

Holographic Cosmology: Dark Energy and the Arrow of Time from One Horizon Entropy Budget

Quantum-Geometric Correspondence

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Abstract

The Bekenstein–Hawking entropy of the cosmological horizon is one budget, and this paper spends it twice. Equipartition of the horizon’s thermal energy against the critical density gives the dark-energy density $\rho_{\text{DE}} = \alpha c^2 H^2 / G$ with $\alpha = 3\Omega_{\text{DE}} / (8\pi) \approx 0.082$; generalized-second-law saturation at the de Sitter attractor pins the equation of state to $w = -1$ exactly, indistinguishable from a cosmological constant and consistent with $w_0 = -1.03 \pm 0.03$. The same budget, applied to the shrinking past horizon, implements the Past Hypothesis: the holographic bound $S(t) \leq S_{\text{dS}}(t)$ with $S_{\text{dS}}(t) \rightarrow 0$ as $t \rightarrow 0$ forces low initial entropy without fine-tuning, reads Penrose’s 10^{123} as the horizon’s *capacity* rather than a probability, makes the smallness of ρ_{DE} in Planck units the reciprocal of that capacity, and orients the Weyl curvature asymmetry along the direction of cosmic decoherence. The two applications are tied by the identity $\rho_{\text{DE}}^\infty S_{\text{dS}}^\infty = 3c^7 / (8G^2 \hbar)$, exact in the de Sitter attractor, both sides depending on H_∞ alone. The limits are stated plainly: α is the measured cosmic entropy budget, fitted rather than predicted; the cosmological constant problem is reparameterized, not solved; generalized-second-law saturation is assumed at the attractor; the operator form of the Weyl hypothesis is heuristic; and no observable discriminates against Λ CDM at current precision. The gain is conceptual: dark energy and the arrow of time are two readings of one horizon entropy budget.

Keywords: dark energy, holographic principle, cosmological constant problem, Past Hypothesis, arrow of time, Weyl curvature hypothesis, de Sitter horizon entropy, quantum-geometric correspondence

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1 Introduction

Two of cosmology’s oldest puzzles concern quantities separated by 120 orders of magnitude, and both are questions about the same horizon.

Why is the dark-energy density small but nonzero? Quantum field theory estimates a vacuum energy $\rho_{\text{vac}} \sim \rho_{\text{Pl}} \sim 10^{113} \text{ J/m}^3$; cosmic acceleration [1, 2, 3, 4] requires $\rho_{\text{DE}} \sim 10^{-9} \text{ J/m}^3$. The ratio $\rho_{\text{DE}}/\rho_{\text{Pl}} \sim 10^{-122}$ is the largest disagreement between theory and observation in physics [5]. Anthropic selection [6], quintessence [7], and modified gravity [8] relocate rather than resolve the question of why the scale is so small.

Why was the early universe in a state of extraordinarily low entropy? The microscopic dynamics of statistical mechanics is time-symmetric, yet the macroscopic world is not. The standard resolution adds an initial-condition postulate by hand—the Past Hypothesis [25, 26, 27]: at some early epoch the universe occupied a microstate of extraordinarily low coarse-grained entropy, from which the Second Law follows by ordinary statistical reasoning. Its sharp geometric form is Penrose’s Weyl Curvature Hypothesis [27, 28]: the Weyl tensor $C_{\mu\nu\rho\sigma}$, the tidal gravitational-wave part of curvature, is extraordinarily close to zero at the Big Bang and diverges at black-hole singularities, an asymmetry Penrose estimated at fine-tuning $\exp(-10^{123})$. The postulate is necessary but unmotivated: nothing in the dynamics demands it.

1.1 One budget, spent twice

The thesis of this paper is that both puzzles are statements about a single quantity, the Bekenstein–Hawking entropy of the cosmological horizon,

$$S_{\text{dS}}(t) = \frac{A_H(t)}{4\ell_{\text{P}}^2} = \frac{\pi c^5}{H(t)^2 \hbar G}, \quad \ell_{\text{P}} = \sqrt{\hbar G/c^3}, \quad (1)$$

the holographic capacity of a comoving observer’s causal patch at Hubble rate $H(t)$. The budget is used twice.

First, for dark energy. Equipartition of the horizon’s thermal energy against the critical energy—equivalently, the holographic bound with the horizon as infrared cutoff—gives

$$\rho_{\text{DE}} = \alpha \frac{c^2 H^2}{G}, \quad \alpha = \frac{3\Omega_{\text{DE}}}{8\pi} \approx 0.082. \quad (2)$$

Extending the generalized second law to the cosmological horizon makes de Sitter space the thermodynamically consistent late-time attractor; saturation there pins the equation of state to $w = -1$ exactly.

Second, for the arrow of time. The same budget, applied to the shrinking past horizon where $H(t) \rightarrow \infty$ and hence $S_{\text{dS}}(t) \rightarrow 0$ as $t \rightarrow 0$, implements the Past Hypothesis as a kinematic consequence of the holographic bound $S(t) \leq S_{\text{dS}}(t)$: there is no room for high entropy in the early universe. Penrose’s 10^{123} becomes S_{dS}^∞ in nats—the *capacity* of the cosmic horizon, not a probability assigned to the Big Bang. The smallness of ρ_{DE} in Planck units is the reciprocal of that capacity. The Weyl asymmetry is the direction of cosmic decoherence: the Big Bang upstream of all branching, black-hole singularities downstream.

The two applications are tied by an exact identity in the de Sitter attractor of any FRW

cosmology,

$$\rho_{\text{DE}}^\infty S_{\text{dS}}^\infty = \frac{3c^7}{8G^2\hbar}, \quad (3)$$

a Planck-scale constant. Both sides depend on H_∞ alone: dark-energy density and cosmic entropy capacity are inverses of each other up to that constant. Reducing ρ_{DE} raises the capacity, and conversely; the two are not independent.

1.2 Scope, stated up front

The construction is a conceptual rearrangement, and its claims carry different weights, stated here and once more in full in Section 7.

Remark 1.1 (Honest scope). The coefficient $\alpha = 3\Omega_{\text{DE}}/(8\pi)$ is the measured cosmic entropy budget in Planck units, fitted to Ω_{DE} , not predicted; its value encodes the same mystery as Λ . The cosmological constant problem is reparameterized, not solved. Generalized-second-law saturation is assumed at the attractor, not derived. The operator form of the Weyl hypothesis is heuristic, with its well-definedness issues flagged and no proof claimed. No observable distinguishes the framework from Λ CDM at current precision; the gain is conceptual.

The identity (3) is a re-labeling, not a reduction: both sides are functions of H_∞ , and the algebra occupies three lines. Versions appear in the holographic-dark-energy literature from Banks [12], Fischler–Susskind [13], and Cohen–Kaplan–Nelson [9] onward. The novelty here is the *coupling*: one horizon budget serves dark energy and the arrow of time within one framework.

This paper belongs to the Quantum-Geometric Correspondence (QGC) series. The canonical core paper [36] presents the three primitive axioms, the gravitational-decoherence prediction, and the holographic bound adopted here as premise; companion papers treat emergent gravity with MOND phenomenology [37], the laboratory decoherence floor [38], and cosmic gravitational decoherence on a de Sitter background [39]. Each paper is self-contained.

Section 2 states the holographic capacity and tabulates the cosmic entropy budget. Section 3 derives the dark-energy formula, the saturation parameter, and $w = -1$, and addresses the coincidence problem. Section 4 gives the observational status against DESI and Planck. Section 5 derives the central identity. Section 6 develops the arrow of time—the Past Hypothesis and the Weyl asymmetry—and why $\Lambda > 0$ closes the budget. Section 7 states the limits once for the whole paper and compares with alternatives; Section 8 concludes.

Conventions. We work in geometric and Planck units interchangeably, with conversions given where ambiguity may arise. The Hubble parameter $H(t) = \dot{a}/a$ is positive throughout the cosmic history considered. The Bekenstein–Hawking entropy is $S = A/(4\ell_{\text{P}}^2)$ with $\ell_{\text{P}} = \sqrt{\hbar G/c^3}$. We write $S_{\text{dS}}(t) \equiv \pi c^5/(H(t)^2 \hbar G)$ for the de Sitter horizon entropy at instantaneous Hubble rate $H(t)$, and $S_{\text{dS}}^\infty \equiv S_{\text{dS}}(H_\infty)$ for its asymptotic value at $H_\infty = H_0 \sqrt{\Omega_{\text{DE}}}$.

2 The Horizon Budget

The central object is the cosmic entropy capacity $S_{\text{dS}}(t)$: the Bekenstein–Hawking entropy of the cosmological horizon, read as the holographic bound on the observer’s causal patch. This section states the two principles the budget rests on, identifies the relevant horizons, establishes the de Sitter attractor at which the budget saturates, and tabulates the budget across cosmic history.

2.1 Two principles from black-hole physics

Both principles are imported from black-hole thermodynamics and extended to the cosmological context. Neither has been rigorously proven for cosmological horizons; each is a working assumption whose consistency the framework tests.

Axiom 2.1 (Holographic bound). For any spacelike region $\mathcal{R}$ with bounding area $A(\mathcal{R})$, the von Neumann entropy of the quantum state supported on $\mathcal{R}$ satisfies

$$S(\mathcal{R}) \leq \frac{A(\mathcal{R})}{4\ell_{\text{P}}^2}, \quad \ell_{\text{P}} \equiv \sqrt{\hbar G/c^3}. \quad (4)$$

The bound originates in black-hole physics: black holes carry entropy proportional to horizon area, and packing more entropy into a region induces gravitational collapse [17, 19, 20]. Within QGC it is a theorem of the framework, following from the first primitive axiom together with the generalized second law [36]; here we adopt it as premise. It underlies the holographic ansatz $\rho_{\text{DE}} \propto 1/L^2$ of Section 3.

Assumption 2.2 (Generalized second law with saturation). For cosmological horizons the generalized entropy $S_{\text{gen}} = S_{\text{matter}} + S_{\text{horizon}}$ satisfies $dS_{\text{gen}}/dt \geq 0$, and approaches its maximum—saturates—as $t \rightarrow \infty$.

The horizon entropy takes the Bekenstein–Hawking form $S_{\text{horizon}} = A/(4\ell_{\text{P}}^2)$ [17, 18]. For black holes the generalized second law (GSL) is well-established by semiclassical arguments [20]. Its extension to cosmological horizons is physically motivated but subtle: such horizons are observer-dependent, and their thermodynamic interpretation remains open. The framework proceeds on the assumption that the thermodynamic description holds, and tests it by consistency.

