-rate suppression $\Delta f\sigma_8 \sim -0.03$ at $z \approx 0.8$—qualitative expectations, not yet quantitatively derived, that are targets for Euclid and DESI.

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1 Introduction

Galaxy rotation curves stay flat far beyond the visible disk, where Newtonian gravity predicts a Keplerian $v \propto r^{-1/2}$ falloff, and the deviation sets in once the local acceleration drops below $a_0 \approx 1.2 \times 10^{-10}$ m/s^2 across galaxies of every size and type. We show that this scale is fixed by the present cosmological horizon: $a_0 = cH_0/(2\pi) \approx 1.08 \times 10^{-10}$ m/s^2, with no adjustable parameter.

Milgrom [3] read the same data as a modification of dynamics: when $g_N = GM/r^2$ falls below a critical a_0, the acceleration crosses over to $g \approx \sqrt{g_N a_0}$. With $a_0 \approx 1.2 \times 10^{-10}$ m/s^2 this single ansatz reproduces flat rotation curves, the baryonic Tully–Fisher relation $v^4 \propto M$, and the galactic scaling laws [8, 9]. The dark-matter alternative fits the same observations but must impose the tightness of the Tully–Fisher relation as a correlation between visible and dark components. What MOND lacks is a value for a_0: it is fitted, not predicted.

1.1 Result and status

We obtain a_0 from the thermodynamics of de Sitter space. Jacobson [1] derived the Einstein equations as the equilibrium condition $\delta S_{\text{gen}} = 0$ for the generalized entropy $S_{\text{gen}} = A/(4\ell_P^2) + S_{\text{bulk}}$ across local causal horizons, treating gravity as an equation of state of entanglement. That derivation uses flat patches at large distance. Our universe is asymptotically de Sitter: a cosmological horizon at $R_{\text{H}} = c/H_0$ radiates at the Gibbons–Hawking temperature $T_{\text{dS}} = \hbar H_0/(2\pi k_B)$ and holds its interior in a thermal state. The thermal bath carries volume-law entropy $S_{\text{vol}} = s_\Lambda V$ in addition to the area term, and this changes the entropy accounting that fixes the dynamics.

A mass M in this bath displaces entropy $\Delta S = Mc^2/T_{\text{dS}}$ from the surrounding volume (Sec. 4). We postulate that when the local volume cannot accommodate this displacement, local thermodynamic equilibrium fails and the gravitational dynamics change. This failure is a physical hypothesis, not a derived result; it occurs precisely when g_N falls below $a_0 = cH_0/(2\pi)$. Above a_0 the displacement is absorbed by small curvature adjustments and general relativity applies; below it the deficit spreads non-locally through the de Sitter volume and produces $g \propto \sqrt{g_N}$.

The acceleration scale and the elastic modulus $\kappa = c^4/(8\pi G a_0)$ are fixed independent of any covariant completion. The interpolation function $\mu(x)$ and the nonlinear kinetic functional needed to reproduce the deep-MOND limit are not; we mark these gaps explicitly where they arise.

1.2 Plan

Section 2 establishes the de Sitter horizon thermodynamics — the Gibbons–Hawking temperature and the volume entropy density. Section 3 shows that the total volume entropy saturates the Bousso covariant entropy bound at the cosmological scale, removing the apparent conflict with holography. Section 4 derives a_0 from entropy equilibrium and recovers the Tully–Fisher relation. Section 5 gives the ϵ-expansion that separates the general-relativistic and MOND regimes and places the solar system safely in the former. Section 6 sketches Entanglement-Elastic Gravity (EEG), a candidate covariant completion in which an elastic field ψ sources a modified Einstein equation. Section 7 collects the testable predictions; Section 8 states open problems and outlook.

Series context. This paper belongs to the Quantum-Geometric Correspondence series. The canonical core paper [15] presents the axiomatic framework and develops gravitational decoherence from the same entropy principles; companion work derives holographic dark energy and the entanglement-decoherence prediction. Each paper is self-contained but cross-references the others.

2 De Sitter Thermodynamics

The positive cosmological constant Λ introduces a horizon that radiates thermally. This section fixes the de Sitter temperature, entropy, and the energy and acceleration scales they set; these underpin the MOND derivation of Sec. 4.

2.1 The Static Patch and Cosmological Horizon

De Sitter space is the maximally symmetric solution of $G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$. In static coordinates centered on a freely falling observer,

$$ds^2 = -\left(1 - \frac{r^2}{R_{\rm H}^2}\right) c^2 dt^2 + \frac{dr^2}{1 - r^2/R_{\rm H}^2} + r^2 d\Omega^2, \tag{1}$$

with de Sitter radius

$$R_{\rm H} = \sqrt{\frac{3}{\Lambda}} = \frac{c}{H_0}. \tag{2}$$

At $r = R_{\rm H}$ the coefficient g_{tt} vanishes and g_{rr} diverges: a coordinate horizon, not a singularity, beyond which events are causally disconnected from $r = 0$. For $H_0 \approx 70$ km/s/Mpc, $R_{\rm H} \approx 14$ billion light-years.

The region $r < R_{\rm H}$ admits the timelike Killing vector ∂_t and hence a conserved energy and a well-defined thermodynamics. It encloses proper volume $V_H = (4\pi/3)R_{\rm H}^3$. The de Sitter horizon is a future boundary: signals approaching it are redshifted to zero frequency.

2.2 The Gibbons-Hawking Temperature

Proposition 2.1 (Gibbons–Hawking temperature)**.** *The de Sitter horizon holds its interior at temperature*

$$T_{\rm dS} = \frac{\hbar H_0}{2\pi k_B} \approx 2.76 \times 10^{-30}\ K. \tag{3}$$

Proof. Three independent computations agree [2].

Surface gravity. The Killing horizon strength $\kappa_{\rm dS}$ is fixed by $\nabla_\mu(k_\nu k^\nu) = -2\kappa_{\rm dS} k_\mu$; for the metric (1), $\kappa_{\rm dS} = cH_0$ (Appendix A.1), and the universal relation $T = \hbar\kappa_{\rm dS}/(2\pi c k_B)$ gives (3).

Euclidean continuation. Under $t \to -i\tau$ the metric is Riemannian with a conical singularity at $r = R_{\rm H}$ unless τ has period $\beta = 2\pi/H_0$; regularity fixes this period, and $T = \hbar/(2\pi k_B \beta) = \hbar H_0/(2\pi k_B)$.

Unruh effect. A static observer at $r < R_{\rm H}$ accelerates against the expansion; as $r \to R_{\rm H}$ the proper acceleration diverges, and the Unruh temperature, redshifted to the observer, approaches (3). □

The symbol κ without subscript is reserved for the elastic modulus $\kappa \equiv c^4/(8\pi G a_0)$ of Sec. 6.

2.3 The First Law and Horizon Entropy

The cosmological horizon carries entropy $S_{dS} = A_H/(4\ell_{\rm P}^2) \approx 10^{122}$ in natural units, the largest entropy in the observable universe. Its thermodynamics is the first law

$$dE_{dS} = T_{\rm dS}\, dS_{dS}, \tag{4}$$

with E_{dS} the vacuum energy in the static patch: adding interior energy increases the horizon area. The horizon acts as a heat reservoir at $T_{\rm dS}$, in equilibrium with the interior.

