Graviton Absorption and the Slope of the Gravitational Form Factor

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Abstract

How strongly can a piece of matter absorb a gravitational wave? For a weak plane wave and any matter with a conserved stress tensor we show that, to first order in Newton's constant and without a multipole expansion, the absorption cross-section is the spectral function of one operator: the quadrupole moment, across the direction of travel, of the stress tensor integrated along the null rays of the wave, the averaged null energy. Its moment $\int \sigma \, d\omega/\omega^2$ gives the energy left by a wave with a sudden kink and is the expectation value of a double commutator. For slow matter in a body small against the wavelength the double commutator is the moment of inertia about the direction of travel, which gives $\int \sigma \, d\omega/\omega^2 = 2\pi^2 GI/c^3$, the gravitational counterpart of the Thomas–Reiche–Kuhn rule and in content the energy-weighted quadrupole sum rule of nuclear physics; we check it on hydrogen and on a bar detector. For relativistic matter in canonical light-front quantization, with interactions that are local across the ray, it is the second transverse moment of the averaged null energy, and for a neutral particle of mass m and spin 0 or 1/2 at rest, $\int \sigma \, d\omega/\omega^2 = 8\pi^2 Gm\, A'(0)$ in units $\hbar = c = 1$, with $A(t)$ the gravitational form factor; the D-term and the trace form factors do not enter. We verify this at second order in the coupling for a scalar and a Yukawa theory by computing both sides with covariant Feynman rules: they agree to the accuracy of the quadrature. At that order the interaction enters only through the particle's wavefunction, so the test confirms the chain of statements and not the algebra behind its extension. Beyond that order the evaluation rests on the canonical algebra of light-ray operators at coincident transverse points, which is known not to be universal; we state the rule with its explicit correction term and leave its status in renormalized perturbation theory open.

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1 Introduction

The absorption of light by matter obeys a sum rule that does not depend on what the matter is made of: the cross-section integrated over frequency is fixed by the charges and masses alone, the Thomas–Reiche–Kuhn rule [1, 2]. The absorption of gravitational waves by matter has been computed many times, for bars and spheres [3, 4, 5], for atoms [6, 7] and for resonators near their quantum ground state [8], in each case mode by mode or line by line. We ask the question the sum rule answers for light: what constrains the absorption of a gravitational wave by an arbitrary piece of matter, taken as a whole?

The answer comes in three steps, and the first is the one that makes the others possible. In the gauge adapted to a plane wave, the wave couples to matter through a single component of the stress tensor, the one obtained by contracting it twice with the wave's null vector. For a transition that absorbs exactly the wave's frequency, the integral of that component along the wave turns into an integral along a null ray. The operator that absorbs the wave is therefore

$$\mathcal{Q} = \int d^2x_\perp \, (x^2 - y^2)\, \mathcal{E}(x_\perp), \qquad \mathcal{E}(x_\perp) = \int d\lambda \, T_{\mu\nu} k^\mu k^\nu, \tag{1}$$

where $\mathcal{E}(x_\perp)$ is the stress tensor integrated along the null ray that pierces the transverse plane at $x_\perp$, λ being the distance along the ray: the averaged null energy operator of quantum field theory [9, 10, 11], weighted by the quadrupole profile of the wave. This holds for a body of any size relative to the wavelength and for matter moving at any speed (Sec. 3).

The second step is the sum rule. Because $\mathcal{Q}$ does not depend on the frequency, the moment $\int \sigma \, d\omega/\omega^2$ is the expectation value of a double commutator, and it has a direct meaning: it gives the energy that a wave with a sudden kink leaves in the matter (Sec. 4). For slow matter held together by potentials the double commutator is the moment of inertia about the direction of travel. For relativistic matter, at the canonical level of light-front quantization, it is the second transverse moment of the averaged null energy, provided the interactions are local across the ray; otherwise an explicit correction term appears. For a single neutral particle of spin 0 or 1/2 the moment is then $8\pi^2 Gm$ times the slope of its gravitational form factor at zero momentum transfer.

The third step is a test. We compute the absorption cross-section of a dressed particle from the tree graphs of covariant perturbation theory, integrate it, and compare with the one-loop slope of the form factor computed separately, for a scalar and for a Yukawa theory (Sec. 5). The two agree to the accuracy of the numerical integration. Nothing from the light front is used in that comparison.

What is and is not new. The ingredients are known. Plane waves in the adapted gauge [12, 13], the averaged null energy and its smeared versions as generators of shifts along null rays [14, 15], the Galilean structure of the light front [16, 17], the transverse radius of the momentum density and its relation to the slope of the form factor [18, 19], and the energy-weighted quadrupole

sum rule of nuclear physics [20] are all in the literature. Absorption as the imaginary part of the dynamical elastic response, and a sum rule for its zeroth moment, were given in Ref. [21]; a sum rule for the tidal Love number in terms of graviton emission rates in Ref. [22]; sum rules for spectral densities that are absorption probabilities of gravitons of each helicity, for the quantum Hall fluid, in Ref. [23]; and a low-energy theorem for the scattering of gravitons in Ref. [24]. What we have not found is the assembly: the cross-section as the spectral function of the light-ray operator (1), and the resulting sum rule with the slope of the gravitational form factor on its right-hand side. The non-relativistic rule is the nuclear quadrupole sum rule read as a statement about gravitational absorption, and we make no claim to it. The standard treatments of detection [3, 4] integrate the cross-section over a single resonance; we have not found it summed over all final states, there or elsewhere.

What is not shown. The evaluation of the double commutator for interacting relativistic matter is carried out at the canonical level of light-front quantization and verified at second order in the coupling. It is equivalent to the canonical form of a commutator of light-ray operators at the same transverse point, an algebra that is theory dependent in interacting conformal field theories [25]. Whether the rule survives where the form factor's slope receives divergent loop corrections is open (Sec. 6). Self-gravity of the matter is neglected throughout, and nothing in this paper is measurable with present means: the interest of the result is structural.