2.2 The two cosmological horizons

An expanding universe possesses horizons that limit causal access and carry thermodynamic properties analogous to black-hole horizons.

The Hubble horizon $r_H = c/H$, with $H = \dot{a}/a$, is the distance at which recession reaches the speed of light. In de Sitter space an observer perceives thermal radiation at $T_H = \hbar H/(2\pi k_B)$ [18], of order 10^{-30} K today.

The future event horizon

$$R_h(t) = a(t) \int_t^\infty \frac{c dt'}{a(t')} \quad (5)$$

is the boundary beyond which events are never observable; it depends on the entire future evolution, not on instantaneous data. It carries temperature $T_E = \hbar c/(2\pi k_B R_h)$, structurally the Hawking form at the event-horizon scale. In de Sitter space $R_h = c/H$ and the two temperatures coincide, $T_H = T_E$; this equality characterizes the late-time attractor (Section 2.3).

Remark 2.1 (Teleological character of the event horizon). $R_h(t)$ depends on the *future* evolution of the universe, whereas physics is normally fixed by initial conditions. This limitation is shared by all event-horizon-based approaches and is not resolved here. The Hubble horizon avoids it but gives $w = 0$, ruled out observationally (Section 3.2). We read the event horizon not as exerting causal influence from the future but as encoding global constraints: the universe, as a solution to Einstein’s equations, is determined as a whole.

The choice of the event horizon over the Hubble horizon as infrared cutoff is empirical. The Hubble cutoff predicts $w = 0$ —dark energy tracks matter, producing no acceleration—ruled out at more than 30σ [3]. The event-horizon cutoff yields $w = -1$, consistent with all current data. It is chosen because it works, not because it is derived from first principles; the theoretical justification remains incomplete.

2.3 The de Sitter attractor

Any cosmology dominated by dark energy with $w < -1/3$ asymptotically approaches de Sitter space—a consequence of the Friedmann equations, not a prediction. In de Sitter space $R_h = c/H$, so

$$HR_h = c \quad (\text{de Sitter geometric identity}). \quad (6)$$

Proposition 2.3 (Thermodynamic consistency of de Sitter). *The horizon entropy $S_E = \pi c^3 R_h^2 / (G\hbar)$ saturates, $dS_E/dt \rightarrow 0$, if and only if $HR_h \rightarrow c$.*

Proof. Differentiating the defining integral (5) gives the event-horizon evolution

$$\frac{dR_h}{dt} = HR_h - c, \quad (7)$$

hence

$$\frac{dS_E}{dt} = \frac{2\pi c^3 R_h}{G\hbar} (HR_h - c). \quad (8)$$

Since $R_h > 0$, the right side vanishes exactly when $HR_h = c$, the de Sitter condition (6). $\square$

Remark 2.2 (Scope of Proposition 2.3). The GSL is *consistent with* de Sitter asymptotics and explains why the de Sitter endpoint maximizes entropy; it does not *derive* de Sitter or predict $w = -1$. The de Sitter endpoint is the attractor of any dark-energy-dominated cosmology; the result shows this attractor is thermodynamically sensible, not that thermodynamics selects it. The temperature equality $T_H = T_E$ is likewise a geometric fact of de Sitter space, not a physical equilibration: the horizons are not in causal contact and no process equilibrates them.

2.4 The budget across cosmic history

Applied to the de Sitter horizon of a comoving observer at instantaneous Hubble rate $H(t)$, the holographic bound (4) becomes a definite finite number. With Hubble area $A_H = 4\pi R_H^2$ and $R_H = c/H$,

$$S_{\text{dS}}(t) = \frac{A_H(t)}{4\ell_{\text{P}}^2} = \frac{4\pi c^2/H(t)^2}{4\hbar G/c^3} = \frac{\pi c^5}{H(t)^2 \hbar G}. \quad (9)$$

This is the *capacity*: the maximum entropy any subsystem in the observer’s causal patch can carry, by Axiom 2.1. It is observer-dependent, but in a homogeneous FRW universe all comoving observers share the same $S_{\text{dS}}(t)$.

Equation (9) tracks the dominant energy component through $H(t)$: $H \propto a^{-2}$ in radiation, $H \propto a^{-3/2}$ in matter, $H \rightarrow H_\infty$ in the de Sitter attractor. The capacity grows monotonically from its smallest value at the Planck epoch to a finite asymptotic value S_{dS}^∞ set by the dark-energy density [21]. Table 1 lists S_{dS} at six epochs; the growth from $\approx \pi$ nats at the Planck time to $\approx 3 \times 10^{122}$ nats in the asymptotic future is the structural fact on which the paper hinges.

Table 1. Cosmic horizon entropy $S_{\text{dS}} = \pi c^5 / (H^2 \hbar G)$ at representative epochs. Hubble rates are standard estimates from Friedmann evolution with Λ CDM parameters; the asymptotic value uses $H_\infty = H_0 \sqrt{\Omega_{\text{DE}}}$ with Planck 2018 values $H_0 = 67.4 \text{ km/s/Mpc}$, $\Omega_{\text{DE}} = 0.689$.

Epoch	$H \text{ (s}^{-1}\text{)}$	$S_{\text{dS}} \text{ (nats)}$	Comment
Planck time	$\sim 1/t_{\text{P}} \approx 1.85 \times 10^{43}$	$\sim \pi$	one Planck area
End of inflation	$\sim 10^{36}$	$\sim 10^{17}$	$H_{\text{inf}} \sim 10^{14} \text{ GeV}$
Matter–radiation eq.	3.40×10^{-13}	9.3×10^{111}	$z_{\text{eq}} \approx 3400$
Recombination	5.11×10^{-14}	4.1×10^{113}	$z_{\text{LSS}} \approx 1100$
Today	2.18×10^{-18}	2.27×10^{122}	$H_0 = 67.4 \text{ km/s/Mpc}$ (Planck 2018)
de Sitter asymptote	1.81×10^{-18}	3.29×10^{122}	$H_\infty = H_0 \sqrt{\Omega_{\text{DE}}}$

The values span 122 orders of magnitude. Two features recur:

- (i) $S_{\text{dS}}(t) \rightarrow 0$ as $t \rightarrow 0$, forced by $H \rightarrow \infty$ in any FRW cosmology with a past singularity. This is the kinematic ingredient of the Past Hypothesis (Section 6).
- (ii) $S_{\text{dS}}(t) \rightarrow S_{\text{dS}}^\infty$ as $t \rightarrow \infty$ with S_{dS}^∞ finite, provided $\Lambda > 0$. Without dark energy the asymptotic capacity is either undefined (recollapse) or infinite (Minkowski limit). The finiteness of S_{dS}^∞ is exactly the structural role of dark energy in this account.

2.5 From bound to budget: saturation

A bound is not a budget. The inequality (4) states $S(t) \leq S_{\text{dS}}(t)$ without requiring equality; whether the actual cosmic entropy tracks the bound or sits below it is a separate question. Assumption 2.2 answers it: the cosmic-horizon entropy saturates in the de Sitter attractor, so the actual entropy tracks the capacity.

Assumption 2.4 (GSL saturation of the cosmic budget). In any FRW cosmology with a de Sitter attractor, the actual entropy of the observer’s causal patch tracks the holographic bound: $S(t) \sim S_{\text{dS}}(t)$ at every epoch, with $S(t)/S_{\text{dS}}(t) \rightarrow 1$ in the asymptotic future.

The assumption is non-trivial. Within QGC it is motivated by the cosmic gravitational decoherence rate [39]: per-mode gravitational decoherence at rate $g(1)H/(4\pi)$ with $g(1) = 1 - (2/\pi)\text{Si}(1) \approx 0.398$, multiplied by the holographically-allowed number of horizon modes, gives a total cosmic entropy production rate scaling as H . That cosmic rate holds within a controlled approximation assuming decoherent-histories branching, holographic finiteness of the mode count, and free/Gaussian factorization of mode contributions [39]. The per-mode rate alone does not saturate the bound: integrated over the age of the universe ($N_{\text{efolds}} \sim 140$), it gives only $\sim N_{\text{efolds}}/(10\pi) \sim 4$ nats per mode, far short of $S_{\text{dS}}^\infty \sim 10^{122}$. The 122-order shortfall is closed by the *mode count*: each Hubble volume contains S_{dS} holographically distinguishable horizon modes, and summing the per-mode contribution over this count multiplies by precisely S_{dS} . The saturation is supplied by the mode count, not the per-mode rate; Assumption 2.4 requires the count to do the structural work. Appendix B works the numerics and flags the residual $O(1)$ ambiguity.

Under Assumption 2.4 the budget $S_{\text{dS}}(t)$ is the actual cosmic entropy at epoch t , not merely an upper bound. Three readings recur, worked in the following sections: the dark-energy density

(Section 3), the reciprocal cosmological-constant hierarchy (Section 5), and the Past Hypothesis with its Weyl-asymmetry corollary (Section 6).

3 Dark Energy from the Budget

The first use of the horizon budget is the dark-energy density. The holographic ansatz with the horizon scale as infrared cutoff fixes the H^2 scaling; the de Sitter attractor of Section 2.3 fixes the coefficient's structure, the saturation parameter, and the equation of state.

3.1 The H^2 scaling and the coefficient α

For a region of characteristic size L the holographic ansatz posits $\rho_{\text{DE}} \propto 1/L^2$. With L the horizon scale c/H , dimensional analysis gives

$$\rho_{\text{DE}} = \alpha \frac{c^2 H^2}{G}, \quad (10)$$

since $c^2 H^2/G$ carries the dimensions of energy density. This is dimensional analysis given the ansatz, not a first-principles derivation; the coefficient α encodes the unexplained physics. Equation (10) is the late-time de Sitter relation, where the event horizon $R_h = c/H$ coincides with the Hubble scale; α is read off at the attractor.