2.4 The Characteristic Energy and Acceleration Scales

The Gibbons–Hawking temperature sets the energy scale

$$E_{dS} = k_B T_{\rm dS} = \frac{\hbar H_0}{2\pi} \approx 3.6 \times 10^{-52}\ {\rm J}. \tag{5}$$

Converting to an acceleration via the Planck mass $m_{\rm P} = \sqrt{\hbar c/G}$ and Planck length $\ell_{\rm P} = \sqrt{\hbar G/c^3}$,

$$a_0 = \frac{E_{dS}}{m_{\rm P}\ell_{\rm P}} = \frac{cH_0}{2\pi} \approx 1.08 \times 10^{-10}\ {\rm m/s}^2. \tag{6}$$

This is the MOND acceleration scale. The coincidence $a_0 \sim cH_0$, noted since Milgrom 1983, is here a consequence of de Sitter thermodynamics: the Gibbons–Hawking temperature fixes the acceleration at which gravity changes character. The entropy-budget argument underlying this identification is given in Sec. 4.

3 Volume-Law Entropy in de Sitter Space

In flat space the entropy of a gravitational region obeys an area law. The de Sitter thermal bath adds a volume-law term. This section computes that term and shows it saturates — rather than violates — the Bousso covariant entropy bound at the cosmological horizon.

3.1 The de Sitter Vacuum as a Thermal State

The Hilbert space of a quantum field in de Sitter factorizes as $\mathcal{H}_{\rm full} = \mathcal{H}_{\rm in} \otimes \mathcal{H}_{\rm out}$ across the cosmological horizon. The global vacuum is entangled across that boundary,

$$|0\rangle = \sum_n e^{-\pi E_n/\hbar H_0} |n\rangle_{\rm in} \otimes |n\rangle_{\rm out}, \tag{7}$$

so tracing over the inaccessible super-horizon modes gives a thermal reduced state,

$$\rho_{\rm in} = {\rm Tr}_{\rm out}|0\rangle\langle 0| = \frac{1}{Z}\sum_n e^{-2\pi E_n/\hbar H_0} |n\rangle_{\rm in}\langle n|, \tag{8}$$

a Boltzmann distribution at the Gibbons–Hawking temperature $T_{\rm dS} = \hbar H_0/(2\pi k_B)$. The horizon holds its interior in thermal equilibrium.

3.2 The Volume Entropy Density

A thermal state of energy density ρ at temperature T has entropy density $s = \rho/T$. The de Sitter vacuum energy density is

$$\rho_\Lambda = \frac{\Lambda c^4}{8\pi G} = \frac{3c^2 H_0^2}{8\pi G}, \tag{9}$$

using $\Lambda = 3H_0^2/c^2$ for a flat universe approaching de Sitter at late times. Dividing by $T_{\rm dS}$,

$$s_\Lambda = \frac{\rho_\Lambda}{T_{\rm dS}} = \frac{3c^2 H_0^2}{8\pi G} \cdot \frac{2\pi k_B}{\hbar H_0} = \frac{3c^2 H_0 k_B}{4G\hbar}, \tag{10}$$

numerically $s_\Lambda \approx 2.2 \times 10^{43}\ k_B\,\mathrm{m}^{-3}$. Because it is uniform, this density is invisible to local experiments and enters only when significant volume is enclosed.

3.3 Total Entropy: Area Plus Volume

A sphere of radius r in the static patch carries area-law and volume-law contributions,

$$S_{dS}(r) = \frac{A(r)c^3}{4G\hbar} + s_\Lambda V(r) = \frac{\pi r^2 c^3}{G\hbar} + \frac{\pi c^2 H_0 k_B r^3}{G\hbar}. \tag{11}$$

The area term dominates for $r \ll R_{\rm H}$, the volume term for $r \gg R_{\rm H}$; the crossover is at $r \sim c/H_0 = R_{\rm H}$.

3.4 Consistency with Holography

For large regions $S_{\rm vol} > S_{\rm area}$, which appears to violate the holographic bound $S \le A/(4\ell_{\rm P}^2)$. The resolution is the Bousso covariant entropy bound [10]: for a surface B, the entropy on a light sheet (a null hypersurface of non-expanding orthogonal geodesics) satisfies $S \le A(B)/(4\ell_{\rm P}^2)$. The bound constrains light sheets, not spatial volumes; a spatial volume may exceed it.

Proposition 3.1 (Saturation at the cosmological horizon)**.** *The total volume entropy within the static patch equals the horizon entropy.*

Proof. The interior volume entropy is

$$S_{\rm vol}^{\rm total} = s_\Lambda \cdot V_H = \frac{3c^2 H_0 k_B}{4G\hbar} \cdot \frac{4\pi c^3}{3H_0^3} = \frac{\pi c^5 k_B}{G\hbar H_0^2}, \tag{12}$$

and the horizon entropy is

$$S_H = \frac{A_H}{4\ell_{\rm P}^2} = \frac{4\pi R_{\rm H}^2 \cdot c^3}{4G\hbar} = \frac{\pi c^5}{G\hbar H_0^2}. \tag{13}$$

These are identical in natural units ($k_B = 1$). □

The volume entropy is thus the holographic entropy of the horizon written in bulk variables; it saturates the Bousso bound rather than violating it. De Sitter space is a maximally entropic configuration: matter placed inside must displace entropy from the thermal bath, and once the displacement becomes significant the gravitational dynamics change (Sec. 4).

4 Emergence of the MOND Acceleration Scale

The MOND scale emerges from entropy equilibrium in the de Sitter bath, with its value fixed entirely by cosmological parameters. The derivation proceeds in three steps: matter displaces volume entropy; equilibrium fails at a critical radius; the acceleration at that scale is $a_0 = cH_0/(2\pi)$.

4.1 Entropy Displacement by Matter

A mass M at rest is bound energy $E = Mc^2$ removed from the thermal background. Removing energy dE from a system at temperature T lowers its entropy by dE/T, so M creates the displacement

$$\Delta S(M) = \frac{Mc^2}{T_{\mathrm{dS}}} = \frac{2\pi k_B M c^2}{\hbar H_0}. \tag{14}$$

Because $T_{\mathrm{dS}} \approx 2.76 \times 10^{-30}$ K is extraordinarily low, even modest masses create large deficits: a galaxy of baryonic mass $M \approx 10^{11} M_\odot$ gives $\Delta S \approx 10^{99} k_B$. This deficit must be accommodated by the surrounding geometry.

4.2 The Entropy Budget

The volume entropy of Sec. 3 fills space at density $s_\Lambda = 3c^2 H_0 k_B/(4G\hbar)$. Within a sphere of radius r the available entropy is

$$S_{\mathrm{available}}(r) = s_\Lambda \cdot \frac{4\pi r^3}{3} = \frac{\pi c^2 H_0 k_B r^3}{G\hbar}, \tag{15}$$

growing as r^3. Local thermodynamic equilibrium requires

$$S_{\mathrm{available}}(r) \geq S_{\mathrm{displaced}}(M). \tag{16}$$

When this holds, small curvature adjustments restore equilibrium and general relativity applies. When it fails, the excess deficit spreads non-locally and the gravitational dynamics change. The failure of local equilibrium is a physical hypothesis, not a result derived from first principles.