Conventions. Sections 2 and 3, Appendix A and the non-relativistic statements keep $\hbar$, c and G and use the metric signature $(-,+,+,+)$ of Eq. (2). The field-theory formulas of Secs. 4 and 5 and of Appendices B and C are in units $\hbar = c = 1$ with signature $(+,-,-,-)$. The wave travels along $+z$, $k^\mu = (1,0,0,1)$, $u = t - z/c$, and $x_\perp = (x, y)$.

2 The idea: a plane wave sees matter along its own rays

A weak plane gravitational wave, $h_+(t - z/c)$, is usually written in transverse-traceless gauge, where it stretches x and squeezes y. There is a second standard way of writing the same wave [12, 13], in which the whole perturbation sits in one component of the metric,

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 + \tfrac{1}{2}\,\ddot{h}_+(u)\,(x^2 - y^2)\,du^2. \tag{2}$$

In this form the wave is a tidal potential, quadratic in the distance from an axis, that travels at the speed of light. A metric perturbation $h_{\mu\nu}$ couples to matter through $\frac{1}{2}h_{\mu\nu}T^{\mu\nu}$, and here $h_{\mu\nu} \propto k_\mu k_\nu$, so the wave couples only to $T_{\mu\nu}k^\mu k^\nu$: for matter at rest the energy density plus the stress along the direction of travel, and in general the density of the null energy that the stress tensor carries along the wave.

Now let the matter make a transition that absorbs a quantum of the wave. The wave's phase is constant on the null planes $u = \text{const}$. Energy conservation ties the frequency to the energy difference of the two matter states, and for exactly that frequency the time dependence of the transition matrix element of $T_{\mu\nu}k^\mu k^\nu$ at height z compensates the phase of the wave at height z. The integral over z at fixed time becomes an integral along the null ray $t = z/c$. What is left is a function of the transverse position only, the stress tensor integrated along the ray, weighted by the tidal profile $x^2 - y^2$. No assumption about the size of the body enters: a body many

wavelengths long is sampled coherently, because the wave and the transition stay in step along the ray.

The picture for the sum rule is the wave with a kink, $\dot{h}_+$ jumping by v at $u = 0$. In the form (2) this is a tidal kick delivered on one null plane. Every particle receives the velocity $\frac{1}{2}v\,(x, -y, 0)$ as the plane passes it, and because the plane crosses the body far faster than the body can react, the matter responds as if its constituents were free. The energy left behind is $v^2 I/8$ for slow matter, I being the moment of inertia about the direction of travel. The fluence of such a wave per unit frequency falls as $1/\omega^2$, so the same energy is $\propto \int \sigma\, d\omega/\omega^2$. That is the sum rule, and it is the gravitational form of the statement behind the Thomas–Reiche–Kuhn rule, that against a sudden electric kick every charge responds as a free particle. The kick is a memory effect of the wave: a permanent change of velocity, as found for plane waves in Ref. [26] (the memory of bursts was introduced as a permanent displacement in Ref. [27]). For quantum matter it is a shift of the null coordinate that depends on the transverse position, generated by the operator $\mathcal{Q}$ of Eq. (1).

3 The cross-section as the spectral function of a light-ray operator

This section derives the first statement. The matter is any system in flat space with a conserved symmetric stress tensor $T^{\mu\nu}$, written in units of energy density; the wave is weak, and we work to first order in it and in G, neglecting the gravitational field of the matter itself.

The coupling. In transverse-traceless gauge the wave is $h_{xx} = -h_{yy} = h_+(u)$, $u = t - z/c$. The change of gauge given in Appendix A brings it, exactly at linear order and for all $x_\perp$, to the form (2), $h_{\mu\nu}dx^\mu dx^\nu = \frac{1}{2}\ddot{h}_+(u)\, w(x_\perp)\, du^2$ with $w = x^2 - y^2$. With the interaction Lagrangian $\frac{1}{2}h_{\mu\nu}T^{\mu\nu}$ this gives the interaction Hamiltonian

$$H_{\rm int}(t) = -\frac{1}{4c^2}\int d^3x\; \ddot{h}_+(t - z/c)\, w(x_\perp)\, T_{kk}(\mathbf{x}, t), \qquad T_{kk} \equiv T_{\mu\nu}k^\mu k^\nu = T^{00} - 2T^{0z} + T^{zz}. \tag{3}$$

For slow matter in a body small against the wavelength, $T_{kk} \to \rho c^2$ and $\ddot{h}_+$ may be taken at $z = 0$; Eq. (3) is then the familiar tidal coupling $-\frac{1}{4}\ddot{h}_{ij}\sum_a m_a x_a^i x_a^j$. The perturbation (2) grows with $x_\perp$, so first-order perturbation theory requires $h\,(\omega L/c)^2 \ll 1$ for a body of transverse size L; for a weak wave this is no restriction on the size of the body relative to the wavelength.

On shell. Let $h_+ = h_0 \cos\omega u$ and let the matter go from an energy eigenstate i to f with $E_f - E_i = \hbar\omega_{fi} > 0$. The golden rule involves $\int d^3x\, e^{i\omega z/c}\, w\, \langle f|T_{kk}(\mathbf{x}, 0)|i\rangle$ at $\omega = \omega_{fi}$. Since $\langle f|T_{kk}(\mathbf{x}, t)|i\rangle = e^{i\omega_{fi}t}\langle f|T_{kk}(\mathbf{x}, 0)|i\rangle$, the phase $e^{i\omega_{fi}z/c}$ is what evaluates the operator at the time $t = z/c$:

$$\int dz\; e^{i\omega_{fi}z/c}\, \langle f|T_{kk}(x_\perp, z, 0)|i\rangle = \langle f|\, \mathcal{E}(x_\perp)\, |i\rangle, \qquad \mathcal{E}(x_\perp) \equiv \int dz\; T_{kk}(x_\perp, z, t = z/c). \tag{4}$$