To determine α , substitute (10) into the Friedmann equation. In a flat universe of matter and dark energy,

$$H^2 = \frac{8\pi G}{3c^2}(\rho_m + \rho_{\text{DE}}). \quad (11)$$

Substituting $\rho_{\text{DE}} = \alpha c^2 H^2/G$ and solving,

$$H^2 = \frac{8\pi G \rho_m}{3c^2(1 - 8\pi\alpha/3)}. \quad (12)$$

Positivity $H^2 > 0$ requires the self-consistency bound $\alpha < 3/(8\pi) \approx 0.119$; larger α would make the dark-energy contribution exceed the total. With $\rho_{\text{crit}} = 3c^2 H^2/(8\pi G)$, the dark-energy fraction is

$$\Omega_{\text{DE}} = \frac{\rho_{\text{DE}}}{\rho_{\text{crit}}} = \frac{\alpha c^2 H^2/G}{3c^2 H^2/(8\pi G)} = \frac{8\pi\alpha}{3}, \quad (13)$$

inverted to

$$\alpha = \frac{3\Omega_{\text{DE}}}{8\pi}. \quad (14)$$

The observed $\Omega_{\text{DE}} = 0.689 \pm 0.006$ [3] yields

$$\alpha \approx 0.082. \quad (15)$$

Propagating the statistical uncertainty in Ω_{DE} alone gives a formal error at the one-percent level; this figure is statistical only and does not account for the H_0 -tension systematic, so we quote $\alpha \approx 0.082$ rather than a sharp ± 0.001 .

Cosmic acceleration requires $\Omega_{\text{DE}} > 1/2$, i.e. $\alpha > 3/(16\pi) \approx 0.060$. With the self-consistency bound,

$$0.060 \lesssim \alpha \lesssim 0.119, \quad (16)$$

a range spanning a factor of two, within which $\alpha \approx 0.082$ falls. The magnitude is not explained: the mystery of why $H_0 \ll H_P$ is equivalent to the mystery of why $\Lambda \ll \Lambda_P$. The claim that $\alpha \sim 0.1$ is “natural” is tautological, since $\alpha = 3\Omega_{\text{DE}}/(8\pi)$ is of order 0.1 whenever $\Omega_{\text{DE}} = O(1)$, which holds by definition at the present epoch. The framework reparameterizes the cosmological constant problem; it does not solve it (Section 7).

3.2 The saturation parameter and the equation of state

The equation of state $w = p_{\text{DE}}/\rho_{\text{DE}}$ governs the dark-energy dynamics. For an event-horizon cutoff, the standard derivation (Appendix A.2) gives

$$w = -\frac{1}{3} - \frac{2\sqrt{\Omega_{\text{DE}}}}{3\xi}, \quad (17)$$

where ξ is the saturation parameter of the ansatz $\rho_{\text{DE}} = 3\xi^2 M_P^2/R_h^2$. Li’s original holographic dark-energy proposal [10] took $\xi = 1$, which with $\Omega_{\text{DE}} = 0.689$ gives $w_0 = -0.88$, in mild tension with observations; the Hubble-scale cutoff gives the degenerate $w = 0$ [11].

Theorem 3.1 (Saturation parameter and equation of state at the attractor). *Imposing the de Sitter condition $HR_h = c$ of Section 2.3 fixes*

$$\xi = \sqrt{\Omega_{\text{DE}}} \approx 0.83, \quad (18)$$

and the equation of state (17) reduces to $w = -1$ exactly at all epochs.

Proof. The holographic relation $\Omega_{\text{DE}} = \xi^2 c^2/(H^2 R_h^2)$ with $HR_h = c$ gives (18), a consistency condition of the attractor, not a fitted parameter. Substituting into (17),

$$w = -\frac{1}{3} - \frac{2\sqrt{\Omega_{\text{DE}}}}{3\sqrt{\Omega_{\text{DE}}}} = -\frac{1}{3} - \frac{2}{3} = -1. \quad (19)$$

□

When the de Sitter saturation condition $\xi = \sqrt{\Omega_{\text{DE}}}$ holds, holographic dark energy has the equation of state of a cosmological constant. Current observations give $w_0 = -1.03 \pm 0.03$, consistent with this value.

The attractor is dynamically stable: a perturbation $\delta\Omega_m$ about the de Sitter fixed point evolves as $\delta\Omega_m \propto e^{\lambda N}$ with $N = \ln a$ and $\lambda = -3\Omega_{\text{DE}} < 0$, decaying over roughly half an e-fold, so initial conditions in the basin converge to the de Sitter endpoint.

3.3 The coincidence problem, ameliorated

The coincidence problem asks why the dark-energy and matter densities are comparable today, when their ratio has varied enormously over cosmic history. In Λ CDM, Λ has a fixed density while $\rho_m \propto a^{-3}$, so $\rho_\Lambda/\rho_m \propto a^3$ grows by a factor $\sim 10^{30}$ from matter–radiation equality to the distant future; observing it at order unity is either explained or accepted as coincidence.

Here $\rho_{\text{DE}} = \alpha c^2 H^2/G$ tracks the expansion rate rather than staying constant. During matter domination $H^2 \propto \rho_m$, so $\rho_{\text{DE}} \propto \rho_m$ and

$$\frac{\rho_{\text{DE}}}{\rho_m} = \frac{\Omega_{\text{DE}}}{1 - \Omega_{\text{DE}}} = \frac{8\pi\alpha/3}{1 - 8\pi\alpha/3}, \quad (20)$$

constant during matter domination. Dark energy and matter keep a fixed proportion, removing the timing coincidence of Λ CDM: no time during matter domination is distinguished. The transition to dark-energy domination occurs when $\Omega_m \sim \Omega_{\text{DE}}$ by definition, an epoch set by α rather than fine-tuned.

This is a rephrasing, not a solution. The timing aspect is removed, but the coincidence is relocated from cosmic time to the parameter α : the puzzle becomes why $\alpha \approx 0.08$. The bounds (16) within which α produces an accelerating universe span a factor of two, and the naturalness argument is circular. The magnitude of α encodes the same mystery as the cosmological constant—why $H_0 \sim 10^{-18} \text{ s}^{-1}$ rather than $H_P \sim 10^{44} \text{ s}^{-1}$, why $\rho_{\text{DE}}^{1/4} \sim 10^{-3} \text{ eV}$ rather than the Planck energy. The 10^{120} discrepancy reappears in disguised form; the coincidence problem is transformed, not solved.

The dark-energy formula $\rho_{\text{DE}} = \alpha c^2 H^2 / G$ thus follows from dimensional analysis given the holographic ansatz; the coefficient $\alpha = 0.082$ is fitted; the de Sitter condition determines $\xi = \sqrt{\Omega_{\text{DE}}}$ as a consistency requirement, yielding $w = -1$. The framework is internally consistent but does not explain the value of α or supply a microscopic mechanism for dark energy.

4 Observational Status

The framework makes definite statements about cosmological observables that, at present precision, do not distinguish it from Λ CDM. The predictions are either already known and Λ CDM-consistent, or in-principle testable but parametrically out of reach.

4.1 The core prediction and its falsifiers

Dark energy arises from the holographic bound on the future event horizon, giving $\rho_{\text{DE}} = \alpha c^2 H^2 / G$ with $\alpha \approx 0.082$ and a de Sitter attractor with $w = -1$. Combined Planck, baryon acoustic oscillation, and Type Ia supernova data give $w_0 = -1.03 \pm 0.03$, consistent with the prediction; the DESI hints of time-varying $w(z)$ remain at low significance and subject to systematics.

Remark 4.1 (Status of the DESI evolving- w preference). DESI DR2 baryon acoustic oscillations combined with the DESY5 supernovae prefer an evolving $w(z)$ at up to $3.9\text{--}4.5\sigma$ in the $w_0 w_a$ CDM parametrization (best fit $w_0 \approx -0.79$, $w_a \approx -0.61$) [22, 23]. The significance is supernova-compilation dependent: it drops to $1.5\text{--}2.3\sigma$ with Pantheon+ or Union3, and DESI+Planck alone remains consistent with $w = -1$ [23]. A dedicated Bayesian comparison of holographic dark energy against these data finds the model statistically as viable as Λ CDM under DESI BAO and supernovae [24]. The leading-order prediction here is therefore not excluded. A robust cross-dataset detection of $w \neq -1$ at greater than 5σ , stable across all supernova compilations, would falsify it.

Three classes of measurement would falsify the framework.

1. *Equation of state.* A $w(z)$ evolution incompatible with a constant near -1 at greater than 3σ would rule out the holographic ansatz as formulated, requiring either $\xi \neq \sqrt{\Omega_{\text{DE}}}$ against the consistency analysis or a failure of the horizon relation at some epoch. Euclid and the Vera Rubin Observatory’s LSST will constrain $w(z)$ at the sub-percent level across redshift bins.

2. *Modified gravity.* The framework operates within general relativity with $G_{\text{eff}} = G_N$ exactly; a measured $G_{\text{eff}}(z)/G_N \neq 1$ at the percent level—via lensing statistics or the integrated Sachs–Wolfe effect—would rule it out. $f(R)$ and scalar-tensor theories generically predict $G_{\text{eff}} \neq G_N$, a clean discriminant.
3. *Early dark energy.* The CMB constrains $\Omega_{\text{DE}}(z = 1100) \lesssim 10^{-4}$, whereas the tracking value during matter domination is ≈ 0.7 . The tracking analysis of Section 3.3 applies only during matter domination; the radiation era and earlier follow a different scaling, requiring a full solution of the Friedmann equations across all epochs, deferred to future work. Early dark energy beyond the CMB bounds would require re-examining whether the framework accommodates it.

Table 2 separates what is derived as a consistency condition, what follows from general relativity, and what is fitted to observations.

Observable	Value	Status
Equation of state $w(z = 0)$	-1	De Sitter attractor consistency
Saturation parameter ξ	$\sqrt{\Omega_{\text{DE}}} = 0.83$	Consistency condition
Holographic coefficient α	$0.082 \pm 0.001_{\text{stat}}$	Fitted to Ω_{DE}
Effective gravitational constant G_{eff}	$= G_N$	Follows from GR
ISW effect	Matches ΛCDM	Follows from $w = -1$

Table 2. *Summary of dark-energy predictions. The equation of state and the saturation parameter follow from the de Sitter attractor consistency condition; the coefficient α is fitted; the gravitational constant follows from operating within general relativity; ISW suppression is predicted but within current cosmic variance.*

4.2 CMB consistency and late-time expansion

With $w = -1$ from the de Sitter attractor, the framework is observationally indistinguishable from a cosmological constant at the level of the equation of state, consistent with $w_0 = -1.03 \pm 0.03$.

The integrated Sachs–Wolfe (ISW) effect matches ΛCDM . For $w = -1$, gravitational potentials decay at the standard rate, so at multipoles $\ell < 30$, where the ISW contributes roughly 10% of the temperature power, the framework predicts no deviation from ΛCDM ; any difference lies within cosmic variance ($\sigma \sim 30\%$ at $\ell = 2$). The model with $\alpha = 0.082$ is consistent with current CMB constraints from DESI 2024 baryon acoustic oscillations and Planck. The modified-gravity parameters from the ISW–lensing bispectrum, $\mu_0 = 0.05 \pm 0.22$ and $\Sigma_0 = 0.008 \pm 0.045$, are consistent with general relativity. The primary acoustic peaks ($30 < \ell < 1000$) show no deviation from ΛCDM , since they probe recombination, where dark energy contributed negligibly.