4.3 The Critical Radius

Lemma 4.1 (Critical radius)**.** *Equilibrium fails outside the radius*

$$r_c = \left(\frac{2GM}{H_0^2}\right)^{1/3}. \tag{17}$$

Proof. Set $S_{\mathrm{displaced}}(M) = S_{\mathrm{available}}(r_c)$:

$$\frac{2\pi k_B M c^2}{\hbar H_0} = \frac{\pi c^2 H_0 k_B r_c^3}{G\hbar}. \tag{18}$$

The factors π, k_B, c^2, $\hbar$ cancel, giving (17). □

At r_c the de Sitter expansion velocity $v = H_0 r$ equals the escape velocity $\sqrt{2GM/r}$: beyond r_c cosmic expansion dominates local binding. For the Sun, $r_c \approx 394$ light-years; for a galaxy

($M = 10^{11} M_\odot$), $r_c \approx 560$ kpc, exceeding the optical radius (10–50 kpc) where flat rotation curves appear. The outer rotation-curve region therefore lies *inside* r_c — in the equilibrium regime but at low enough acceleration ($g_N \lesssim a_0$) to show MOND dynamics, since r_c marks the boundary of Newtonian dominance, not the onset of MOND. For a cluster ($M = 10^{14} M_\odot$), $r_c \approx 5.6$ Mpc. These scales track where non-Newtonian behavior is observed; the quantitative correspondence requires the ϵ-expansion of Sec. 5.

4.4 The Universal Acceleration Scale

The Newtonian acceleration at r_c,

$$a_c = \frac{GM}{r_c^2} = \frac{GM}{(2GM/H_0^2)^{2/3}} = \frac{(GM)^{1/3} H_0^{4/3}}{2^{2/3}}, \tag{19}$$

still depends on M and is not universal. The universal scale comes instead from the intrinsic de Sitter bath. The horizon energy scale $E_{\rm dS} = k_B T_{\rm dS} = \hbar H_0/(2\pi)$, converted via $m_{\rm P} = \sqrt{\hbar c/G}$ and $\ell_{\rm P} = \sqrt{\hbar G/c^3}$, gives

$$a_0 = \frac{E_{\rm dS}}{m_{\rm P}\ell_{\rm P}} = \frac{\hbar H_0/(2\pi)}{\sqrt{\hbar c/G}\cdot\sqrt{\hbar G/c^3}} = \frac{cH_0}{2\pi}. \tag{20}$$

Two logically distinct steps enter: the entropy-budget argument establishes cH as the relevant acceleration scale, and the coefficient $1/(2\pi)$ is the Gibbons–Hawking temperature assignment $T_{\rm dS} = \hbar H_0/(2\pi k_B)$ — a dimensional identification rather than an independent derivation, in the spirit in which Bekenstein separated the black-hole area law from the value of its coefficient.

Prediction: The characteristic acceleration below which gravitational dynamics deviate from Newtonian behavior is

$$a_0 = \frac{cH_0}{2\pi} \approx 1.08 \times 10^{-10}\ \text{m/s}^2. \tag{21}$$

This contains no adjustable parameters—it is a direct prediction from de Sitter thermodynamics.

With $c = 2.998 \times 10^8$ m/s and $H_0 = 2.268 \times 10^{-18}$ s^{-1} ($H_0 = 70$ km/s/Mpc),

$$a_0 = \frac{(2.998 \times 10^8)(2.268 \times 10^{-18})}{2\pi} \approx 1.08 \times 10^{-10}\ \text{m/s}^2. \tag{22}$$

The observed value is $a_0^{\rm obs} \approx (1.2 \pm 0.2) \times 10^{-10}$ m/s^2 [8]: agreement to within 10% for a parameter-free prediction linking cosmology to galactic dynamics.

4.5 Modified Gravitational Dynamics

When $g_N < a_0$, equilibrium $\delta S_{\rm gen} = 0$ must include the volume entropy. At high acceleration ($g_N \gg a_0$) the displaced entropy is absorbed locally and the Einstein equations apply; at low acceleration ($g_N \ll a_0$) the deficit spreads through the de Sitter volume as a non-local strain that supplements the Newtonian field. The strain is expressed by the modified Poisson equation

$$\nabla \cdot \left[\mu\left(\frac{|\nabla\Phi|}{a_0}\right)\nabla\Phi\right] = 4\pi G\rho, \tag{23}$$

with interpolation function $\mu(x)$ satisfying

- $\mu(x) \to 1$ for $x \gg 1$ (Newtonian regime), and
- $\mu(x) \to x$ for $x \ll 1$ (deep MOND regime).

The entropy arguments fix these limits but not the shape of $\mu(x)$; the phenomenological ansatz $\mu(x) = x/\sqrt{1+x^2}$ fits rotation-curve data well [8]. Deriving $\mu(x)$ from the microscopic entropy dynamics is an open problem and a real gap in the framework.

The effective field $g_{\text{eff}} = -\nabla\Phi$ recovers the inverse-square law for $g_N \gg a_0$,

$$g_{\text{eff}} = g_N = \frac{GM}{r^2}, \tag{24}$$

and in the deep MOND regime $g_N \ll a_0$,

$$g_{\text{eff}} = \sqrt{g_N \cdot a_0} = \sqrt{\frac{GMa_0}{r^2}} = \frac{\sqrt{GMa_0}}{r}. \tag{25}$$

The $1/r$ falloff gives flat rotation curves: $v^2/r = g_{\text{eff}} \propto 1/r$ implies $v = \text{const}$.

4.6 The Baryonic Tully-Fisher Relation

Theorem 4.2 (Baryonic Tully–Fisher relation)**.** *In the deep MOND regime the asymptotic rotation velocity satisfies*

$$v^4 = GMa_0 = GM \cdot \frac{cH_0}{2\pi}, \tag{26}$$

with normalization fixed by cosmology and no free parameter.

Proof. For a circular orbit in the deep MOND regime the centripetal acceleration equals the field,

$$\frac{v^2}{r} = g_{\text{eff}} = \sqrt{\frac{GMa_0}{r^2}}. \tag{27}$$

Squaring,

$$\frac{v^4}{r^2} = \frac{GMa_0}{r^2}, \tag{28}$$

and the r^2 factors cancel, leaving (26). □

The relation $v^4 \propto M$ has slope 4 on a log–log plot; the observed slope is 3.98 ± 0.12 [9]. For $M = 10^{11} M_\odot = 2 \times 10^{41}$ kg,

$$\begin{aligned} v^4 &= (6.67 \times 10^{-11}\ \text{m}^3\text{kg}^{-1}\text{s}^{-2})(2 \times 10^{41}\ \text{kg})(1.08 \times 10^{-10}\ \text{m/s}^2) \\ &\approx 1.5 \times 10^{21}\ \text{m}^4/\text{s}^4, \end{aligned} \tag{29}$$

giving $v \approx 200$ km/s, the rotation velocity of Milky Way-sized galaxies. In dark-matter models the tight scatter requires a correlation between visible and dark matter imposed by hand; here the normalization follows from de Sitter thermodynamics, leaving no room for scatter from varying halo properties.