$\mathcal{E}(x_\perp)$ is the stress tensor integrated along the null ray through $x_\perp$, with the dimension of energy per area. Dividing the absorbed power by the flux $c^3\omega^2 h_0^2/32\pi G$ of the wave gives

$$\boxed{\sigma(\omega) = \frac{\pi^2 G\,\omega^3}{\hbar c^7} \sum_f \left|\langle f|\mathcal{Q}|i\rangle\right|^2 \delta(\omega - \omega_{fi}), \qquad \mathcal{Q} = \int d^2x_\perp\, w(x_\perp)\, \mathcal{E}(x_\perp).} \tag{5}$$

For the cross polarization $w = 2xy$, and for circular polarizations of the same amplitude $w = (x \pm iy)^2/\sqrt{2}$. For slow matter in a small body $\mathcal{Q} \to c^2 \sum_a m_a(x_a^2 - y_a^2)$ and Eq. (5) is the standard quadrupole result [3, 6]. No multipole expansion has been made: Eq. (5) holds for a body of any size and for relativistic matter. If the initial state is not the ground state, the same expression with $\omega_{fi} < 0$ gives stimulated emission, and σ below denotes absorption minus stimulated emission.

The operator. Smeared averaged null energies $\int d^2x_\perp f(x_\perp)\mathcal{E}(x_\perp)$ are familiar objects: they generate translations along the null rays by an amount $f(x_\perp)$ that varies from ray to ray, and have been studied as supertranslations of a null plane [14, 15]. $\mathcal{Q}$ is the member of this family with the quadrupole profile of a plane gravitational wave. Two properties will be used. The centre-of-mass motion separates: with P the total null momentum $\int d^2x_\perp\, \mathcal{E}$ and R the light-front centre of mass, $\mathcal{Q} = P\, w(R) + \mathcal{Q}_{\rm int}$ exactly, because w is quadratic and the first moment about R vanishes; only the intrinsic part $\mathcal{Q}_{\rm int}$ causes transitions between internal states. And in a one-particle matrix element only one form factor survives (Sec. 4).

4 The sum rule

4.1 The identity, and what it means

The operator $\mathcal{Q}$ in Eq. (5) does not depend on ω. For the ground state, or for net absorption in any stationary state, completeness therefore gives

$$\int_0^\infty \frac{\sigma(\omega)}{\omega^2}\, d\omega = \frac{\pi^2 G}{\hbar^2 c^7}\, \tfrac{1}{2}\langle[\mathcal{Q},[H,\mathcal{Q}]]\rangle, \tag{6}$$

provided the integral converges (Sec. 6). The right-hand side is also the energy left by a sudden wave. If $\dot h_+$ jumps by v at $u = 0$, Eq. (3) integrates to the unitary $U = \exp(iv\mathcal{Q}/4\hbar c^2)$, which shifts the matter along each null ray by $(v/2c)\, w(x_\perp)$, and the energy of the state changes by

$$\Delta E = \frac{v}{4c^2}\left\langle \frac{d\mathcal{Q}}{du}\right\rangle + \frac{v^2}{32\hbar^2 c^4}\langle[\mathcal{Q},[H,\mathcal{Q}]]\rangle + \ldots, \tag{7}$$

the first term vanishing in a stationary state. Equation (6) is the statement that the fluence of this wave per unit frequency, $c^3 v^2/16\pi^2 G\omega^2$, times σ, integrates to the second term.

4.2 Slow matter: the moment of inertia

For particles with $H = \sum_a p_a^2/2m_a + V$, V a function of the positions, and a body small against the wavelengths it absorbs, $\mathcal{Q} = c^2 \sum_a m_a(x_a^2 - y_a^2)$ and $[\mathcal{Q},[H,\mathcal{Q}]] = 4\hbar^2 c^4 \sum_a m_a(x_a^2 + y_a^2)$.

Hence

$$\boxed{\int_0^\infty \frac{\sigma(\omega)}{\omega^2}\,d\omega = \frac{2\pi^2 G\,I}{c^3}, \qquad I = \Big\langle \sum_a m_a(x_a^2 + y_a^2)\Big\rangle,} \tag{8}$$

the moment of inertia about the direction of travel, taken about the centre of mass for the body's own absorption. Planck's constant has dropped out, the forces do not enter, and the result is the same for both polarizations. The kick energy (7) is $v^2 I/8$, as in Sec. 2. Since $\sigma \geq 0$ for a system without population inversion, Eq. (8) bounds the strength of any set of lines: a resonance at ω_0 has $\int \sigma\, d\omega \leq 2\pi^2 G I \omega_0^2/c^3$. Written per logarithmic bandwidth the strength of a line is $2\pi^2 \ell_P^2\,(I\omega_0/\hbar)$ times the fraction of the rule it carries, ℓ_P the Planck length; for a system with $I\omega_0/\hbar$ of order one, such as an atom, this is the observation that gravitons are absorbed with a cross-section of the order of the Planck area [7, 28]. Equation (8) is the energy-weighted quadrupole sum rule of nuclear physics [20] read as a statement about gravitational waves. The neighbouring moment, $\int \sigma\, d\omega$, is fixed by the kinetic and interaction energies [21].