The saturated attractor admits a sharper test than consistency. Any small departure of $w_{\text{eff}}(z)$ from -1 at $z \sim 0.5$ – 1 —the generic signature of a budget not yet fully saturated—would modify the late-time ISW effect at $\ell < 30$ by 1–2%, below current Planck cosmic variance but potentially accessible to a combined CMB-S4 + large-scale-structure analysis this decade. The saturated reading, for which $w = -1$ holds exactly, requires the absence of that signature; a detection would localize the epoch at which saturation completes and falsify the exact- $w = -1$ statement.

4.3 The negative result: CMB non-Gaussianity

The cosmic gravitational decoherence underlying the entropy budget (Section 2.5) might have imprinted a measurable CMB non-Gaussianity. The calculation gives a negative verdict.

The per-mode accumulated decoherence functional from horizon exit during inflation to last scattering, with the de Sitter form factor $g(x) = 1 - (2/\pi) \text{Si}(x)$ [39], is

$$D_k(\eta_{\text{LSS}}) = \frac{1}{4\pi} \int_{\ln a_*(k)}^{\ln a_{\text{LSS}}} g\left(\frac{aH}{ck}\right) d \ln a \approx 0.35\text{--}0.51 \quad \text{per mode}, \quad (21)$$

the dominant contribution coming from the deep sub-horizon period during inflation. The induced non-Gaussianity is a multiplicative suppression of the standard inflationary value,

$$f_{\text{NL}}^{\text{QGC}} = e^{-D_k(\eta_{\text{LSS}})} f_{\text{NL}}^{\text{standard}}, \quad |\Delta f_{\text{NL}}^{\text{QGC}}| \sim 5 \times 10^{-3}\text{--}1.5 \times 10^{-2}. \quad (22)$$

The current Planck 2018 bound is $f_{\text{NL}}^{\text{local}} = -0.9 \pm 5.1$ [32]; the CMB-S4 target sensitivity is $\sigma(f_{\text{NL}}) \sim 1$ [33]. The prediction is 60–200 times below CMB-S4 sensitivity: the induced non-Gaussianity is not observable at any planned CMB experiment.

Compared to the standard squeezing-induced classicality mechanism of Kiefer, Polarski, and Starobinsky [30]: the squeezing parameter $r_k(\eta) = \ln(a/a_*)$ reaches $r \sim 60$ for CMB pivot modes by the end of inflation, an $e^{2r} \sim 10^{52}$ phase-space elongation that classicalizes inflationary perturbations *without* genuine decoherence. The squeezing rate $r_{\text{dot}} = H$ is ~ 30 times the per-mode decoherence rate $g(1) H/(4\pi) \approx 0.032 H$:

$$\frac{\Gamma_{\text{QGC}}}{r_{\text{dot}}} = \frac{g(1)}{4\pi} \approx 0.032. \quad (23)$$

Squeezing-induced classicality dominates the apparent quantum-to-classical transition of inflationary perturbations; the gravitational-decoherence contribution is real but observationally redundant.

4.4 The decisive tests are elsewhere

The decisive laboratory test of the underlying framework is gravitational decoherence at the microgram scale. The predicted rate

$$\Gamma_{\text{dec}} = \frac{GM^2}{\hbar d} \equiv \frac{E_G}{\hbar} \quad (24)$$

applies to two coherent mass distributions of mass M separated by distance d .¹ The Bose–Marletto–Vedral experiments [34, 35] test whether two masses become gravitationally entangled. For $M \sim 10^{-9}$ kg and $d \sim 10^{-3}$ m, $\Gamma_{\text{dec}}^{-1} \sim 10^{-9}$ s [38]. Confirmation at the predicted rate supports the gravitational-decoherence mechanism underlying the cosmic-decoherence argument (Section 2.5); the present framework inherits its testability through that connection.

Table 3 collects the predictions with their status.

¹The energy scale $E_G = GM^2/d$ is established as the classical gravitational potential energy of the two configurations, independent of the rate identification. The assertion that the decoherence rate equals $E_G/\hbar$ follows from the QGC saturation principle and Hamiltonian-constraint argument of the canonical core and companion papers [36, 38]; the energy scale and the rate identification are logically distinct steps, and only the former is rigorous in the classical sense.

Table 3. *Observational status of the predictions relevant to this framework, graded from “testable now” to “not testable in the foreseeable future.” Below-sensitivity predictions are listed for completeness; they do not constitute observational signatures against Λ CDM.*

Prediction	Value	Status
Lab gravitational decoherence rate	$\Gamma = GM^2/(\hbar d)$	Testable this decade; companion setup [38].
Equation of state $w(z=0)$	-1	Consistent with $w_0 = -1.03 \pm 0.03$; DESI at low significance.
ISW deviation (if saturation incomplete)	$1\text{--}2\%$ at $\ell < 30$	Below Planck cosmic variance; possibly CMB-S4 + LSS.
Identity $\rho_{\text{DE}}^\infty S_{\text{dS}}^\infty = 3c^7/(8G^2\hbar)$	Exact in attractor	Structural consistency relation, not a novel test.
$S_{\text{dS}}^\infty \approx 3 \times 10^{122}$	Predicted from H_∞	Standard; informational interpretation new.
$\Lambda > 0$ requirement (framework formulation)	Sign-determination	Consistent with observation; not a discriminating test.
CMB non-Gaussianity $ \Delta f_{\text{NL}} $	$\sim 5 \times 10^{-3}\text{--}10^{-2}$	60–200 $\times$ below CMB-S4 sensitivity; not testable.
$a_0(z) = cH(z)/(2\pi)$ MOND scale	Galaxy dynamics at $z > 2$	Distinguishes the instantaneous-horizon reading of a_0 , itself not an established prediction [37]; JWST/Euclid/Rubin, 5–10 yr.

At present precision the framework predicts nearly identically to Λ CDM: both give $G_{\text{eff}} = G_N$ and w within current uncertainties. It remains falsifiable in principle— w significantly different from -1 , or $G_{\text{eff}} \neq G_N$, would rule it out—and will face stringent tests as precision improves. The value is conceptual organization, a thermodynamic language connecting dark energy to horizon physics, developed further in the identity of Section 5 and the arrow-of-time reading of Section 6.

5 The Dark-Energy / Horizon-Entropy Identity

The two uses of the horizon budget—dark energy (Section 3) and, in Section 6, the arrow of time—are tied by a single exact identity, the conjunction of the holographic horizon entropy (9) with the dark-energy formula (10) in the de Sitter attractor.

5.1 The central identity

In the late-time attractor, dark energy dominates the Friedmann equation:

$$H_\infty^2 = \frac{8\pi G}{3c^2} \rho_{\text{DE}}^\infty, \quad (25)$$

the standard de Sitter expansion rate set by a cosmological constant $\Lambda \equiv 8\pi G \rho_{\text{DE}}^\infty / c^4$. Substituting (25) into the horizon entropy (9) at $H = H_\infty$,

$$S_{\text{dS}}^\infty = \frac{\pi c^5}{H_\infty^2 \hbar G} = \frac{\pi c^5}{(8\pi G \rho_{\text{DE}}^\infty / (3c^2)) \hbar G} \quad (26)$$

$$= \frac{3 c^7}{8 G^2 \hbar \rho_{\text{DE}}^\infty}. \quad (27)$$

Rearranging gives the central identity:

Dark-energy / horizon-entropy identity. In the de Sitter attractor of any FRW cosmology,

$$\rho_{\text{DE}}^\infty S_{\text{dS}}^\infty = \frac{3 c^7}{8 G^2 \hbar} \quad (28)$$

The right-hand side is a Planck-scale constant. Numerically,

$$\frac{3 c^7}{8 G^2 \hbar} \approx 1.74 \times 10^{113} \text{ J/m}^3, \quad (29)$$

the dark-energy density at one nat of cosmic entropy capacity.

The derivation occupies three algebraic lines, and both sides of (28) are functions of the single variable H_∞ , so the identity is a re-labeling of one quantity, not a relation between two independent ones. The substance lies not in the identity but in what it permits: coupling the dark-energy density to the arrow of time within one framework. Versions of (28) appear in the holographic-dark-energy literature from Banks [12], Fischler–Susskind [13], and Cohen–Kaplan–Nelson [9], and most explicitly in Li [10]; the originality claimed here is the conceptual coupling of Section 6.

The identity is a conservation law in Planck units. With the Planck density $\rho_{\text{Pl}} \equiv c^7 / (\hbar G^2)$, the right-hand side of (28) is $(3/8) \rho_{\text{Pl}}$, and the identity becomes

$$\frac{\rho_{\text{DE}}^\infty}{\rho_{\text{Pl}}} = \frac{3/8}{S_{\text{dS}}^\infty}. \quad (30)$$

5.2 Numerical check

Substituting measured values is a consistency check; today’s product and the asymptotic product differ because today is not yet at the attractor.

Today. Using Planck 2018 values ($H_0 = 67.4 \text{ km/s/Mpc}$, $\Omega_{\text{DE}} = 0.689$, $\rho_{\text{crit}}^{\text{today}} = 7.67 \times 10^{-10} \text{ J/m}^3$), the observed dark-energy density is

$$\rho_{\text{DE}}^{\text{today}} = \Omega_{\text{DE}} \rho_{\text{crit}}^{\text{today}} \approx 0.689 \times 7.67 \times 10^{-10} \text{ J/m}^3 \approx 5.28 \times 10^{-10} \text{ J/m}^3.$$

Today’s horizon entropy is $S_{\text{dS}}^{\text{today}} \approx 2.27 \times 10^{122}$ (Table 1), so

$$\rho_{\text{DE}}^{\text{today}} S_{\text{dS}}^{\text{today}} \approx 1.20 \times 10^{113} \text{ J/m}^3.$$

This differs from the Planck-scale constant (29) by a factor $\Omega_{\text{DE}} = 0.689$, as expected: the identity (28) holds only at the attractor where $\Omega_{\text{DE}} \rightarrow 1$, while today is still in the matter–dark-energy transition.