5 Regime Separation: GR versus MOND

Solar-system tests constrain departures from general relativity to parts per billion, so the framework must reduce to GR there while modifying gravity at low acceleration. The single ratio $\epsilon = g_N/a_0$ controls the crossover, and its enormous value in bound systems places them in the GR regime.

5.1 The Control Parameter

The dimensionless control parameter is

$$\epsilon(r) \equiv \frac{g_N(r)}{a_0} = \frac{GM}{r^2 a_0} = \frac{2\pi GM}{r^2 c H_0}. \tag{30}$$

For $\epsilon \gg 1$ Newtonian gravity dominates; for $\epsilon \lesssim 1$ the volume entropy becomes dynamically important. The equilibrium condition $\delta S_{\text{gen}} = 0$ involves the area, bulk-matter, and volume contributions, which for a spherical perturbation at radius r scale as

$$\delta S_{\text{area}} \sim \frac{r}{G\hbar}\,\delta r, \tag{31}$$

$$\delta S_{\text{bulk}} \sim \frac{c^2 r^2 \epsilon}{G\hbar}\,\delta r, \tag{32}$$

$$\delta S_{\text{vol}} \sim \frac{c^2 H_0 r^2}{G\hbar}\,\delta r. \tag{33}$$

Since $\delta S_{\text{vol}}/\delta S_{\text{bulk}} \sim 1/\epsilon$, the equilibrium condition organizes as an expansion in inverse powers of ϵ:

$$\delta S_{\text{gen}} = \underbrace{\delta S_{\text{area}} + \delta S_{\text{bulk}}}_{O(1)} + \underbrace{\epsilon^{-1}\delta S^{(1)}_{\text{vol}}}_{O(\epsilon^{-1})} + O(\epsilon^{-2}). \tag{34}$$

5.2 The Three Dynamical Regimes

The expansion yields three regimes.

High acceleration ($\epsilon \gg 1$): the $1/\epsilon$ terms drop, leaving $\delta S_{\text{area}} + \delta S_{\text{bulk}} = 0$ — Jacobson's derivation of the Einstein equations. General relativity holds exactly, with corrections of order a_0/g_N.

Transition ($\epsilon \sim 1$): all three contributions compete, and $\delta S_{\text{area}} + \delta S_{\text{bulk}} + \delta S_{\text{vol}} = 0$ gives the interpolated dynamics

$$g_{\text{eff}} \cdot \mu\left(\frac{g_{\text{eff}}}{a_0}\right) = g_N. \tag{35}$$

Low acceleration ($\epsilon \ll 1$): the volume term dominates and the deep MOND result $g_{\text{eff}} = \sqrt{g_N a_0}$ follows, with $g_{\text{eff}} \propto 1/r$ producing flat rotation curves.

5.3 Precision Tests in the Solar System

The inner solar system has $\epsilon \gg 1$, suppressing MOND corrections below measurement precision. At Earth's surface ($g_N \approx 10$ m/s^2), $\epsilon \approx 10^{11}$, so corrections are at the 10^{-11} level; at Mercury's perihelion, $\epsilon \approx 4 \times 10^8$, corrections at 10^{-9}, below perihelion-precession sensitivity; at Voyager 2's 120 AU ($g_N \approx 4 \times 10^{-6}$ m/s^2), $\epsilon \approx 10^4$, corrections at 10^{-4}, below deep-space tracking precision.

The critical radius where $\epsilon = 1$ marks where the inverse-square law begins to fail. For the Sun,

$$r_c = \left(\frac{2GM_\odot}{H_0^2}\right)^{1/3} \approx 394 \text{ light years,} \tag{36}$$

far outside the solar system. For a galaxy ($M = 10^{11} M_\odot$), $r_c \approx 560$ kpc, larger than the rotation-curve region (< 50 kpc): the rotation-curve regime lies inside r_c but at low enough $g_N \lesssim a_0$ to show MOND dynamics, where volume-entropy corrections dominate. The critical radius is a necessary but not sufficient condition; MOND effects appear when $\epsilon \lesssim 1$, which occurs within r_c for sufficiently diffuse mass distributions.

5.4 The Spectrum of Gravitational Environments

Astrophysical systems span ϵ from 10^{13} (binary pulsars) to 0.01 (cosmic voids), and the single scale $a_0 = cH_0/(2\pi)$ orders them without tuning: GR to parts in 10^7–10^{13} in the solar system and pulsars; near-Newtonian with small corrections in galactic bulges ($\epsilon \sim 10$); full MOND in galaxy outskirts and halos ($\epsilon \lesssim 1$); deep MOND with possible higher-order corrections in clusters ($\epsilon \sim 0.3$) and voids ($\epsilon \sim 0.01$).

5.5 Cosmological Evolution of the MOND Scale

The robust statement is that a_0 is locked to a cosmological horizon rate, $a_0 = cH/(2\pi)$ (the horizon-clock lock; see the Falsification Criteria in Sec. 7). Whether a_0 then evolves with cosmic time depends on which horizon rate enters H: the *instantaneous*-horizon reading makes the scale larger in the past, the event-horizon reading keeps it constant. Current high-redshift rotation curves favour an approximately constant a_0 over $z = 0$–2, so the evolving case below is the signature that *would* distinguish the instantaneous reading, not an established prediction. In that reading $a_0 = cH(z)/(2\pi)$, so

$$a_0(z) = \frac{cH(z)}{2\pi} = a_0(0) \cdot \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}. \tag{37}$$

At $z = 1$ the scale was ≈ 1.78 times larger; at $z = 3$, ≈ 4.54 times; at recombination ($z \sim 1100$), $\approx 2.4 \times 10^4$ times (including $\Omega_r \approx 9.4 \times 10^{-5}$). A larger a_0 pushes more systems into the Newtonian regime, so rotation-curve anomalies appear at smaller radii at high redshift and extreme-redshift systems may appear Newtonian — testable by JWST and Euclid rotation curves at $z \sim 1$–3.

5.6 Consistency with the Cosmic Microwave Background

At recombination, $a_0(z = 1100) \approx 2.5 \times 10^{-6}$ m/s^2, far above any CMB-relevant acceleration. The critical radius for a proto-cluster shrinks to ~ 1 kpc while acoustic oscillations occur on

150 Mpc scales, so all CMB physics lies deep in the Newtonian regime where the framework reduces to standard gravity. The acoustic peaks therefore match ΛCDM, resolving the standard objection that modified gravity cannot reproduce them without dark matter: the CMB probes accelerations far above $a_0(z_{\rm rec})$.

5.7 The Galaxy Cluster Tension

Clusters retain a factor 2–5 mass discrepancy after MOND corrections. In this framework they occupy the transition regime $\epsilon \sim 0.1$–1, where the ϵ-expansion requires higher-order terms: the leading correction scales as ϵ^{-1}, but ϵ^{-2} and higher terms, sensitive to the density gradient $\nabla^2\rho/\rho$ rather than the local density, become non-negligible as $\epsilon \to 1$ in the extended, non-uniform cluster environment. Additional contributions may close part of the gap: triaxiality breaks the spherical symmetry assumed here; the intracluster medium at $T \sim 10^8$ K carries its own thermodynamics affecting the effective temperature; neutrinos with $\sum m_\nu \sim 0.1$ eV supply 1–5% of the cluster mass. A complete resolution requires detailed calculation of these effects and remains future work.