4.3 Relativistic matter: the transverse moment of the averaged null energy

From here on $\hbar = c = 1$. The natural variables are those of the light front [16, 17]: u plays the role of time, its generator contains the transverse momenta quadratically, as a Galilean kinetic energy with the null momentum in the place of mass, and the density T_{kk} on a null plane is the density of that "mass". In these variables Eq. (6) is the Thomas–Reiche–Kuhn rule of a Galilean system in the transverse plane. The sudden wave (7) shifts the fields on the plane $u = 0$ along the rays by $2a\,w(x_\perp)$, $a = v/4$, so the only change in the light-front Hamiltonian P_u, the generator of translations in u, at second order comes from terms with two transverse derivatives: $\partial_i \to \partial_i + 2a(\partial_i w)\partial_\lambda$, with ∂_λ the derivative along the ray. For a scalar field with any potential, for the Maxwell field in light-cone gauge, and for Yukawa, QED and Yang–Mills theories in canonical light-front quantization (constraints eliminated, renormalization and zero modes set aside), those terms are the free kinetic terms: potentials and the instantaneous terms of gauge theories are local across the ray, and the cubic vertices carry at most one transverse derivative. The second-order change is then proportional to $\int |\nabla_\perp w|^2 T_{kk}$ with the coefficient of free matter, $|\nabla_\perp w|^2 = 4x_\perp^2$, that is,

$$\tfrac{1}{2}\langle[\mathcal{Q},[H,\mathcal{Q}]]\rangle = 2\int d^2x_\perp\; x_\perp^2\,\langle\mathcal{E}(x_\perp)\rangle \;+\; \tfrac{1}{2}\langle[\mathcal{Q},[P_u^{\rm int},\mathcal{Q}]]\rangle, \tag{9}$$

where $P_u^{\rm int}$ is the interaction part of P_u; the last term vanishes for the theories just listed at this level. The computation is given for a scalar field in Appendix B, with the changes needed for fermions and gauge fields. Then

$$\boxed{\int_0^\infty \frac{\sigma(\omega)}{\omega^2}\,d\omega = 2\pi^2 G\int d^2x_\perp\; x_\perp^2\,\langle\mathcal{E}(x_\perp)\rangle,} \tag{10}$$

for a neutral body, with the moment taken about the light-front centre of mass. For a static body $\mathcal{E} = \int dz\,(T^{00} + T^{zz})$: the wave weighs the energy and the stress along its direction of travel. For slow matter the right-hand side is $2\pi^2 GI$, Eq. (8). The correction term is the analogue of the enhancement of the nuclear dipole sum rule by exchange currents: an interaction that is not local across the ray carries null energy that the constituents' $\mathcal{E}$ does not count. It is non-zero for a vertex with a form factor (Sec. 6).

4.4 A single particle: the slope of the gravitational form factor

For a particle of mass m the matrix elements of the stress tensor between momentum eigenstates are parametrized by gravitational form factors [29]; for spin 0,

$$\langle p'|T_{\mu\nu}|p\rangle = 2P_\mu P_\nu\, A(t) + \tfrac{1}{2}(\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2)D(t), \tag{11}$$

with $P = (p+p')/2$, $\Delta = p' - p$, $t = \Delta^2$, and for spin 1/2 the same structures appear between spinors, together with a form factor B whose term is linear in Δ and flips the light-front helicity. The integral along the ray in Eq. (4) sets $k\cdot\Delta = 0$, and $k^2 = 0$, so

$$\langle p'|T_{kk}|p\rangle\big|_{k\cdot\Delta=0} = 2\,(k\cdot P)^2 A(t)\,: \tag{12}$$

the D structure, and any term proportional to $g_{\mu\nu}$ such as the trace form factors of a partial stress tensor, drop out identically. For spin 1/2 this is the helicity-conserving part; the B term is odd in $\Delta_\perp$, so it does not change the second transverse moment; since $B(0) = 0$ [30] it does not move the centre either, and for an unpolarized target it averages out. The transverse profile of $\langle\mathcal{E}\rangle$ is the Fourier transform of A, the momentum density of the light front, and its mean square radius is $\langle b^2\rangle = 4A'(0)$, with b the transverse distance from the light-front centre of mass, $A' = dA/dt$ and $A(0) = 1$ [18, 19]. For a neutral particle of spin 0 or 1/2 at rest Eq. (10) becomes

$$\boxed{\int_0^\infty \frac{\sigma(\omega)}{\omega^2}\,d\omega = 8\pi^2 G\, m\, A'(0).} \tag{13}$$

The left-hand side is the inelastic absorption of gravitons by the particle, in which it is broken up or excited; the right-hand side is a property of its elastic form factor. In ordinary units the right-hand side is $8\pi^2 G\hbar^2 m A'(0)/c^3$, with t a squared momentum.

5 Checks

5.1 Slow matter: hydrogen and the bar

Hydrogen. For the $1s$ state, averaged over orientation, $I = 2m_e a_0^2$, and Eq. (8) reads $\int \sigma\, d\omega/\omega^2 = 4\pi^2 G m_e a_0^2/c^3$. With σ in units of $G\hbar/c^3$ and $x = \hbar\omega$ in units of 13.6 eV the prediction is $\sum_{\text{lines}} + \int \sigma\, dx/x^2 = 2\pi^2 = 19.739209$. The spectrum consists of the lines $1s \to nd$ and the continuum. The continuum cross-section with the exact Coulomb final state is

$$\sigma(x) = \frac{2048\pi^2}{15}\,\frac{4x-3}{x^4}\,\frac{e^{-4\eta\,\mathrm{arccot}\,\eta}}{1-e^{-2\pi\eta}}\,\frac{G\hbar}{c^3}, \qquad \eta = (x-1)^{-1/2}, \tag{14}$$

and it contributes 15.983778. The lines $n = 3, \ldots, 60$, from the exact radial integrals, contribute 3.748688, and the remaining lines, estimated from the n^{-3} fall-off of their strength, add 0.006744. The total is 19.739210. The zeroth moment is fixed by the kinetic and interaction energies [21]; for hydrogen this gives $\int \sigma\, dx = (56/15)\cdot 2\pi^2$ in the same units, 73.693046; the same spectrum gives 73.693047 with the lines up to $n = 60$ and the estimated remainder.