Asymptote. The dark-energy density does not dilute, so $\rho_{\text{DE}}^\infty = \rho_{\text{DE}}^{\text{today}}$. The asymptotic horizon entropy is $S_{\text{dS}}^\infty = S_{\text{dS}}^{\text{today}}/\Omega_{\text{DE}} \approx 3.29 \times 10^{122}$, and

$$\rho_{\text{DE}}^\infty S_{\text{dS}}^\infty \approx 1.74 \times 10^{113} \text{ J/m}^3,$$

matching the Planck-scale constant (29) to better than 1% (saturating exactly in the limit), with residual at the level of present cosmological-parameter uncertainties. Both sides are computed from the same cosmological parameters: the identity is a structural relation, not an empirical test.

5.3 The cosmological constant problem reframed

The cosmological constant problem [5] asks why $\rho_{\text{DE}}/\rho_{\text{Pl}} \sim 10^{-122}$ rather than $O(1)$ as quantum field theory naively predicts. The identity (30) inverts the question:

$$\frac{\rho_{\text{DE}}^\infty}{\rho_{\text{Pl}}} = \frac{3/8}{S_{\text{dS}}^\infty} \sim \frac{1}{S_{\text{dS}}^\infty}. \quad (31)$$

For $S_{\text{dS}}^\infty \sim 10^{122}$, $\rho_{\text{DE}}^\infty/\rho_{\text{Pl}} \sim 10^{-122}$ *automatically*: the dark-energy hierarchy is the reciprocal of the cosmic entropy capacity, not a separate number requiring a separate explanation.

This does not solve the cosmological constant problem—why the cosmic horizon holds 10^{122} nats and not 10^{60} or 10^{200} remains open—but it makes it *one* question. “Why is ρ_{DE} so small” is “why is S_{dS}^∞ so large.” The identity (28) links ρ_{DE}^∞ and S_{dS}^∞ ; it derives neither separately. The mystery survives in dimensionless form: one number, $\alpha = 3\Omega_{\text{DE}}/(8\pi)$, equivalently $1/S_{\text{dS}}^\infty$ in Planck units, fitted to observation. Whether it derives from a deeper principle is open. Section 6 shows that this same number controls the arrow of time.

6 The Arrow of Time from the Budget

The second use of the horizon budget is the arrow of time. The same bound (4) that fixes the dark-energy density, applied to the shrinking past horizon, implements the Past Hypothesis, recasts Penrose’s 10^{123} as a capacity, and orients the Weyl curvature asymmetry.

6.1 The Past Hypothesis as a corollary of the bound

The Past Hypothesis [25, 26]—the early universe in a microstate of extraordinarily low coarse-grained entropy, the boundary condition that selects the thermodynamic arrow from time-symmetric dynamics—is normally an external postulate. The holographic bound applied to the shrinking cosmic horizon implements it automatically. The bound reads

$$S(t) \leq S_{\text{dS}}(t) = \frac{\pi c^5}{H(t)^2 \hbar G}. \quad (32)$$

In any FRW cosmology with a past singularity, $H(t) \rightarrow \infty$ as $t \rightarrow 0$, so

$$S_{\text{dS}}(t) \rightarrow 0 \quad \text{and hence} \quad S(t) \rightarrow 0 \quad \text{as } t \rightarrow 0. \quad (33)$$

Low initial entropy is not fine-tuned; it is forced. There is no room for high entropy in the early universe because there is no horizon area to support it.

Equation (33) is an upper bound; that the actual $S(t)$ tracks $S_{\text{dS}}(t)$ rather than sitting far below it is Assumption 2.4 (GSL saturation), motivated within QGC by the cosmic decoherence rate $\Gamma_{\text{total}} = g(1) H/(4\pi) \approx H/(10\pi)$ [39]: the cosmic decoherence rate scales as H , so entropy production keeps pace with capacity growth, making saturation generic rather than fine-tuned. The QGC Past Hypothesis is thus not merely “ $S(0) \approx 0$ ” (trivial from the bound) but the stronger statement that $S(t)$ tracks $S_{\text{dS}}(t)$ throughout. The arrow of time is the direction of this growth. Against the conventional axiomatic statement—“the macrostate of the early universe had entropy far below the maximum compatible with its energy”—the framework substitutes “the cosmic entropy budget at early times was small because the holographic capacity was small,” following from Axiom 2.1 plus saturation. One postulate is eliminated at the cost of an axiom QGC accepts anyway.

6.2 Penrose’s 10^{123} as a capacity

Penrose’s fine-tuning estimate $\exp(-10^{123})$ for the initial state [27, 28] is, in this reading, not a probability but the value of S_{dS}^∞ in nats: the *capacity* of the cosmic horizon. The asymptotic budget $S_{\text{dS}}^\infty \approx 3 \times 10^{122}$ (Table 1) is the maximum information any observer can ever encode in their causal patch. It is large because the universe is large; it is not a measure of fine-tuning but the size of the cosmic address book. Through the identity (30), the smallness of $\rho_{\text{DE}}/\rho_{\text{Pl}} \sim 10^{-122}$ is the reciprocal of this largeness: Penrose’s 10^{123} and the cosmological constant hierarchy are the same number in two languages.

6.3 Why $\Lambda > 0$ closes the budget

A sharper claim is available: the QGC reading of the Past Hypothesis, as a *finite, two-sided* statement about the cosmic entropy budget, requires $\Lambda > 0$. This is a self-consistency condition on the chosen formulation, not a universal necessity. Three cases:

Case (i): $\Lambda < 0$ (AdS-like). The universe recollapses. There is no asymptotic cosmic horizon and no well-defined S_{dS}^∞ ; the budget’s maximum is reached only in the recollapse phase, a turnaround, not a stable asymptote.

Case (ii): $\Lambda = 0$ (Minkowski asymptote). $H(t) \rightarrow 0$ as $t \rightarrow \infty$, so $S_{\text{dS}}(t) \rightarrow \infty$ and the bound becomes vacuous in the future. The Past Hypothesis is one-sided: bounded below in the past, unbounded above in the future.

Case (iii): $\Lambda > 0$ (de Sitter attractor; our case). The Hubble rate asymptotes to finite $H_\infty > 0$, and the horizon entropy to finite S_{dS}^∞ . The Past Hypothesis is two-sided: $S(t)$ grows from ~ 0 at the Planck epoch to S_{dS}^∞ in the asymptotic future. Both endpoints are finite; the

arrow of time has a beginning (no room for entropy) and an asymptotic destination (capacity saturated).

Only case (iii) gives the closed, finite, two-sided budget developed here.

Framework requirement on dark energy. For the Past Hypothesis to have the closed two-sided holographic-capacity implementation developed in this paper, the universe must have $\Lambda > 0$. Dark energy is the structural condition that closes the cosmic entropy budget under this formulation; without it, the Past Hypothesis would have to be re-stated differently—one-sided in the Minkowski case, turnaround-bounded in the AdS case.

Remark 6.1 (Scope). This is a requirement of *this formulation*, not a universal theorem. A $\Lambda < 0$ recollapsing universe admits “entropy near the Big Bang is small relative to the Big Crunch,” two-sided but turnaround-bounded; a $\Lambda = 0$ Minkowski-asymptote universe admits the conventional one-sided Boltzmann/Penrose form. The framework chose the two-sided finite-budget reading because dark energy supplies a natural upper bound and the identity (28) pairs cleanly with it; that choice then selects $\Lambda > 0$ for internal consistency. The argument fixes the sign of Λ conditional on the formulation, not absolutely; the magnitude $|\Lambda|$ remains fitted in all three cases.

6.4 The Weyl asymmetry as the direction of decoherence

The Weyl tensor $C_{\mu\nu\rho\sigma}$ vanishes at the Big Bang and diverges at black-hole singularities, two qualitatively different generic singular outcomes of gravitational dynamics [27, 28]. Penrose’s Weyl Curvature Hypothesis (WCH) states this as the classical initial-condition postulate

$$C_{\mu\nu\rho\sigma}(t \rightarrow 0) \rightarrow 0, \quad C_{\mu\nu\rho\sigma}(\text{BH singularity}) \rightarrow \infty. \quad (34)$$

As a phenomenological observation this is hard to dispute—the CMB is extraordinarily isotropic, gravitational collapse generically produces BKL-like curvature divergences. As an explanation it is unsatisfying: a boundary condition on the classical field, not derived from a dynamical principle, requiring a past/future distinction with no analogue in the time-symmetric Einstein equations.

In QGC the Weyl tensor is an operator $\hat{C}_{\mu\nu\rho\sigma}$ on the universal Hilbert space. A state with $\langle \hat{C} \rangle = 0$ but large $\text{Var}(\hat{C})$ is a superposition of non-zero values, qualitatively different from a sharp zero; the classical statement (34) is incomplete. This motivates:

Conjecture 6.1 (QGC Weyl Curvature Hypothesis, heuristic). *Any solution $|\Psi\rangle$ of the Wheeler–DeWitt constraint $\hat{H}|\Psi\rangle = 0$ with no environment satisfies, at every past conformal boundary point of $|\Psi\rangle$ ’s support,*

$$\langle \hat{C}_{\mu\nu\rho\sigma} \rangle_{|\Psi\rangle} = 0, \quad \text{Var}(\hat{C}_{\mu\nu\rho\sigma})_{|\Psi\rangle} = 0. \quad (35)$$

The first condition is the operator analogue of Penrose’s statement; the vanishing variance is the new content, forbidding the Weyl tensor from being in a superposition of non-zero values at the past boundary.

Remark (Well-definedness caveats). Conjecture 6.1 is offered as *heuristic*; a precise formulation requires three specifications beyond the present scope. (i) The operator $\hat{C}_{\mu\nu\rho\sigma}$ requires a constraint-quantization choice; the simplest implementation is a minisuperspace truncation, a

toy quantization, not a full quantum-gravitational construction. (ii) The variance depends on the choice of frame and on which scalar contractions are tracked; a gauge-invariant formulation (e.g. through the Weyl scalars $\Psi_0, \dots, \Psi_4$ in a Newman–Penrose frame) is needed. (iii) The “past conformal boundary of $|\Psi\rangle$ ’s support” presupposes a definite support structure with an identifiable past boundary, itself requiring a clock variable and a notion of conformal compactification at the quantum level. None is addressed here.

Support comes from three complementary readings of one underlying argument, not logically independent derivations. All rely on the same core step—the past conformal boundary has vanishing bounding area, $A_{\text{boundary}} \rightarrow 0$ —and differ only in emphasis. *Holographic*: the bound $S \leq A/(4\ell_{\text{P}}^2) \rightarrow 0$ forces any entropy-carrying branching of the metric to vanish at the boundary. *Kinematic vacuum*: sharp Weyl values require environment-induced decoherence; at a past boundary no decoherence has occurred, so $\hat{C}$ has neither a definite eigenvalue nor a definite superposition of non-zero values. *No-environment plus global purity*: the universe-as-a-whole is pure, so its von Neumann entropy is zero, the matter entropy is bounded by the vanishing boundary area, and the realized branching that would give nonzero $\text{Var}(\hat{C})$ goes to zero. The three are three vocabularies for one heuristic; the cumulative weight is one motivating argument, not three independent ones.

