A Thermodynamic Framework for Holographic Dark Energy

Quantum-Geometric Correspondence

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Abstract

Holographic dark energy with an event-horizon cutoff gives a dark energy density $\rho_{\text{DE}} = \alpha c^2 H^2/G$ with $\alpha \approx 0.082$. The de Sitter attractor of any dark-energy-dominated cosmology fixes the saturation parameter through $\xi = \sqrt{\Omega_{\text{DE}}}$ and pins the equation of state to $w = -1$ as a consistency requirement: extending the generalized second law to cosmological horizons makes de Sitter space thermodynamically consistent, the saturation condition $HR_h = c$ coinciding with the de Sitter geometric identity. The ratio ρ_{DE}/ρ_m remains stable during matter domination, ameliorating the timing aspect of the coincidence problem. The scope is limited. The cosmological constant problem is not solved—the magnitude $\alpha \sim 0.08$ encodes the same mystery as Λ, merely reparameterized. The claim that $\alpha \sim 0.1$ is natural is tautological: $\alpha = 3\Omega_{\text{DE}}/(8\pi)$ is of order 0.1 whenever $\Omega_{\text{DE}} = O(1)$, which holds by definition at the present epoch. No microscopic mechanism is provided, and at current observational precision no measurement distinguishes this framework from standard ΛCDM. The framework offers thermodynamic language for organizing holographic dark energy, not a solution to its fundamental mysteries.

Keywords: dark energy, holographic principle, cosmological constant problem, horizon thermodynamics, event horizon

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1 Introduction

Quantum field theory predicts a vacuum energy density of order $\rho_{\text{vac}} \sim 10^{113}$ J/m^3, the Planck-scale contribution of zero-point energies up to the Planck cutoff; cosmic acceleration [1, 2, 3, 4] reveals a dark energy density of $\rho_\Lambda \sim 10^{-9}$ J/m^3. The discrepancy spans 120 orders of magnitude—arguably the worst prediction in the history of physics [5]. The coincidence problem compounds it: the ratio ρ_Λ/ρ_m varies by some thirty orders of magnitude across cosmic history, yet we observe it today at ≈ 2.3, because in ΛCDM the constant Λ and the diluting $\rho_m \propto a^{-3}$ are equal only during a fleeting epoch. Anthropic selection [6], quintessence [7], and modified gravity [8] relocate rather than resolve the question of why the dark energy scale is so small.

1.1 Result and scope

This paper casts holographic dark energy (HDE)—the effective-field-theory bound of Cohen, Kaplan and Nelson [9], given dynamical form by Li [10]—as a thermodynamic consistency framework. The holographic bound caps the entropy of a region by its boundary area, so dark energy tied to the information content of a cosmological horizon scales on dimensional grounds as $\rho_{\text{DE}} \propto 1/L^2$ for an infrared cutoff L. With the horizon as cutoff,

$$\rho_{\text{DE}} = \alpha \frac{c^2 H^2}{G}, \tag{1}$$

where $\alpha = 3\Omega_{\text{DE}}/(8\pi) \approx 0.082$. The H^2 scaling follows from dimensional analysis given the horizon cutoff; the coefficient is fitted to the observed fraction $\Omega_{\text{DE}} = 0.689$. The magnitude is not explained: the mystery of why $H_0 \ll H_P$ is equivalent to the mystery of why $\Lambda \ll \Lambda_P$. We have reparameterized, not solved, the cosmological constant problem.

Two assumptions—the generalized second law extended to cosmological horizons, and the holographic entropy bound—make de Sitter space the unique late-time attractor of the cosmic evolution. The condition $HR_h = c$ for the future event horizon R_h is a geometric identity of de Sitter space, hence a consistency requirement rather than an independent prediction; it fixes the saturation parameter $\xi = \sqrt{\Omega_{\text{DE}}} \approx 0.83$ and gives equation of state $w = -1$ exactly at all epochs. The ratio ρ_{DE}/ρ_m is stable during matter domination, removing the timing aspect of the coincidence problem while leaving the magnitude of α unexplained.

Remark 1.1 (What the framework does not achieve). The cosmological constant problem is not solved: why $\alpha \sim 0.08$ rather than some other value encodes the same mystery as Λ. The claim that $\alpha \sim 0.1$ is "natural" is tautological, since $\alpha = 3\Omega_{\text{DE}}/(8\pi)$ is of order 0.1 whenever $\Omega_{\text{DE}} = O(1)$, which holds by definition at the present epoch. No microscopic mechanism is provided. At present precision no observation distinguishes this framework from standard ΛCDM.