A bar. For a thin bar of mass M and length L across the wave, with the favourable polarization, $I = ML^2/12$. Its longitudinal modes of odd order n carry the fractions $96/\pi^4 n^4$ of the rule, which sum to one; the fundamental carries $96/\pi^4 = 0.9855$. This corresponds to the standard

integrated cross-section of the fundamental mode [4], and the $1/n^2$ fall-off of the unweighted integral agrees with Ref. [5]. For this orientation and polarization a bar therefore comes within 1.5% of the most that a body of the same moment of inertia, without population inversion, can absorb in a line at its frequency.

Size. Equation (8) was derived for a body small against the wavelengths it absorbs. Equations (5) and (6) have no such condition, and a slow body of any size is expected to obey Eq. (8) up to corrections of relative order v^2/c^2; in Eq. (10) these are its kinetic energy and its internal stress. The reason is visible in the spectrum: a mechanical mode has a wavelength shorter than the gravitational wavelength at the same frequency by the ratio of the speed of sound to that of light, so the modes that carry the rule see a nearly uniform tidal field.

5.2 A dressed particle, with covariant Feynman rules

Equation (13) can be tested where both sides are computable from first principles. Take a particle of mass m coupled with strength g to a scalar field ϕ of mass μ, and work to order g^2. The left-hand side is then the cross-section for graviton + particle $\to$ particle + ϕ at tree level, integrated with weight $1/\omega^2$. The right-hand side is the one-loop slope of the particle's gravitational form factor. We compute both with the covariant Feynman rules of linearized gravity, $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, $\kappa^2 = 32\pi G$, coupled through $-\frac{\kappa}{2} h_{\mu\nu} T^{\mu\nu}$ (Appendix C). Nothing from the light front is used.

Scalar theory. $\mathcal{L}_{\rm int} = -g\,\Phi^\dagger\Phi\,\phi$ with a complex scalar Φ. For a transverse-traceless polarization $\varepsilon_{\mu\nu}$ the amplitude is

$$\mathcal{M} = \kappa g \left[\frac{\varepsilon_{\mu\nu} p^\mu p^\nu}{2p\cdot k} - \frac{\varepsilon_{\mu\nu} p'^\mu p'^\nu}{2p'\cdot k} - \frac{\varepsilon_{\mu\nu} q^\mu q^\nu}{2q\cdot k} \right], \tag{15}$$

with p, p' the momenta of the particle before and after, q that of ϕ and k that of the graviton; it is invariant under $\varepsilon_{\mu\nu} \to \varepsilon_{\mu\nu} + k_\mu \xi_\nu + k_\nu \xi_\mu$ with $k\cdot\xi = 0$, because $p - p' - q = -k$. The two triangle graphs give

$$A'(0) = \frac{g^2}{96\pi^2} \int_0^1 dx\, \frac{x^2(1-x)^2}{\Delta(x)^2}, \qquad \Delta(x) = (1-x)^2 m^2 + x\mu^2. \tag{16}$$

Yukawa theory. $\mathcal{L}_{\rm int} = -g\,\bar\psi\psi\,\phi$ with a Dirac fermion, the symmetric stress tensor, an unpolarized target. The slope is ultraviolet finite and unaffected by wavefunction renormalization,

$$A'(0) = \frac{g^2}{96\pi^2} \int_0^1 dx\, x(1-x)^2 \left[\frac{m^2(1+x)^2}{\Delta(x)^2} + \frac{2}{\Delta(x)} \right], \tag{17}$$

which follows from the light-front wavefunctions of Ref. [30] and agrees with the Feynman-parameter form to 10^{-13}.

Result. Table 1 compares the two sides. They agree to the accuracy of the quadrature in all six cases. The cross-section rises as the fifth power of the final momentum at threshold, as a quadrupole transition should, and at high energy tends to $Gg^2/2s$ in the scalar theory and

Theory	μ/m	$A'(0)$	$\int \sigma\, d\omega/\omega^2$	ratio -1
scalar	0.5	$2.890447434 \times 10^{-4}$	$2.282205818 \times 10^{-2}$	2×10^{-16}
scalar	0.1	$4.609538525 \times 10^{-3}$	$3.639545737 \times 10^{-1}$	-2×10^{-15}
scalar	2	$8.215019517 \times 10^{-6}$	$6.486319422 \times 10^{-4}$	-7×10^{-16}
Yukawa	0.5	$1.690720157 \times 10^{-3}$	$1.334939128 \times 10^{-1}$	-4×10^{-12}
Yukawa	0.1	$1.961611737 \times 10^{-2}$	1.548826546	-3×10^{-12}
Yukawa	2	$1.430447546 \times 10^{-4}$	$1.129436112 \times 10^{-2}$	-1×10^{-12}

Table 1. *The sum rule (13) at order g^2. Third column: the one-loop slope of the gravitational form factor, Eqs. (16) and (17). Fourth column: the tree-level cross-section for graviton + particle → particle + ϕ, target at rest, one linear polarization, integrated numerically with weight $1/\omega^2$. Last column: the ratio of the fourth column to $8\pi^2$ times the third, minus one. Units: $A'(0)$ in g^2/m^4 and the integral in Gg^2/m^3 for the scalar theory, where g has the dimension of mass; g^2/m^2 and Gg^2/m for the Yukawa theory.*

to the constant $Gg^2/8$ in the Yukawa theory, so the integral converges in both. Both linear polarizations give the same $\sigma(\omega)$.

In the scalar theory the agreement is pointwise and not only integrated: at every energy tested, to 10^{-14}, the covariant cross-section equals Eq. (5) evaluated with the two-particle light-front wavefunction of the dressed particle. There $\mathcal{Q}_{\rm int} = \sqrt{2}\,\mathcal{Q}_\varepsilon$, where $\mathcal{Q}_\varepsilon$ acts as $m\,x(1-x)\,\varepsilon_{ab} r^a r^b$ on a pair with momentum fraction x and transverse separation r, and ε_{ab} is the transverse polarization tensor normalized to $\varepsilon_{ab}\varepsilon_{ab} = 1$. Two further moments have simple values in this theory, found numerically to ten digits: $\int \sigma\, d\omega/\omega^3 = 2\pi^2 G\langle \mathcal{Q}_\varepsilon^2 \rangle$, the fluctuation of the transverse quadrupole, and $\int \sigma\, d\omega/\omega = (8\pi^2/3)\, G\,(1-Z)$, with $1-Z$ the probability of the two-particle component of the dressed particle.