Conjecture 6.1 is symmetric in past and future conformal boundaries and so does not by itself distinguish the Big Bang from the future heat death. The Big Bang / black-hole asymmetry is not a past-versus-future boundary of the global $|\Psi\rangle$: both are local singularities within a single cosmic aeon. The asymmetry is in the *decoherence direction*. Within a single aeon:

- The *Big Bang* is upstream of all decoherence. No semiclassical clock yet exists, no environment has measured the Weyl operator into definite branches. The Weyl operator is in its kinematic vacuum, $\langle \hat{C} \rangle = \text{Var}(\hat{C}) = 0$, matching (34) at the Big Bang.
- The *black-hole singularity* is downstream. Substantial gravitational entanglement has accumulated; the Weyl operator has been measured many times into definite branches; the BKL-like chaos is the classical description of high-variance branches each with large $\hat{C}$ eigenvalue. Both $\langle \hat{C} \rangle$ and $\text{Var}(\hat{C})$ are large, matching (34) at the singularity.

The asymmetry follows from a single fact—emergent time as the direction of cosmic decoherence—applied at both endpoints. There is no separate past–future-asymmetry postulate; the asymmetry *is* the arrow of time, with the mechanism cosmic gravitational decoherence at per-mode rate $g(1)H/(4\pi)$ [39].

Remark 6.2 (What is established and what is conjectured). Established: the classical WCH is recoverable as the semiclassical limit of Conjecture 6.1, and within a single aeon the Big-Bang/black-hole asymmetry is upstream/downstream of cosmic decoherence with no postulate beyond the QGC arrow of time. Conjectured: Conjecture 6.1 itself. A full derivation requires a precise operator construction for $\hat{C}_{\mu\nu\rho\sigma}$ beyond minisuperspace, a derivation of $A_{\text{boundary}} \rightarrow 0$ from the Wheeler–DeWitt constraint rather than as an assumption, and either an explicit Hartle–Hawking-like construction of $|\Psi\rangle$ near the past boundary or a proof that the no-environment condition forces the operator-vacuum form. All three remain open.

7 Scope and Comparison

Both uses of the horizon budget carry the same caveats, and this section states them once for the whole paper, then compares the construction with the main alternatives.

7.1 The limits, stated once

Remark 7.1 (What the framework does not achieve). 1. **α is fitted, not predicted.** The coefficient $\alpha = 3\Omega_{\text{DE}}/(8\pi) \approx 0.082$ is the measured cosmic entropy budget in Planck units, fitted to Ω_{DE} . It controls the dark-energy density and, via the identity (28), the cosmic entropy capacity $1/S_{\text{dS}}^\infty$ simultaneously—one free parameter, not two—but its value is not derived.

2. **The cosmological constant problem is reparameterized, not solved.** Why H_0 is 60 orders of magnitude below the Planck scale—equivalently why ρ_{DE} is 120 orders below quantum field theory expectations, why the cosmic horizon holds 10^{122} nats and not 10^{60} or 10^{200} —remains open. The 10^{120} discrepancy reappears in disguised form as the smallness of α . The claim that $\alpha \sim 0.1$ is “natural” is tautological, since α is of order 0.1 whenever $\Omega_{\text{DE}} = O(1)$, which holds by definition at the present epoch.

3. **GSL saturation is assumed at the attractor.** The holographic bound is an upper bound; that the actual cosmic entropy saturates it is Assumption 2.4. Within QGC it is motivated by the per-mode cosmic decoherence rate, but the mode count must do the structural work of closing the 122-order gap between per-mode rate and total budget, and that combination is not derived from first principles (Appendix B).

4. **The operator Weyl hypothesis is heuristic.** Conjecture 6.1 states $\langle \hat{C} \rangle = \text{Var}(\hat{C}) = 0$ at past conformal boundaries. Three motivating readings are given (holographic, kinematic-vacuum, no-environment); they are one argument in three vocabularies, not a proof, and the operator-theoretic well-definedness issues remain open.

5. **No observational discriminant at current precision.** The framework predicts the same expansion history, the same CMB power spectrum, $G_{\text{eff}} = G_N$, and CMB non-Gaussianity too small to measure. It is falsifiable in principle— w significantly different from -1 , or $G_{\text{eff}} \neq G_N$ —but at present precision no observation distinguishes it from standard Λ CDM. The decisive tests are elsewhere (laboratory decoherence, high- z dynamics).

Additionally, the event-horizon cutoff is teleological, depending on the future evolution; we read this as the global character of solutions to Einstein’s equations, not as future causation. No microscopic mechanism is provided: the holographic ansatz is postulated, and although string theory furnishes examples of holography (AdS/CFT), no derivation of holographic dark energy from string theory exists. The framework does not explain why quantum field theory vacuum energy, $\rho_{\text{vac}} \sim 10^{113} \text{ J/m}^3$, fails to gravitate down to $\rho_{\text{DE}} \sim 10^{-9} \text{ J/m}^3$; it takes ρ_{DE} as given.

7.2 Comparison with alternatives

Λ CDM and Li’s HDE. Λ CDM treats Λ as a fundamental parameter, fitting the data with high precision but explaining nothing about its origin or magnitude, fine-tuned at 10^{-120} relative to naive quantum field theory. The framework here replaces the unexplained Λ with the

unexplained dimensionless $\alpha \sim 0.08$; neither explains the magnitude, but the Past Hypothesis becomes a consequence rather than a separate postulate. Li’s HDE [10] posits $\rho_{\text{DE}} = 3\xi^2 M_P^2/L^2$ with ξ free; $\xi = 1$ gives $w_0 \approx -0.88$, in mild tension. Here $\xi = \sqrt{\Omega_{\text{DE}}}$ follows as a consistency condition of the de Sitter attractor, removing the freedom and yielding $w = -1$ exactly. Both share the limitation that the holographic ansatz is not derived from first principles.

Quintessence and modified gravity. Quintessence [7] drives acceleration with a scalar field and arbitrary potential, giving dynamical $w(t)$; it is underconstrained, trading Λ for the function $V(\phi)$ and generically predicting $w \neq -1$. Modified gravity [8] alters the Einstein equations and generically gives $G_{\text{eff}} \neq G_N$; the framework here, within general relativity, predicts $G_{\text{eff}} = G_N$ exactly, so $G_{\text{eff}}(z)/G_N \neq 1$ at the percent level would falsify it and support modified gravity.

Prior holographic dark energy and the N-bound. Prior work linked the cosmological constant to horizon entropy via essentially the algebraic identity (28). Banks [12] and Fischler–Susskind [13] proposed (1998–2000) that the cosmological constant is fundamentally an entropy quantity: the de Sitter horizon carries $N = S_{\text{dS}}^\infty$ degrees of freedom, and the smallness of Λ in Planck units is the largeness of N . The principle rests on ’t Hooft’s dimensional-reduction proposal [14] and Susskind’s “world as a hologram” [15]. Cohen–Kaplan–Nelson [9] gave the effective-field-theory bound $\rho_{\text{vac}} \sim H^2 M_P^2$; Li [10] sharpened it into the dynamical $\rho_{\text{DE}} \propto c^2 H^2/G$ inherited here. The closest prior synthesis is Padmanabhan’s thermodynamic-aspects-of-gravity program [16]. The structural content of (28) is twenty-five years old; the novelty is the *coupling* of this prior-art identity to the Past Hypothesis (the shrinking past horizon making $S_{\text{dS}}(t \rightarrow 0) \rightarrow 0$ automatic) and to Penrose’s WCH (the upstream/downstream-of-decoherence reading). Neither connection appears in those threads.

Bousso’s covariant entropy bound. Bousso’s bound [19, 20] is the most general holographic principle, yielding for the de Sitter horizon exactly the $S_{\text{dS}} = A/(4\ell_P^2)$ used here. The framework specializes it to the cosmic horizon and adds the dynamical-dark-energy link (28); it elaborates Bousso into a dynamical statement, and does not contradict it.

Penrose’s Conformal Cyclic Cosmology. CCC [28] identifies the future conformal boundary of one aeon with the past boundary of the next, so the WCH holds automatically at every junction. The framework endorses Penrose’s mass-as-clock and conformal-symmetry intuitions [29] and the reframing of 10^{123} as geometric, but rejects cyclicity: the would-be Thermofield-Double identification between aeons fails for lack of a shared Hamiltonian (the Wheeler–DeWitt constraint is $\hat{H}|\Psi\rangle = 0$ globally), a shared inverse temperature (the spacelike past conformal boundary has no timelike Killing vector), and a canonical mode pairing. The surviving picture is block-static, not cyclic: every conformal boundary of $|\Psi\rangle$ has the same kinematic-vacuum form, but “aeons” are not chronologically connected as Penrose proposed. CCC’s ontological mass-shedding (erebons) is replaced by operational horizon dilution: massive particles persist but are diluted past the holographic bound of any observer’s horizon.

Hartle–Hawking and anthropic explanations. The Hartle–Hawking no-boundary proposal [31] fixes a specific boundary condition; the capacity argument here provides a bound for any state, so the two are consistent and complementary—a Hartle–Hawking state satisfies the

capacity bound at small scale factor, both predicting $S \rightarrow 0$ as $A \rightarrow 0$. Anthropic resolutions [6] appeal to a landscape of universes; the framework offers a non-multiverse alternative in which the Past Hypothesis is forced by the bound and Λ is reciprocal to the capacity, without an unobservable ensemble. This refutes neither—both can be true—but is more economical.

The framework does not replace the need for a microscopic theory of quantum gravity. The holographic bound, the GSL, and the dark-energy formula are phenomenological inputs—well-motivated within QGC, none derived from a quantum-gravity Lagrangian. It organizes these inputs economically; it is not a final theory.

8 Conclusions

The Bekenstein–Hawking entropy of the cosmological horizon is one budget, and this paper spends it twice.