6 Covariant Completion: Entanglement-Elastic Gravity

A covariant completion of the thermodynamic MOND framework must reduce to the modified Poisson equation (23) in the Newtonian limit. This section sketches one candidate — Entanglement-Elastic Gravity (EEG) — in which an elastic field ψ encoding entanglement strain sources a modified Einstein equation.

Remark 6.1 (Scope). EEG is presented as a schematic proposal, not a finished theory. The quadratic elastic action below is a linearized caricature; a consistent completion reproducing the deep-MOND limit requires a *nonlinear* (Bekenstein–Milgrom/AQUAL-type) kinetic functional [4] (Sec. 6.5). What the framework fixes independent of that completion are the acceleration scale $a_0 = cH_0/(2\pi)$ and the elastic modulus $\kappa = c^4/(8\pi G a_0)$.

6.1 Conceptual Foundation

Spacetime carries two interacting forms of entanglement. Local entanglement obeys an area law: across any infinitesimal surface, short-range Bell pairs contribute $S_{\rm area} = A/(4\ell_P^2)$, the entanglement that yields the Einstein equations under $\delta S_{\rm gen} = 0$ [1], encoded by the metric $g_{\mu\nu}$. Global entanglement obeys a volume law: the de Sitter horizon fills the interior with long-range entanglement at density $s_\Lambda = 3c^2 H_0 k_B/(4G\hbar)$, a cosmological contribution with no flat-space analog.

Matter couples to both. A mass M requires area-law equilibrium on surrounding surfaces (local curvature) and displaces volume-law entropy $\Delta S \sim Mc^2/T_{\rm dS}$ (global strain). At $g \gg a_0$ the area-law term dominates and general relativity applies; at $g \lesssim a_0$ the volume-law strain modifies the dynamics. The crossover is at $a_0 = cH_0/(2\pi)$.

Definition 6.1 (Displacement field)**.** The scalar field $\psi(x)$ quantifies the local displacement of volume-law entanglement: ψ is large where matter is present and approaches zero far from matter. Its gradients are the strain in the entanglement network, which contributes to the field equations through an elastic stress tensor.

6.2 Microscopic Model

A toy model fixes the elastic modulus while remaining agnostic about the ultraviolet completion. Discretize a closed spacelike surface $\mathcal{H}$ of area A into $N = A/\ell_P^2$ Planck plaquettes, each carrying horizon qubits $(q_i^{(L)}, q_i^{(R)})$ maximally entangled in the vacuum:

$$|\Psi_{\text{area}}\rangle = \bigotimes_{i=1}^{N} |\Phi^+\rangle_i\,, \quad |\Phi^+\rangle_i = \frac{|00\rangle_i + |11\rangle_i}{\sqrt{2}}, \tag{38}$$

with von Neumann entropy $S = N\log 2 = A/(4\ell_P^2)$, the Bekenstein–Hawking value. A mass M at the center removes Bell pairs in proportion to its rest energy,

$$n = \frac{2\pi Mc}{\hbar H_0}, \tag{39}$$

creating a deficit that coarse-grains to the displacement field ψ. The associated elastic energy is

$$E_{\text{el}} = \frac{\kappa}{2}\int (\nabla\psi)^2\, d^3x, \tag{40}$$

with the modulus fixed by dimensional analysis and the requirement that the elastic force reproduce MOND,

$$\kappa = \frac{c^4}{8\pi G a_0}. \tag{41}$$

The model ignores gauge constraints, assumes instantaneous equilibration, and leaves the scrambling dynamics unspecified, but it reproduces the entropy-displacement formula and the elastic modulus without fine-tuning.

6.3 Effective Action

The covariant action combines the Einstein–Hilbert term, the matter Lagrangian, and an elastic sector for ψ:

$$\begin{aligned} S_{\text{eff}} = {} & \frac{1}{16\pi G}\int R\sqrt{-g}\, d^4x + \int \mathcal{L}_{\text{matter}}\sqrt{-g}\, d^4x \\ & + \int \left[-\frac{c^4}{8\pi G a_0}(\partial\psi)^2 - \lambda\left(\psi - \frac{8\pi G}{a_0 c^2}\rho_{\text{rest}}\right)\right]\sqrt{-g}\, d^4x, \end{aligned} \tag{42}$$

with Lagrange multiplier $\lambda(x)$ enforcing the constraint relating ψ to the matter density, and the kinetic coefficient $c^4/(8\pi G a_0)$ fixed by the non-relativistic limit. The kinetic term has the standard sign: the Hamiltonian density

$$\mathcal{H}_\psi = \frac{c^4}{8\pi G a_0}\left[(\partial_t\psi)^2 + (\nabla\psi)^2\right] \tag{43}$$

is non-negative, so the elastic sector is ghost-free and stable. The constraint couples ψ algebraically, not dynamically: ψ is slaved to the matter distribution, like the Newtonian potential. As $a_0 \to 0$ (equivalently $H_0 \to 0$) the action reduces to general relativity, so EEG is an extension rather than a replacement.

6.4 Field Equations

Varying (42) with respect to $g_{\mu\nu}$ gives the modified Einstein equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G \left(T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^{\text{elastic}}\right), \tag{44}$$

with elastic stress tensor

$$T_{\mu\nu}^{\text{elastic}} = \frac{c^4}{4\pi G a_0} \left(\partial_\mu\psi\partial_\nu\psi - \frac{1}{2} g_{\mu\nu}(\partial\psi)^2\right), \tag{45}$$

the form of a massless scalar but with ψ constrained to track matter. Varying with respect to ψ and λ gives

$$\nabla^2\psi = \frac{4\pi G a_0}{c^4}\lambda(x), \tag{46}$$

$$\psi = \frac{8\pi G}{a_0 c^2}\rho_{\text{rest}}(x), \tag{47}$$

which determine λ in terms of known quantities and close the system.

6.5 Toward the MOND Limit

The Newtonian limit is where the schematic action is deliberately incomplete. In the weak-field, static limit ($g_{00} \approx -(1 + 2\Phi/c^2)$), the time–time component of (44) reads

$$\nabla^2\Phi = 4\pi G\rho + \frac{c^4}{a_0}(\nabla\psi)^2. \tag{48}$$

With ψ slaved to the rest-mass density by $\psi = 8\pi G\rho/(a_0 c^2)$, the source $(\nabla\psi)^2 \propto (\nabla\rho)^2$ depends on gradients of the *matter distribution*, not on the field $\nabla\Phi$. It therefore does *not* reproduce the MOND structure

$$\nabla \cdot \left[\mu\left(\frac{|\nabla\Phi|}{a_0}\right)\nabla\Phi\right] = 4\pi G\rho, \tag{49}$$

in which the modification is controlled by $|\nabla\Phi|/a_0$. Recovering it requires promoting ψ to a dynamical field with a *nonlinear* kinetic functional $\mathcal{F}((\partial\psi)^2/a_0^2)$ whose deep-MOND limit $\mathcal{F}(z) \propto z^{3/2}$ yields the $\sqrt{a_0\, g_N}$ behaviour — the route of the Bekenstein–Milgrom AQUAL theory [4] and its relativistic descendants (TeVeS [7], the Skordis–Złośnik model [21]). The quadratic action above is the linearization of such a functional about the high-acceleration regime, given only to fix notation and the modulus.