6 Conditions, the correction term, and what is open

Conditions. Equations (6)–(13) assume the ground state or net absorption; no population inversion; first order in G; and a neutral body. The last is an infrared condition: with a massless field attached to a net charge the transverse moment of $\langle \mathcal{E} \rangle$ diverges with the tail of the long-range field. The centre-of-mass part $P\,w(R)$ of $\mathcal{Q}$ sits at zero frequency, where it describes the kick to the body as a whole, and requires wave packets. A rotating body has a T^{0z} term in $\mathcal{E}$. Improvement terms of the stress tensor of the form $(\partial_\mu\partial_\nu - g_{\mu\nu}\partial^2)\phi^2$ are total derivatives along the ray and do not contribute to $\mathcal{E}$.

Convergence. The integral in Eq. (6) converges if σ grows more slowly than ω. In the two theories of Sec. 5 it falls or tends to a constant. Beyond these we have an argument, not a calculation. For hadrons one expects $\sigma \propto \omega^{\alpha(0)-1}$ with $\alpha(0)$ the leading Regge intercept, close to one, so the integral should converge. In light-front terms a constituent couples to the wave through its share of the null momentum, and the momentum sum rule then keeps the right-hand side of Eq. (10) finite. The corresponding integral for photons [31] weights the constituents by the square of the charge divided by that share, and by the same argument does not converge without subtraction.

The correction term. The last term of Eq. (9) is not zero in general. In the two-particle sector of a light-front model with vertex function $V(k)$, k the relative transverse momentum, the energy-weighted sum and the radius side differ by $-g\,\mathrm{Re}\langle\psi|\mathcal{Q}_{\mathrm{int}}^2|V\rangle$ in suitable units, which vanishes when V is a constant or linear in k, because $\mathcal{Q}_{\mathrm{int}}^2 \propto r^4$ annihilates a vertex that is local in the transverse separation. For a vertex with a form factor, $V = \Lambda^2/(k^2+\Lambda^2)$ with $\Lambda = 2m$, $\mu = m/2$ and $x = 0.3$, the difference is 7.8% of the radius side; for a vertex quadratic in k both sides diverge and their difference depends on the regulator. Effective descriptions with non-local vertices, and couplings with two transverse derivatives, are therefore outside Eq. (10) and inside Eq. (9).

What is open. The step from Eq. (6) to Eq. (10) uses canonical light-front commutators. It is equivalent to the statement that the commutator of $\mathcal{E}$ with the integrated transverse momentum density on a null plane has its canonical form at coincident transverse points. For conformal field theories this algebra has been derived under conditions [14, 32], and it is known that the algebra of light-ray operators at coincident points is not universal: in interacting conformal theories it receives contributions that depend on the operator content [25], and commutators of light-ray operators at coincident points require care already in free theories [33]. What enters here is not such a commutator on its own but the double commutator with the Hamiltonian in a state, which for free particles and the free scalar field is evaluated explicitly in Appendix B. Canonical double commutators of currents are also known to fail beyond lowest order in other sum rules [34, 35]. At order g^2 both sides of Eq. (13) are finite and none of this is tested. We therefore regard Eq. (9), with its correction term, as the general statement, and Eqs. (10) and (13) as shown for free particles and the free scalar field, for slow matter, at the canonical level for the theories listed, and at order g^2 in the two theories of Sec. 5. A calculation at the order where the slope of A receives divergent subgraphs, or in a gauge theory with a massless quantum, would test whether a correction appears.

7 Discussion

The result can be said in one sentence: a plane gravitational wave reads matter along its own null rays, and how much the matter can absorb in total is fixed, at the level shown here, by how far its null momentum is spread across them.

Equation (13) relates an inelastic integral to the slope of an elastic form factor. It belongs with the Cabibbo–Radicati sum rule, which does the same for the isovector charge radius and magnetic moment of the nucleon [36], and with the relativistic forms of the Thomas–Reiche–Kuhn rule [31]. Its real-photon relatives are the sum rules that tie an inelastic integral to a static property of the target, such as that of Ref. [37]. Two things set the gravitational rule apart. Its integral is expected to converge without subtraction, because the wave couples to null momentum, which is bounded by the total (Sec. 6); this is an expectation, shown only in the two theories of Sec. 5. And only the form factor A appears: the D-term, which carries the pressure and shear inside a hadron [29], and the trace form factors drop out, because the wave's null vector annihilates their tensor structures on the ray. The total absorption of a gravitational wave thus measures the second moment of the light-front momentum density of Refs. [18, 19].

The same operator governs sudden waves. The unitary $\exp(iv\mathcal{Q}/4)$ of Sec. 4 is the action of an impulsive plane wave on quantum matter; unitaries of this kind, with general profiles,

generate shifts along null generators that vary from one generator to the next [15], and the plane wave selects the quadrupole profile. The velocity memory of the wave, the energy it leaves and the sum rule all come from the one operator $\mathcal{Q}$.

For ordinary matter the relativistic part of the right-hand side of Eq. (10) is a correction of relative order pressure over energy density, and the absorption itself is far below anything measurable [28, 38]. What the rule offers is a constraint. Under the conditions of Sec. 6, a computed absorption spectrum of a body, from a bar to an atom to a model hadron, should integrate to the moment of its null energy. One that does not contains an error, has a correction term to exhibit, or lies where the canonical evaluation fails.