This paper belongs to the Quantum-Geometric Correspondence series. The canonical core paper [11] presents the axiomatic framework, the gravitational-decoherence predictions, and the information-theoretic bounds; companion papers treat emergent gravity with MOND phenomenology and the entanglement-decoherence prediction. Each paper is self-contained.

Section 2 states the two foundational principles and the cosmological horizons. Section 3 derives the dark energy formula and equation of state. Section 4 addresses the coincidence problem; Section 5, predictions and falsifiability; Section 6, alternative approaches; Section 7, conclusions and limitations.

2 Theoretical Framework

The framework rests on two principles, both from black hole physics, extended here to the cosmological context. Neither has been rigorously proven for cosmological horizons; each is a working assumption whose consistency the framework tests.

2.1 The two foundational principles

Assumption 2.1 (Generalized Second Law with Saturation)**.** For cosmological horizons, the generalized entropy $S_{\text{gen}} = S_{\text{matter}} + S_{\text{horizon}}$ satisfies $dS_{\text{gen}}/dt \geq 0$, with the entropy approaching a maximum (saturating) as $t \to \infty$.

The horizon entropy takes the Bekenstein-Hawking form $S_{\text{horizon}} = A/(4\ell_P^2)$, with horizon area A and Planck length $\ell_P = \sqrt{G\hbar/c^3}$ [12, 13]. For black holes the GSL is well-established by semiclassical arguments [14]. Its extension to cosmological horizons is physically motivated but subtle: such horizons are observer-dependent, and their thermodynamic interpretation remains open. The framework proceeds on the assumption that the thermodynamic description holds and tests it by consistency.

Assumption 2.2 (Holographic Bound)**.** The maximum entropy contained within a region is bounded by its boundary area: $S_{\text{max}} \leq A/(4\ell_P^2)$.

The bound originates in black hole physics: black holes carry entropy proportional to horizon area, and packing more entropy into a region induces gravitational collapse [15, 14]. The degrees of freedom of a region are encoded on its boundary, against the volume-scaling of local quantum field theory. This underlies the holographic ansatz $\rho_{\text{DE}} \propto 1/L^2$.

2.2 The two cosmological horizons

An expanding universe possesses horizons that limit causal access and carry thermodynamic properties analogous to black hole horizons.

The Hubble horizon $r_H = c/H$, with $H = \dot{a}/a$, is the distance at which recession reaches the speed of light. In de Sitter space an observer perceives thermal radiation at $T_H = \hbar H/(2\pi k_B)$ [13], of order 10^{-30} K today.

The future event horizon

$$R_h(t) = a(t) \int_t^{\infty} \frac{c\, dt'}{a(t')} \tag{2}$$

is the boundary beyond which events are never observable. It depends on the entire future evolution, not on instantaneous data. It carries temperature $T_E = \hbar c/(2\pi k_B R_h)$, structurally the Hawking form at the event-horizon scale. In de Sitter space, $R_h = c/H$ and the two temperatures coincide, $T_H = T_E$; this equality is the consistency condition characterizing the late-time attractor (Sec. 2.3).

Remark 2.1 (Teleological character of the event horizon). $R_h(t)$ depends on the *future* evolution of the universe, whereas physics is normally fixed by initial conditions. This limitation is shared by all event-horizon-based approaches and is not resolved here. The Hubble horizon avoids it but gives $w = 0$, ruled out observationally. We read the event horizon not as exerting causal influence from the future but as encoding global constraints: the universe, as a solution to Einstein's equations, is determined as a whole.

The choice of the event horizon over the Hubble horizon as the infrared cutoff is empirical. The Hubble cutoff predicts $w = 0$—dark energy tracks matter and produces no acceleration—ruled out at more than 30σ [3]. The event-horizon cutoff yields $w = -1$, consistent with all current data. The event-horizon cutoff is chosen because it works, not because it is derived from first principles; the theoretical justification remains incomplete.