Used forward, against the critical density, the budget gives the dark-energy density $\rho_{\text{DE}} = \alpha c^2 H^2 / G$ with $\alpha = 3\Omega_{\text{DE}} / (8\pi) \approx 0.082$; GSL saturation at the de Sitter attractor fixes $\xi = \sqrt{\Omega_{\text{DE}}}$ as a consistency requirement and pins the equation of state to $w = -1$ exactly, indistinguishable from a cosmological constant and consistent with $w_0 = -1.03 \pm 0.03$. The ratio $\rho_{\text{DE}} / \rho_m$ is stable during matter domination, removing the timing aspect of the coincidence problem.

Used backward, against the shrinking past horizon, the same budget implements the Past Hypothesis: $S_{\text{dS}}(t) \rightarrow 0$ as $t \rightarrow 0$ makes low initial entropy a corollary of the holographic bound, not a postulate. Penrose’s 10^{123} becomes S_{dS}^∞ in nats—the capacity of the cosmic horizon, not a probability. The Weyl curvature asymmetry is the direction of cosmic decoherence: the Big Bang upstream of all branching (kinematic vacuum, $\langle \hat{C} \rangle = \text{Var}(\hat{C}) = 0$), black-hole singularities downstream (large mean and variance).

The two uses are tied by the identity

$$\rho_{\text{DE}}^\infty S_{\text{dS}}^\infty = \frac{3c^7}{8G^2\hbar} = \frac{3}{8} \rho_{\text{Pl}}, \quad (36)$$

exact in the de Sitter attractor of any FRW cosmology with $\Lambda > 0$, both sides functions of H_∞ alone. The smallness of ρ_{DE} in Planck units is the reciprocal of the largeness of S_{dS}^∞ ; the cosmological constant hierarchy and the cosmic entropy capacity are two faces of one number. Dark energy is the structural condition that closes the two-sided budget: $\Lambda > 0$ is required for the finite-capacity formulation developed here.

One axiom at one horizon gives dark energy and the arrow of time in parallel: dark energy makes the asymptotic capacity finite; the finite capacity implements the Past Hypothesis; the Past Hypothesis defines the arrow of time; the arrow produces the Weyl asymmetry as upstream-versus-downstream of decoherence; and the smallness of ρ_{DE} is the reciprocal of the capacity. One axiom, one identity, one parameter (α), two vocabularies.

The contribution is conceptual, not empirical. None of the puzzles is solved in the strong sense of derivation from a deeper principle; rather, explaining $\alpha \approx 0.082$ would simultaneously explain the smallness of the cosmological constant, the Past Hypothesis, and the Weyl asymmetry. The identity (36) is a tautology known in the holographic-dark-energy literature [12, 13, 10, 16]; the novelty is its coupling of dark energy to the arrow of time. The limits are real and flagged throughout (Section 7.1): α is fitted, GSL saturation is assumed, the operator Weyl hypothesis is heuristic, and no observable discriminates against Λ CDM at current precision.

The framework is a step, not a destination. The destination—a first-principles derivation of α from microscopic quantum gravity—requires a theory we do not have. What we have is one organization of the puzzle landscape: the holographic bound at the cosmic horizon, coupled to the dark-energy density via the de Sitter attractor, makes dark energy and the arrow of time two readings of one horizon entropy budget. Whether that organization survives deeper scrutiny decides whether it is the right one or one among several.

Appendices

A Numerical Values, the Hubble Cutoff, and the Identity Check

This appendix collects the numerical values underlying the main text—using the CODATA 2018 fundamental constants and the Planck 2018 cosmological parameters—derives the equation of state for both infrared cutoffs, and checks the central identity epoch by epoch.

A.1 Fundamental constants and cosmological parameters

Table 4. *Fundamental constants used throughout.*

Quantity	Symbol	Value
Gravitational constant	G	$6.67430 \times 10^{-11} \text{ m}^3/(\text{kg s}^2)$
Reduced Planck constant	$\hbar$	$1.054571817 \times 10^{-34} \text{ J s}$
Speed of light	c	$2.99792458 \times 10^8 \text{ m/s}$
Boltzmann constant	k_B	$1.380649 \times 10^{-23} \text{ J/K}$
Planck length	ℓ_P	$1.6162550 \times 10^{-35} \text{ m}$
Planck time	t_P	$5.3912464 \times 10^{-44} \text{ s}$
Planck mass	m_P	$2.1764340 \times 10^{-8} \text{ kg}$
Planck density	$\rho_{\text{Pl}} = c^7/(\hbar G^2)$	$4.633 \times 10^{113} \text{ J/m}^3 = 5.155 \times 10^{96} \text{ kg/m}^3$

Table 5. *Cosmological parameters from Planck 2018 [3] (TT,TE,EE+lowE+lensing).*

Quantity	Symbol	Value
Hubble constant	H_0	$(67.4 \pm 0.5) \text{ km/s/Mpc}$
Hubble rate today (SI)	H_0	$(2.184 \pm 0.016) \times 10^{-18} \text{ s}^{-1}$
Dark energy fraction	Ω_{DE}	0.689 ± 0.006
Matter fraction	Ω_m	0.311 ± 0.006
Critical density today	$\rho_{\text{crit}} = 3c^2 H_0^2/(8\pi G)$	$(7.67 \pm 0.11) \times 10^{-10} \text{ J/m}^3$
Dark energy density today	$\rho_{\text{DE}}^{\text{today}} = \Omega_{\text{DE}} \rho_{\text{crit}}$	$(5.28 \pm 0.10) \times 10^{-10} \text{ J/m}^3$
Asymptotic Hubble rate	$H_\infty = H_0 \sqrt{\Omega_{\text{DE}}}$	$1.81 \times 10^{-18} \text{ s}^{-1}$
Holographic dark energy coefficient	$\alpha = 3\Omega_{\text{DE}}/(8\pi)$	0.0822 ± 0.0007

The critical density and dark-energy density are derived from H_0 and Ω_{DE} via $\rho_{\text{crit}} = 3c^2 H_0^2/(8\pi G)$, ensuring consistency with the body-text Planck 2018 value $H_0 = 67.4 \text{ km/s/Mpc}$. Propagating the Planck uncertainty in Ω_{DE} alone gives $\delta\alpha/\alpha = \delta\Omega_{\text{DE}}/\Omega_{\text{DE}} = 0.006/0.689 \approx 0.9\%$, i.e. $\alpha = 0.082 \pm 0.001_{\text{stat}}$; the H_0 tension is a larger unmodeled systematic, so we quote $\alpha \approx 0.082$ in the body.

A.2 Why the Hubble cutoff fails and the event horizon succeeds

The infrared cutoff determines the equation of state. With the Hubble cutoff $r_H = c/H$, the Friedmann equation gives

$$H^2 = \frac{8\pi G}{3c^2}\rho_m + \frac{8\pi\alpha}{3}H^2 \implies H^2 = \frac{8\pi G\rho_m}{3c^2(1 - 8\pi\alpha/3)}. \quad (37)$$

During matter domination $\rho_m \propto a^{-3}$, so $H^2 \propto a^{-3}$ and $\dot{H}/H^2 = -3/2$. From $w = -1 - 2\dot{H}/(3H^2)$,

$$w_{\text{intrinsic}} = -1 - \frac{2 \times (-3/2)}{3} = 0. \quad (38)$$

With the Hubble cutoff the intrinsic equation of state is $w = 0$: dark energy dilutes like matter, $\rho_{\text{DE}} \propto a^{-3}$, producing no acceleration. Current data give $w_0 = -1.03 \pm 0.03$, ruling out $w = 0$ at more than 30σ .

The future event horizon (5) has $dR_h/dt = HR_h - c$. With $\rho_{\text{DE}} = 3\xi^2 M_P^2/R_h^2$, the dark-energy fraction is $\Omega_{\text{DE}} = \xi^2/(H^2 R_h^2)$, and the continuity equation with the R_h evolution gives

$$w = -\frac{1}{3} - \frac{2\sqrt{\Omega_{\text{DE}}}}{3\xi}. \quad (39)$$

Li's $\xi = 1$ gives $w_0 = -1/3 - 2\sqrt{0.689}/3 \approx -0.88$ at $\Omega_{\text{DE}} = 0.689$, in mild tension. Two equivalent routes fix ξ . Temperature equality $T_H = T_E$ gives $HR_h = c$; GSL saturation gives $dS_E/dt \rightarrow 0 \Rightarrow HR_h \rightarrow c$ (Section 2.3). Both are geometric identities of de Sitter space. With $HR_h = c$, the relation $\Omega_{\text{DE}} = \xi^2 c^2/(HR_h)^2$ gives

$$\xi = \sqrt{\Omega_{\text{DE}}} \approx 0.83 \quad \text{at } \Omega_{\text{DE}} = 0.689, \quad (40)$$

and $w = -1/3 - 2/3 = -1$ exactly, indistinguishable from a cosmological constant, arising as a consistency requirement rather than imposed by hand.

The normalized Hubble parameter $E(z) = H(z)/H_0$ separates the cases at $z = 1$: Λ CDM with $\Omega_m = 0.311$ gives $E(1) = \sqrt{0.311 \times 8 + 0.689} \approx 1.78$; Hubble-cutoff HDE with $w = 0$ gives $E(1) = (1+z)^{3/2} \approx 2.83$, a 60% deviation ruled out; event-horizon HDE with de Sitter saturation gives $w = -1$ and $E(1) \approx 1.78$, identical to Λ CDM within observational uncertainties. The framework is degenerate with Λ CDM at the level of the background expansion history. In the CPL parameterization $w(a) = w_0 + w_a(1-a)$, current data give $w_a = -0.1 \pm 0.3$, consistent with no evolution.

A.3 Numerical check of the central identity

Table 6. Numerical check of the dark-energy / horizon-entropy identity (28). The product $\rho_{\text{DE}} S_{\text{dS}}$ equals the Planck-scale constant $3c^7/(8G^2\hbar)$ in the de Sitter attractor only. “Ratio” is the product divided by the constant; it approaches unity in the asymptote.

Epoch	ρ_{DE} (J/m ³)	S_{dS} (nats)	$\rho_{\text{DE}} S_{\text{dS}}$ (J/m ³)	Ratio
Today	5.28×10^{-10}	2.27×10^{122}	1.20×10^{113}	$0.689 = \Omega_{\text{DE}}$
Cosmic noon ($z = 1$)	5.28×10^{-10}	7.13×10^{121}	3.76×10^{112}	0.217
de Sitter asymptote	5.28×10^{-10}	3.29×10^{122}	1.74×10^{113}	1.00
Planck-scale constant $3c^7/(8G^2\hbar) = (3/8)\rho_{\text{Pl}}$:				1.74×10^{113} J/m ³

The asymptotic ratio matches unity within rounding, residuals at the level of present cosmological-parameter uncertainties. Pre-asymptotic ratios are smaller because $S_{\text{dS}}(t)$ has not yet saturated at S_{dS}^∞ .