Appendix A: From transverse-traceless gauge to the adapted gauge

With $c = 1$, signature $(-,+,+,+)$ and $u = t - z$, the transverse-traceless wave is $h_{xx} = -h_{yy} = h_+(u)$. Under $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$ with

$$\xi^t = \xi^z = -\tfrac{1}{4}\,\dot{h}_+(u)\,(x^2 - y^2), \qquad \xi^x = -\tfrac{1}{2}\,h_+(u)\,x, \qquad \xi^y = +\tfrac{1}{2}\,h_+(u)\,y, \tag{18}$$

the perturbation becomes $h_{\mu\nu}dx^\mu dx^\nu = \tfrac{1}{2}\ddot{h}_+(u)\,(x^2 - y^2)\,du^2$, exactly at linear order and for all x, y. In this gauge $h_{tt} = h_{zz} = -h_{tz} = \tfrac{1}{2}\ddot{h}_+ w$, so $\tfrac{1}{2}h_{\mu\nu}T^{\mu\nu} = \tfrac{1}{4}\ddot{h}_+\, w\,(T^{tt} - 2T^{tz} + T^{zz})$ and $H_{\rm int} = -\int d^3x\, \tfrac{1}{2}h_{\mu\nu}T^{\mu\nu}$ is Eq. (3). A free particle at rest obeys $\ddot{x} = \tfrac{1}{2}\ddot{h}_+ x$, $\ddot{y} = -\tfrac{1}{2}\ddot{h}_+ y$, the geodesic deviation of the wave. For a jump v of $\dot{h}_+$ at $u = 0$ the gauge vector itself jumps by $\delta\xi^t = \delta\xi^z = -\tfrac{1}{4}vw$: the null coordinate $t + z$ of matter crossing the plane is shifted by $\tfrac{1}{2}v\,w(x_\perp)$, the junction condition of an impulsive plane wave.

Appendix B: The double commutator on the light front

Units $\hbar = c = 1$, $u = t - z$, $v = t + z$. For a real scalar field with any potential $U(\phi)$, $S = \int du \int \tfrac{1}{2}dv\, d^2x_\perp\, [2\,\partial_u\phi\,\partial_v\phi - \tfrac{1}{2}(\nabla_\perp\phi)^2 - U(\phi)]$. With u as time the fields on a null plane obey $[\phi(v,x_\perp), \partial_{v'}\phi(v',x'_\perp)] = \tfrac{i}{2}\,\delta(v - v')\,\delta^2(x_\perp - x'_\perp)$, and the generators of translations in u and v are

$$P_u = \int \tfrac{1}{2}dv\, d^2x_\perp \left[\tfrac{1}{2}(\nabla_\perp\phi)^2 + U(\phi)\right], \qquad P_v = \int dv\, d^2x_\perp\, (\partial_v\phi)^2, \qquad H = P_u + P_v. \tag{19}$$

The interaction sits in P_u only. On the plane $T_{kk} = 4(\partial_v\phi)^2$, so $\mathcal{E}(x_\perp) = 2\int dv\,(\partial_v\phi)^2$, $\int d^2x_\perp\,\mathcal{E} = 2P_v$, and

$$\mathcal{Q} = 2\int dv\, d^2x_\perp\; w(x_\perp)\,(\partial_v\phi)^2, \qquad [\mathcal{Q}, \phi(v,x_\perp)] = -2i\,w(x_\perp)\,\partial_v\phi(v,x_\perp). \tag{20}$$

Hence $e^{ia\mathcal{Q}}\phi(v,x_\perp)e^{-ia\mathcal{Q}} = \phi(v + 2a\,w(x_\perp), x_\perp)$: the fields are shifted along each ray by $2a\,w$. P_v and the potential term are unchanged by a shift of the integration variable v at fixed $x_\perp$, so $[P_v, \mathcal{Q}] = 0$ and the potential does not contribute. The transverse gradient of the shifted field is $\nabla_\perp\phi + 2a\,(\nabla_\perp w)\,\partial_v\phi$, and

$$e^{-ia\mathcal{Q}}P_u e^{ia\mathcal{Q}} = P_u + a\int dv\, d^2x_\perp\; \nabla_\perp w\cdot\nabla_\perp\phi\;\partial_v\phi + a^2\int dv\, d^2x_\perp\; |\nabla_\perp w|^2(\partial_v\phi)^2. \tag{21}$$

The last term is $\frac{1}{2}a^2\int d^2x_\perp\,|\nabla_\perp w|^2\,\mathcal{E} = 2a^2\int d^2x_\perp\,x_\perp^2\,\mathcal{E}$ for $w = x^2 - y^2$, and it equals $\frac{1}{2}a^2[\mathcal{Q},[H,\mathcal{Q}]]$. This is Eq. (9) without the correction term, with the coefficient 2, for a scalar field with an arbitrary potential, at the level of the canonical commutator on the null plane.

For a Dirac field the independent component obeys an equation of the same form, with P_u containing $\psi_+^\dagger\,(m^2-\nabla_\perp^2)\,(i\partial_v)^{-1}\psi_+$; under the shift the second-order term is again $|\nabla_\perp w|^2$ times the density of P_v, and no ordering term arises because w is harmonic. A Yukawa or gauge coupling enters the constraint that determines the dependent component with at most one transverse derivative per vertex, and the instantaneous terms it generates are local in $x_\perp$. For a gauge field in light-cone gauge the two transverse components behave as two scalars once the constrained component is eliminated; both the F_{ij}^2 and the $(\partial_i A_i)^2$ terms are needed for this. An interaction with two transverse derivatives acting on different fields, or one that is not local in $x_\perp$, changes P_u at second order and gives the correction term of Eq. (9). For free relativistic particles the same result follows with the points at which the particles cross the null plane as coordinates: $[\mathcal{Q},[H,\mathcal{Q}]] = 4\sum_a (E_a - p_{a,z})\,b_a^2$.