Independent of that completion, EEG fixes the acceleration scale $a_0 = cH_0/(2\pi)$ from de Sitter thermodynamics (Sec. 4) and the modulus $\kappa = c^4/(8\pi G a_0)$ from the strain energy above. Constructing the nonlinear functional, then verifying ghost-freedom and the gravitational-wave speed in the resulting theory, is left to future work.

6.6 Gravitational Waves

GW170817 and GRB 170817A [22] established that gravitational waves travel at c to one part in 10^{15}, ruling out modified-gravity theories that predict deviations. A scalar elastic sector is compatible by construction: ψ enters only through an elastic stress tensor that sources curvature without mixing with the transverse-traceless kinetic term, so tensor perturbations propagate on null geodesics at speed c. This is a design requirement that any nonlinear completion (Sec. 6.5) must preserve, not an established property of a finished theory — unlike vector–tensor MOND extensions, where the GW speed must be checked explicitly [7, 21].

6.7 Comparison with Alternative Approaches

Against dark matter, EEG requires no new particles, no halo-profile tuning, and no cosmic conspiracy for the Tully–Fisher relation, whose scaling follows from the thermodynamic origin of a_0. Against phenomenological MOND, it offers a relativistic framework with a microscopic foundation in entanglement, natural incorporation of Λ, and compatibility with the GW-speed bound. Its limitations: $\mu(x)$ is phenomenological rather than derived; the strong-field regime (black holes, neutron stars) is not yet analyzed; cosmological perturbation theory requires numerical implementation in a Boltzmann code.

6.8 Summary

EEG sketches a candidate covariant completion: a scalar displacement field ψ encoding the strain in the cosmological entanglement network, contributing an elastic stress tensor to the Einstein equations, ghost-free and designed to preserve the GW speed c. Reproducing the modified Poisson equation in the Newtonian limit requires replacing the linearized quadratic action with a nonlinear (AQUAL-type) functional (Sec. 6.5). Already fixed, independent of that completion, are $a_0 = cH_0/(2\pi)$ and $\kappa = c^4/(8\pi G a_0)$, from the same de Sitter thermodynamics and with no new free parameters.

7 Testable Predictions

The framework's observational consequences range from already-confirmed galactic phenomenology to novel signatures that distinguish it from dark matter and from phenomenological MOND. This section collects them, together with the criteria that would falsify the framework.

7.1 Parameter-Free Predictions Already Confirmed

Two central predictions carry no adjustable parameters and match observation.

The MOND acceleration scale: from de Sitter thermodynamics,

$$a_0 = \frac{cH_0}{2\pi} \approx 1.08 \times 10^{-10}\ \mathrm{m/s^2}, \tag{50}$$

against the observed $a_0^{\mathrm{obs}} = (1.2 \pm 0.2) \times 10^{-10}\ \mathrm{m/s^2}$ [8] — agreement to within 10% for a quantity 11 orders of magnitude below terrestrial gravity.

The baryonic Tully–Fisher relation: $v^4 = GMa_0$ predicts slope exactly 4 against the observed 3.98 ± 0.12 [9], with normalization fixed by the same a_0 and scatter consistent with measurement error alone. The tight correlation, which dark-matter models must tune, follows here from thermodynamics.

7.2 Predictions for Precision Tests

Deviations from GR scale as a_0/g_N. Lunar laser ranging ($\epsilon \sim 10^{11}$) should show GR to parts in 10^{11}; Mercury's perihelion ($\epsilon \sim 10^8$) to parts in 10^8; binary pulsar timing ($\epsilon \sim 10^{13}$) is the most stringent test of pure GR. All current precision tests are consistent. The framework predicts *no* anomalous effect in these high-ϵ systems: any MOND-like effect in the inner solar system would falsify it.

7.3 Cosmological Evolution of the Acceleration Scale

The robust prediction is that a_0 is locked to a cosmological horizon frequency, $a_0 = cH/(2\pi)$, with the same horizon rate H fixing the cosmic gravitational decoherence rate through $a_0/(c\,\Gamma_{\text{cosmic}}) = 2/g(1)$ (the horizon-clock lock; see the Falsification Criteria below). Which horizon rate enters H — and hence whether a_0 evolves — is less certain: the *instantaneous*-horizon reading gives $a_0(z) = cH(z)/(2\pi)$, growing in the past, so high-redshift galaxies would show MOND effects at smaller radii; the *asymptotic* event-horizon reading gives constant a_0. The evolving case is retained here because it would distinguish the instantaneous reading, but current high-redshift rotation curves favour an approximately constant a_0.

At $z = 1$ the scale was ≈ 1.78 times larger, so a galaxy of fixed baryonic mass would have had an asymptotic velocity $\approx 19\%$ higher than today. At $z = 2$ (a_0 about 3.0 times larger) many galaxies should appear nearly Newtonian. A measured $a_0(z) \propto H(z)$ would single out the instantaneous reading over both phenomenological MOND and the asymptotic reading (both giving constant a_0).

Priority targets. The transition regime at $z \sim 2$–2.5 gives optimal leverage: zC406690 ($z = 2.196$) and BX482 ($z = 2.26$) lie where $a_0(z)$ suppresses MOND effects substantially but not entirely, and dwarf galaxies at $z > 2$ should appear Newtonian, unlike local dwarfs. JWST observations in 2027–2030 can reach 3–5σ sensitivity to the predicted evolution; ELT commissioning in 2030+ will give $> 10\sigma$ measurements.

7.4 CMB Consistency

At recombination, $a_0(z_{\text{rec}}) \approx 2.5 \times 10^{-6}$ m/s^2 (computed as $a_0 \times \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda + \Omega_r(1+z)^4}$ with $\Omega_r \approx 9.4 \times 10^{-5}$), far above any CMB-relevant acceleration. All perturbations lie deep in the Newtonian regime, so CMB acoustic peaks, lensing, and polarization match ΛCDM identically, with no modification to the standard analysis. The one subtle prediction is the integrated Sachs–Wolfe effect: modified late-time growth can suppress ISW at the 10–20% level, within cosmic variance for current data and not a strong discriminator, though future cross-correlations with large-scale structure may provide a consistency check. CMB observations thus constrain the framework at recombination — where it predicts standard physics — not in the MOND regime.

7.5 Falsification Criteria

A measured $a_0 \neq cH_0/(2\pi)$ at greater than 20% discrepancy would falsify the thermodynamic derivation. MOND-like effects in high-ϵ systems exceeding a_0/g_N — e.g. anomalous perihelion precession at parts in 10^6 rather than 10^9 — would violate the regime separation. The framework ties a_0 to a cosmological horizon, $a_0 = cH/(2\pi)$; whether the relevant H is the instantaneous apparent-horizon rate $H(z)$ (evolving $a_0(z) \propto H(z)$) or the asymptotic event-horizon rate H_Λ (constant a_0) is open, and current high-redshift rotation curves favour constant a_0 over $z = 0$–2. A measured $a_0(z)$ tracking $H(z)$ confirms the instantaneous reading; a constant a_0 selects the asymptotic reading. The latter does *not* falsify the $a_0 = cH/(2\pi)$ derivation — which holds with $H = H_\Lambda$ — nor the rest of the framework. The horizon-clock lock $a_0/(c\,\Gamma_{\mathrm{cosmic}}) = 2/g(1)$ holds independent of which horizon is chosen; that locked ratio, not a particular $a_0(z)$ law, is the robust prediction.