A.4 Cosmological constant problem in Planck units

The standard statement is

$$\frac{\rho_{\text{DE}}^{\text{obs}}}{\rho_{\text{Pl}}} = \frac{5.28 \times 10^{-10} \text{ J/m}^3}{4.63 \times 10^{113} \text{ J/m}^3} \approx 1.14 \times 10^{-123}, \quad (41)$$

the famous 122–123 orders-of-magnitude hierarchy. (The exact exponent depends on whether the comparison is to ρ_{Pl} or to the QFT zero-point estimate; the difference is at most one order of magnitude.) Equivalently,

$$S_{\text{dS}}^\infty = \frac{3/8}{\rho_{\text{DE}}^\infty/\rho_{\text{Pl}}} \approx \frac{3/8}{1.14 \times 10^{-123}} \approx 3.29 \times 10^{122}, \quad (42)$$

in nats, agreeing with the directly-computed value (Table 7) to within input precision.

A.5 Consistency with $\alpha = 0.082$

By $\alpha = 3\Omega_{\text{DE}}/(8\pi)$, the today-to-asymptote entropy ratio is

$$\frac{S_{\text{dS}}^{\text{today}}}{S_{\text{dS}}^\infty} = \left(\frac{H_\infty}{H_0}\right)^2 = \Omega_{\text{DE}} = \frac{8\pi\alpha}{3} \approx \frac{8\pi \times 0.0822}{3} \approx 0.689,$$

so $S_{\text{dS}}^\infty/S_{\text{dS}}^{\text{today}} = 1/\Omega_{\text{DE}} = 3/(8\pi\alpha) \approx 1.45$. From the numerical values, $3.29 \times 10^{122}/2.27 \times 10^{122} \approx 1.45$, matching exactly. The universe today is approximately 69% of the way to its asymptotic entropy capacity—which is exactly Ω_{DE} .

B Cosmic Entropy History

This appendix traces the cosmic entropy capacity $S_{\text{dS}}(t)$ from the Planck epoch to the de Sitter asymptote, showing how a single quantity governed by $H(t)$ produces the entropy budget underlying both dark energy and the arrow of time.

B.1 The capacity as a function of cosmic time

The cosmic entropy capacity is

$$S_{\text{ds}}(t) = \frac{\pi c^5}{H(t)^2 \hbar G} = \frac{\pi}{[H(t) t_{\text{P}}]^2}, \quad (43)$$

the last form making the Planck-units structure manifest: S_{ds} is the inverse square of the dimensionless ratio $H(t) t_{\text{P}}$. At the Planck epoch $H \sim 1/t_{\text{P}}$ and $S_{\text{ds}} \sim \pi$; today $H t_{\text{P}} \sim 10^{-61}$ and $S_{\text{ds}} \sim 10^{122}$.

Table 7. *The cosmic horizon entropy $S_{\text{ds}} = \pi c^5/(H^2 \hbar G)$ at representative epochs. Hubble rates are standard Λ CDM estimates with the Planck 2018 parameters of Appendix A.*

Epoch	Redshift z	H (s^{-1})	S_{ds} (nats)
Planck epoch	$\sim 10^{32}$	$\sim 1.85 \times 10^{43}$	$\pi \approx 3.14$
End of inflation	$\sim 10^{30}$	$\sim 10^{36}$	$\sim 1 \times 10^{17}$
QCD transition	$\sim 10^{12}$	$\sim 10^{12}$	$\sim 1 \times 10^{65}$
Matter–radiation eq.	3.4×10^3	3.40×10^{-13}	9.3×10^{111}
Recombination	1.1×10^3	5.11×10^{-14}	4.1×10^{113}
$z = 1$ (cosmic noon)	1	3.89×10^{-18}	7.13×10^{121}
Today	0	2.18×10^{-18}	2.27×10^{122}
de Sitter asymptote	$z \rightarrow -1$ (infinite future)	1.81×10^{-18}	3.29×10^{122}

The cosmic entropy capacity grows from $\approx \pi$ nats at the Planck epoch to $\approx 3.3 \times 10^{122}$ nats in the de Sitter asymptote, 122 orders of magnitude over the full history of the universe.

B.2 Epoch-by-epoch evolution

Planck epoch ($t \sim t_{\text{P}}$). $H \sim 1/t_{\text{P}} \sim 1.85 \times 10^{43} \text{ s}^{-1}$; $S_{\text{ds}} \sim \pi$ nats; horizon area $\sim 1 \ell_{\text{P}}^2$. Approximately one Planck area’s worth of capacity.

Inflationary epoch. If inflation occurs with energy scale $H_{\text{inf}} \sim 10^{14} \text{ GeV}$ ($H_{\text{inf}} \sim 10^{36} \text{ s}^{-1}$), the capacity is $S_{\text{ds}} \sim 10^{17}$ nats, approximately constant during the de Sitter inflationary phase.

Radiation era. After reheating $H \propto a^{-2}$, so $S_{\text{ds}} \propto a^4$. Evaluating $S_{\text{ds}} = \pi c^5/(H^2 \hbar G)$ at matter–radiation equality ($z_{\text{eq}} \approx 3400$) with the full Λ CDM rate $H \approx 3.40 \times 10^{-13} \text{ s}^{-1}$ gives $S_{\text{ds}} \approx 9.3 \times 10^{111}$ nats.

Matter era. $H \propto a^{-3/2}$, so $S_{\text{ds}} \propto a^3$. At recombination ($z = 1100$, $H \approx 5.11 \times 10^{-14} \text{ s}^{-1}$), $S_{\text{ds}} \approx 4.1 \times 10^{113}$ nats; at $z = 1$ ($H \approx 3.89 \times 10^{-18} \text{ s}^{-1}$), $S_{\text{ds}} \approx 7.1 \times 10^{121}$ nats. Each value is obtained by direct evaluation of the closed-form formula at the appropriate $H(z)$.

Dark-energy era. Today ($z = 0$), $H_0 \approx 2.18 \times 10^{-18} \text{ s}^{-1}$ gives $S_{\text{ds}}^{\text{today}} \approx 2.27 \times 10^{122}$ nats. The asymptotic value $H_{\infty} = H_0 \sqrt{\Omega_{\text{DE}}}$ gives $S_{\text{ds}}^{\infty} \approx 3.29 \times 10^{122}$ nats. The universe today is approximately 69% of the way (in entropy budget) to this asymptote, consistent with $H_0^2/H_{\infty}^2 = 1/\Omega_{\text{DE}} \approx 1.45$.

B.3 Entropy production from gravitational decoherence

The actual entropy $S(t)$ tracks $S_{\text{dS}}(t)$ under Assumption 2.4. The mechanism is cosmic gravitational decoherence, whose *per-mode* rate is [39]

$$\Gamma_{\text{mode}}(t) = \frac{g(1) H(t)}{4\pi} \approx \frac{H(t)}{10\pi}, \quad g(1) = 1 - \frac{2}{\pi} \text{Si}(1) \approx 0.398. \quad (44)$$

Integrated over the age of the universe this produces only ~ 4 nats per mode; the cosmic budget is reached by summing over the $S_{\text{dS}}(t)$ holographically-allowed horizon modes,

$$\dot{S}(t) \sim S_{\text{dS}}(t) \Gamma_{\text{mode}}(t) = \frac{g(1) S_{\text{dS}}(t) H(t)}{4\pi}, \quad (45)$$

the per-mode rate supplying the H -scaling and the mode count S_{dS} closing the ~ 122 -order gap. At each epoch the rate at which cosmic entropy is realized scales as H , tracking the rate at which the capacity grows.

The naive insertion of $M \sim 10^{53}$ kg (the observable-universe mass) into the lab-scale formula $\Gamma = GM^2/(\hbar d)$ [36] gives a super-causal rate $\sim 10^{113} \text{ s}^{-1}$. This is resolved [39] by (i) redoing the calculation on a de Sitter background rather than Minkowski, giving the form factor $g(x) = 1 - (2/\pi) \text{Si}(x)$ that vanishes for super-horizon mass separations; (ii) selecting the cosmic “ Δm ” as the de Sitter thermal mass $\Delta m_{\text{dS}} = \hbar H/(2\pi c^2)$; (iii) bounding mode additivity at $O(G^2)$. The cosmic rate is then $\Gamma_{\text{total}} = g(1) H/(4\pi) \approx 7 \times 10^{-20} \text{ Hz}$ today, sub-causal.

B.4 Cross-check: the saturation gap

If saturation (Assumption 2.4) is generic, the total entropy produced from the Planck epoch to today should equal $S_{\text{dS}}^{\text{today}} \sim 10^{122}$. The naive per-mode integral is

$$\Delta S_{\text{produced}}^{\text{per-mode}} \sim \int_{t_{\text{P}}}^{t_0} \Gamma_{\text{mode}}(t) dt \sim \frac{N_{\text{efolds}}}{10\pi}, \quad (46)$$

where $N_{\text{efolds}} \sim \ln(a_0/a_{\text{Pl}}) \sim 140$ (60 from inflation +60 subsequent +20 pre-inflation if any), giving $\Delta S \sim 4$ nats per mode, 122 orders short of S_{dS}^∞ . The gap is closed by the mode count: each Hubble volume contains S_{dS} holographically distinguishable modes, so the total rate scales as $S_{\text{dS}} \cdot H/(10\pi)$ and the accumulated entropy as S_{dS} .

Status: structural, not closed. The picture is internally consistent, but a first-principles derivation of why the per-mode rate combines with the holographic count to give precisely S_{dS} nats (rather than $0.1 S_{\text{dS}}$ or $10 S_{\text{dS}}$) is not available, and the mapping between “mode-decoherence events” and “nats realized” carries unresolved $O(1)$ ambiguities. We claim no better than order-of-magnitude agreement; the qualitative match between capacity-growth rate ($\sim H S_{\text{dS}}$) and entropy-production rate ($\sim H S_{\text{dS}}$) supports Assumption 2.4 but is one step short of a derivation.

B.5 Summary

The cosmic entropy capacity $S_{\text{dS}}(t)$ grows monotonically from $\sim \pi$ nats at the Planck epoch to $\sim 3 \times 10^{122}$ nats in the de Sitter asymptote, with entropy production tracking it through

per-mode gravitational decoherence at rate $g(1) H/(4\pi)$. Dark energy, the Past Hypothesis, the Weyl asymmetry, and the cosmological constant problem are different facets of this single arc of capacity growth.

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