Appendix C: Feynman rules, amplitudes and form factors

Units $\hbar = c = 1$, metric $(+,-,-,-)$, $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, $\kappa^2 = 32\pi G$, interaction $-\frac{\kappa}{2}h_{\mu\nu}T^{\mu\nu}$. A graviton of polarization $\varepsilon_{\mu\nu}$, normalized to $\varepsilon_{\mu\nu}\varepsilon^{\mu\nu} = 1$, attaches to a scalar line of mass M with incoming momentum p_1 and outgoing p_2 through $-\frac{i\kappa}{2}\varepsilon_{\mu\nu}\big[p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - \eta^{\mu\nu}(p_1\cdot p_2 - M^2)\big]$, and to a fermion line through the symmetric (Belinfante) tensor, $-\frac{i\kappa}{2}\varepsilon_{\mu\nu}\big[\frac{1}{2}\gamma^{(\mu}(p_1+p_2)^{\nu)} - \eta^{\mu\nu}\big(\frac{1}{2}\gamma\cdot(p_1+p_2) - m\big)\big]$. The interaction Lagrangian contributes a contact vertex proportional to $\eta^{\mu\nu}$. For a transverse-traceless polarization every $\eta^{\mu\nu}$ term drops, and the amplitude is the sum of the three pole graphs in which the graviton attaches to the incoming particle, the outgoing particle and the scalar; for the scalar theory this is Eq. (15). With the target at rest $\varepsilon_{\mu\nu}p^\nu = 0$ and the first graph vanishes. Under a general gauge change $\varepsilon_{\mu\nu} \to \varepsilon_{\mu\nu} + k_\mu\xi_\nu + k_\nu\xi_\mu$ the contact term is needed for the Ward identity, which we have checked numerically with explicit Dirac matrices in boosted frames.

The cross-section for a target at rest is $\sigma = (4m\omega)^{-1}\,|\mathbf{p}'|\,(16\pi^2\sqrt{s})^{-1}\int d\Omega\,\overline{|\mathcal{M}|^2}$, with $s = m^2 + 2m\omega$ and $\mathbf{p}'$ the final momentum in the centre-of-mass frame. In that frame, for the scalar theory and $\varepsilon_{xx} = -\varepsilon_{yy} = 1/\sqrt{2}$, $\mathcal{M} = -(\kappa g/2\sqrt{2})\,p_\perp'^2\cos 2\varphi\,[1/(p'\cdot k) + 1/(q\cdot k)]$.

For the form factor of the scalar theory, the graph with the graviton on the Φ line and the graph with the graviton on the ϕ line give, with Feynman parameters and x the share of the particle's momentum carried by Φ,

$$A(t) - A(0) = \frac{g^2}{16\pi^2}\int_0^1 dx\,\Big[x^2\int_0^{1-x} dy\,\Big(\frac{1}{\Delta - y(1-x-y)t} - \frac{1}{\Delta}\Big) + (1-x)^2\int_0^x dy\,\Big(\frac{1}{\Delta - y(x-y)t} - \frac{1}{\Delta}\Big)\Big], \tag{22}$$

whose derivative at $t = 0$ is Eq. (16). Wavefunction renormalization is independent of t and fixes $A(0) = 1$. For the Yukawa theory the Feynman-parameter result is

$$A'(0) = \frac{g^2}{96\pi^2}\int_0^1 dx\,\left[\frac{m^2x(1-x)^2(1+x)(1+x^2)}{\Delta^2} + \frac{2x(1-x)\big((1-x)^2+x^2\big)}{\Delta}\right], \tag{23}$$

which differs from the integrand of Eq. (17) by the total derivative $\frac{d}{dx}[-x^3(1-x)^2/\Delta]$.

Appendix D: Numerical values

Hydrogen $1s$ (units $G\hbar/c^3$ for σ, 13.6 eV for $\hbar\omega$). Continuum, Eq. (14), weight $1/x^2$: 15.983778. Lines $1s \to nd$, $n = 3, \ldots, 60$: 3.748688; $n > 60$, from the n^{-3} fall-off with coefficient 49.371: 0.006744. Sum 19.739210; $2\pi^2 = 19.739209$. Weight 1: continuum 70.458614, lines to $n = 60$ 3.227691, $n > 60$ estimated 0.006742, sum 73.693047; $(56/15)\,2\pi^2 = 73.693046$.

Bar. Fraction of Eq. (8) in the mode of odd order n: $96/\pi^4 n^4$; $n = 1$: 0.985534.

Scalar theory, other moments ($G = g = m = 1$; $\mu = 0.5$, 0.1, 2). $\int \sigma\, d\omega/\omega^3 = 2\pi^2 G\langle \mathcal{Q}_\varepsilon^2 \rangle$ with $\langle \mathcal{Q}_\varepsilon^2 \rangle = (g^2 m^2/30\pi^2) \int_0^1 dx\, x^3(1-x)^3/\Delta^3$: $1.211306654 \times 10^{-2}$, 1.049089398, $5.497500263 \times 10^{-5}$. $\int \sigma\, d\omega/\omega = (8\pi^2/3) G(1-Z)$ with $1-Z = (g^2/16\pi^2) \int_0^1 dx\, x(1-x)/\Delta$: $7.596411294 \times 10^{-2}$, $2.511999840 \times 10^{-1}$, $1.324025695 \times 10^{-2}$.

High energy. Scalar theory: $\sigma \to Gg^2/2s$. Yukawa theory: $\sigma \to Gg^2/8$.

Vertex with a form factor. Two-particle sector at fixed momentum fraction $x = 0.3$, $m = 1$, $\mu = 0.5$, $V = \Lambda^2/(k^2 + \Lambda^2)$, $\Lambda = 2$: energy-weighted sum minus radius side = 0.02124, which is 7.8% of the radius side and equals the correction term.

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