The galaxy cluster tension is an incomplete, not falsified, prediction. If detailed calculation of higher-order corrections, non-spherical geometry, hot-gas effects, and neutrino contributions fails to close the factor 2–5 gap, additional physics is needed — though not necessarily that the framework is wrong at the galactic level.

7.6 Non-Equilibrium Cluster Dynamics

One mechanism addresses the cluster tension partially: equilibration in clusters is slow compared to the dynamical time. The elastic stress derives from vacuum entanglement entropy, which requires thermal equilibrium to saturate. For $M_{\mathrm{cl}} \sim 10^{15}\ M_\odot$ and $R \sim 1$ Mpc,

$$\tau_{\mathrm{eq}} \gg \tau_{\mathrm{dyn}} \sim \frac{R}{v_{\mathrm{circ}}} \sim 1\ \mathrm{Gyr}, \tag{51}$$

with $v_{\mathrm{circ}} \sim 10^3$ km/s. A rough estimate gives $\tau_{\mathrm{eq}} \sim 100$–300 Gyr; a complete derivation of τ_{eq} from the EEG equations of motion has not been carried out and this should be treated as an order-of-magnitude guide. What matters is $\tau_{\mathrm{eq}} \gg \tau_{\mathrm{dyn}}$: clusters are far from equilibrium.

The elastic stress has therefore not fully developed since formation at $z \sim 1$–2, so clusters show a factor ~ 2 MOND boost rather than the equilibrium ~ 3.5. This predicts a testable correlation: relaxed, cool-core clusters should show *larger* MOND boosts than recently merged ones — testable by combining X-ray morphology with weak-lensing masses, and distinct from both ΛCDM and phenomenological MOND.

7.7 The Bullet Cluster Challenge

The Bullet Cluster (1E 0657-558) is the most cited evidence against MOND-like theories: in this merger at $z = 0.296$, weak lensing shows the gravitational mass offset from the dominant baryonic component (hot X-ray gas). Two mechanisms apply. The collision occurred ≈ 150 Myr ago, far shorter than $\tau_{\mathrm{eq}} \sim 100$–300 Gyr, so the violent merger disrupted the pre-existing equilibrium and the elastic enhancement has not re-developed. At $z = 0.296$, $a_0 \approx 1.3 \times a_0(0)$, slightly reducing the expected enhancement.

The Bullet Cluster requires ≈ 5–6 times the baryonic mass to fit lensing. This framework with standard contributions (massive neutrinos 5–15%, hydrostatic bias 10–30%, triaxial geometry 10–20%) accounts for ≈ 3–4 times, leaving a factor ~ 2 residual. We assess this honestly: the Bullet Cluster challenges but does not definitively falsify the framework. The discrepancy is smaller than often claimed (factor ~ 2, not ~ 6), and the framework predicts that the discrepancy should *decrease* with relaxation state — the Bullet Cluster, a recent merger, should show *larger* discrepancy than relaxed systems. This correlation is testable and distinguishes the non-equilibrium mechanism from particle dark matter.

7.8 Comparison with Alternatives

Standard MOND treats a_0 as an empirical constant, with no account of its value or evolution. TeVeS [7] gives a relativistic completion but requires multiple fields and free parameters and typically assumes constant a_0; more recent relativistic MOND theories [21] reproduce the CMB and matter power spectra at the cost of additional fields. Verlinde's emergent gravity [5, 6] derives an acceleration scale from holography but does not specify its redshift evolution. This framework is unique in predicting $a_0 = cH/(2\pi)$ from first principles with zero free parameters and an explicit tie to the horizon rate. The robust prediction is the horizon-clock lock $a_0/(c\,\Gamma_{\text{cosmic}}) = 2/g(1)$, which holds in either horizon reading; a measured $a_0(z) \propto H(z)$ would additionally select the instantaneous reading, though current data favour constant a_0.

7.9 Implications for Dark Matter Searches

If the framework is correct, the galactic "missing mass" is a gravitational effect of cosmological entropy, not a particle. Direct-detection experiments and collider searches would find no dark matter candidate responsible for rotation curves. This does not preclude dark-matter particles — sterile neutrinos, axions, or other relics may exist — but suggests they do not drive galactic dynamics. The cluster tension may require residual dark matter at the 1–10% level, possibly massive neutrinos with $\sum m_\nu \sim 0.1$–0.5 eV.

8 Conclusions

The cosmological horizon acts as a thermal reservoir, filling the observable universe with volume-law entropy that competes with the standard area-law contribution to gravitational dynamics. The framework rests on physically motivated assumptions — in particular that local entropy equilibrium fails at low accelerations — that are not rigorously derived from first principles.

Matter in this bath displaces entropy from the background, a strain that contributes to the gravitational field. At high accelerations the local curvature absorbs the displacement and general relativity applies; at low accelerations the deficit spreads non-locally through the de Sitter volume and modified dynamics emerge. The crossover is at $a_0 = cH_0/(2\pi) \approx 1.08 \times 10^{-10}$ m/s^2, matching the observed MOND scale with no adjustable parameters. The framework reduces to GR in the solar system ($\epsilon \gg 1$), produces flat rotation curves and the Tully–Fisher relation in galaxies ($\epsilon \lesssim 1$), and preserves standard cosmology at the CMB epoch, where all relevant scales lie deep in the Newtonian regime.

8.1 Relation to the Broader Framework

This paper belongs to the Quantum-Geometric Correspondence series — the thesis that quantum mechanics and general relativity are complementary descriptions of one underlying reality. The canonical core paper [15] presents the axiomatic framework and develops gravitational decoherence from entanglement equilibrium; companion work shows that holographic dark energy follows from the same thermodynamic principles. The present work extends these ideas to de Sitter backgrounds, where the cosmological horizon introduces thermodynamic effects absent in asymptotically flat spacetime.

The volume entropy is a background-dependent phenomenon present only when $\Lambda > 0$, not a modification of the universal axioms. As $\Lambda \to 0$ the de Sitter temperature vanishes, the volume entropy disappears, and general relativity is recovered exactly. The MOND-like modifications are thus a cosmological effect of the finite size of the observable universe, not new short-distance physics.

8.2 Open Problems

The interpolation function $\mu(x)$ is introduced phenomenologically; deriving it from the entropy dynamics would strengthen the framework. The galaxy cluster tension — a factor 2–5 mass discrepancy after MOND corrections — likely involves higher-order ϵ-expansion terms, non-spherical geometry, and hot-gas thermodynamics, but detailed calculation is needed. The framework as developed assumes quasi-static configurations; dynamical situations (mergers, structure formation, time-dependent fields) require a time-dependent entropy analysis not yet attempted. The volume entropy is a semiclassical concept; a full quantum-gravity treatment might confirm the volume law from explicit entanglement calculations, reveal the microscopic origin of s_Λ, and supply quantum corrections that bear on residual tensions.

8.3 Experimental Outlook

JWST and Euclid can measure rotation curves at $z \sim 1$–3, testing whether $a_0(z) \propto H(z)$. Gaia DR4 and DR5 provide precision dynamics for wide binaries and the outer Milky Way, probing the transition regime. DESI and future spectroscopic surveys can measure the growth rate, testing the predicted suppression $\Delta f\sigma_8 \approx -0.03$ at $z = 0.8$. Next-generation gravitational-wave detectors probe the strong-field predictions of the covariant extension.

8.4 Final Remarks

The framework answers a 40-year-old question — why the MOND scale coincides with cH_0 — by deriving both from the thermodynamics of the cosmological horizon: $a_0 = cH_0/(2\pi)$ emerges with no free parameters, connecting cosmology to galaxy dynamics through entropy. The framework does not eliminate dark matter, but suggests that whatever dark component exists is not responsible for the galactic regularities — the tight Tully–Fisher relation, the universal acceleration scale, the flat rotation curves — which here follow with no new particles and no free parameters beyond those already in cosmology. The framework is precise enough to be falsified and predictive enough to be tested; whether it survives confrontation with data remains to be seen.

Appendices

A Derivation Details

A.1 Surface Gravity of de Sitter Horizon

For the metric

$$ds^2 = -f(r)c^2dt^2 + f(r)^{-1}dr^2 + r^2d\Omega^2 \tag{52}$$

with $f(r) = 1 - r^2/R_{\rm H}^2$ and Killing vector $k^\mu = (1, 0, 0, 0)$, the acceleration of a static observer at radius r is

$$a^\mu = k^\nu \nabla_\nu k^\mu = \frac{c^2 r/R_{\rm H}^2}{\sqrt{f(r)}}\hat{r}. \tag{53}$$

The surface gravity (denoted $\kappa_{\rm dS}$ to distinguish it from the elastic modulus $\kappa = c^4/(8\pi G a_0)$ of Sec. 6) is

$$\kappa_{\rm dS} = \lim_{r\to R_{\rm H}} \sqrt{f(r)} \cdot a = \frac{c^2}{R_{\rm H}} = cH_0 \tag{54}$$

A.2 Volume Entropy from Partition Function

The partition function for a scalar field in de Sitter is

$$\ln Z = -\sum_n \ln(1 - e^{-\beta\omega_n}), \tag{55}$$

or in the continuum limit with density of states $D(\omega)$,

$$\ln Z = -\int_0^\infty d\omega\, D(\omega) \ln(1 - e^{-\beta\omega}). \tag{56}$$

With $D(\omega) \approx \omega^2 V$ for low frequencies,

$$\ln Z \approx \frac{\pi^2}{90}\frac{V}{(\beta\hbar c)^3}, \tag{57}$$

and the entropy is

$$S = \beta^2 \frac{\partial}{\partial\beta}(\beta^{-1}\ln Z) = \frac{2\pi^2}{45} g_* k_B \left(\frac{k_B T}{\hbar c}\right)^3 V. \tag{58}$$

For $g_* \sim O(1)$ and $T = T_{\rm dS}$ this is the volume entropy $S_{\rm vol} = s_\Lambda V$.

A.3 Modified Poisson Equation from $\delta S_{\rm gen} = 0$

The equilibrium condition $\delta S_{\rm gen} = 0$ including the volume term is

$$\frac{\delta A}{4G\hbar} + \delta S_{\rm bulk} + s_\Lambda \delta V = 0, \tag{59}$$

with

- $\delta A/(4G\hbar) \approx -\ell^{d+1} G_{00}/(G\hbar)$ (Raychaudhuri)
- $\delta S_{\rm bulk} \approx \ell^{d+1} T_{00}/T$ (first law)

- $\delta V = \ell^3$ for perturbation scale ℓ.

For a localized mass in the low-acceleration limit this becomes

$$\nabla^2\Phi = 4\pi G\rho + \lambda_{\rm MOND}\nabla\cdot\left(\frac{\nabla\Phi}{|\nabla\Phi|}\sqrt{|\nabla\Phi|}\right), \tag{60}$$

with $\lambda_{\rm MOND} = \sqrt{a_0}$, equivalent to the standard MOND form

$$\nabla\cdot\left[\mu\left(\frac{|\nabla\Phi|}{a_0}\right)\nabla\Phi\right] = 4\pi G\rho. \tag{61}$$

A.4 Response Kernel from Modular Hamiltonian

For the vacuum restricted to a causal diamond D in de Sitter, the modular Hamiltonian is

$$K = 2\pi\int_D \frac{(\ell^2 - |\mathbf{x}|^2)}{2\ell} T_{00}(\mathbf{x})\, d^3x + K_{\rm horizon}. \tag{62}$$

The fluctuation–dissipation relation gives the response kernel

$$\chi(x, x') = \frac{1}{T_{\rm dS}}\langle \delta K(x)\delta\rho_M(x')\rangle, \tag{63}$$

and the de Sitter Green's function $G(r) \propto e^{-r/R_{\rm H}}/r$ yields

$$\chi(r) = \chi_0\cdot\frac{R_{\rm H}}{r}\cdot e^{-r/R_{\rm H}}, \tag{64}$$

with normalization χ_0 fixed by

$$\int d^3x\,\chi(x) = \frac{2\pi k_B c^2}{\hbar H_0}. \tag{65}$$

B Numerical Constants and Values

B.1 Fundamental Constants

Quantity	**Symbol**	**Value**
Speed of light	c	2.998×10^{8} m/s
Planck constant	$\hbar$	1.055×10^{-34} J·s
Newton's constant	G	6.674×10^{-11} m^3/(kg·s^2)
Boltzmann constant	k_B	1.381×10^{-23} J/K
Planck length	$\ell_{\rm P}$	1.616×10^{-35} m
Planck mass	$m_{\rm P}$	2.176×10^{-8} kg

B.2 Cosmological Parameters

Quantity	Symbol	Value
Hubble constant	H_0	2.27×10^{-18} s^{-1} (70 km/s/Mpc)
de Sitter radius	R_{H}	1.32×10^{26} m (14 Gly)
de Sitter temperature	T_{dS}	2.76×10^{-30} K
Matter density	Ω_m	0.31
Dark energy density	Ω_Λ	0.69

B.3 Derived Quantities

Quantity	Symbol	Value
MOND acceleration	a_0	1.08×10^{-10} m/s^2
Volume entropy density	s_Λ	3.0×10^{20} J/(K·m^3)
Horizon entropy	S_H	$\sim 10^{122} k_B$

B.4 System-Specific Values

System	Mass	r_c	ϵ (edge)
Sun	2.0×10^{30} kg	394 ly	10^4 at 50 AU
Milky Way (baryonic)	$10^{11} M_\odot$	560 kpc	~ 1 at 30 kpc
Massive galaxy	$10^{12} M_\odot$	1.2 Mpc	~ 1 at 50 kpc
Galaxy cluster	$10^{14} M_\odot$	5.6 Mpc	~ 0.3 at 1 Mpc

B.5 Comparison with Observations

Quantity	Predicted	Observed
a_0	1.08×10^{-10} m/s^2	$(1.2 \pm 0.2) \times 10^{-10}$ m/s^2 [8]
BTFR slope	4.00	3.98 ± 0.12 [9]
Tully-Fisher normalization	Fixed by a_0	Consistent

Agreement is within 10% with no adjustable parameters.

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