2.3 The de Sitter attractor

Any cosmology dominated by dark energy with $w < -1/3$ asymptotically approaches de Sitter space—a consequence of the Friedmann equations, not a prediction. In de Sitter space $R_h = c/H$, so

$$HR_h = c \quad \text{(de Sitter geometric identity).} \tag{3}$$

Proposition 2.3 (Thermodynamic consistency of de Sitter)**.** *The horizon entropy* $S_E = \pi c^3 R_h^2/(G\hbar)$ *saturates,* $dS_E/dt \to 0$*, if and only if* $HR_h \to c$*.*

Proof. Differentiating the defining integral gives the event-horizon evolution

$$\frac{dR_h}{dt} = HR_h - c, \tag{4}$$

hence

$$\frac{dS_E}{dt} = \frac{2\pi c^3 R_h}{G\hbar}(HR_h - c). \tag{5}$$

Since $R_h > 0$, the right side vanishes exactly when $HR_h = c$, the de Sitter condition (3). □

Remark 2.2 (Scope of Proposition 2.3)*.* The GSL is *consistent with* de Sitter asymptotics and explains why the de Sitter endpoint maximizes entropy; it does not *derive* de Sitter or predict $w = -1$. The de Sitter endpoint is the attractor of any dark-energy-dominated cosmology; the result shows this attractor is thermodynamically sensible, not that thermodynamics selects it.

The condition $HR_h = c$ is equivalent to the temperature equality $T_H = T_E$, sometimes called "thermodynamic equilibrium" between the horizons. The Hubble and event horizons are not in causal contact and no process equilibrates them; the equality is a geometric fact of de Sitter space, a consistency condition, not a physical equilibration.

3 The Dark Energy Formula

The holographic ansatz with the horizon scale as infrared cutoff fixes the H^2 scaling of the dark energy density by dimensional analysis. This section determines the coefficient α, the saturation parameter ξ, and the equation of state w.

3.1 The H^2 scaling

For a region of characteristic size L, the holographic ansatz posits $\rho_{\rm DE} \propto 1/L^2$. With L the Hubble scale c/H, dimensional analysis gives

$$\rho_{\rm DE} = \alpha \frac{c^2 H^2}{G}, \tag{6}$$

since c^2H^2/G carries the dimensions of energy density. This is dimensional analysis given the ansatz, not a first-principles derivation; the coefficient α encodes the unexplained physics. Equation (6) is the late-time de Sitter relation, where the event horizon $R_h = c/H$ coincides with the Hubble scale: α is read off only at the attractor, so the Hubble-cutoff result $w = 0$ of Appendix C does not apply there. The value of α and the equation of state are evaluated with cutoffs that coincide only at the de Sitter attractor.

3.2 The coefficient α

To determine α, substitute (6) into the Friedmann equation. In a flat universe of matter and dark energy,

$$H^2 = \frac{8\pi G}{3c^2}(\rho_m + \rho_{\rm DE}). \tag{7}$$

Substituting $\rho_{\rm DE} = \alpha c^2 H^2/G$ and solving for H^2,

$$H^2 = \frac{8\pi G \rho_m}{3c^2(1 - 8\pi\alpha/3)}. \tag{8}$$

Positivity $H^2 > 0$ requires the self-consistency bound $\alpha < 3/(8\pi) \approx 0.119$; larger α would make the dark energy contribution exceed the total.

With $\rho_{\rm crit} = 3c^2H^2/(8\pi G)$, the dark energy fraction is

$$\Omega_{\rm DE} = \frac{\rho_{\rm DE}}{\rho_{\rm crit}} = \frac{\alpha c^2 H^2/G}{3c^2H^2/(8\pi G)} = \frac{8\pi\alpha}{3}, \tag{9}$$

inverted to

$$\alpha = \frac{3\Omega_{\rm DE}}{8\pi}. \tag{10}$$

The observed $\Omega_{\rm DE} = 0.689 \pm 0.006$ [3] yields

$$\alpha \approx 0.082. \tag{11}$$

Propagating the statistical uncertainty in $\Omega_{\rm DE}$ alone gives a formal error at the one percent level; this figure is statistical only and does not account for the H_0-tension systematic, so we quote $\alpha \approx 0.082$ rather than a sharp ± 0.001.

Cosmic acceleration requires $\Omega_{\rm DE} > 1/2$, i.e. $\alpha > 3/(16\pi) \approx 0.060$. With the self-consistency bound,

$$0.060 \lesssim \alpha \lesssim 0.119, \tag{12}$$

a range spanning a factor of two, within which $\alpha \approx 0.082$ falls. The claim that $\alpha \sim 0.1$ is "natural" is tautological: $\alpha = 3\Omega_{\rm DE}/(8\pi)$ is of order 0.1 whenever $\Omega_{\rm DE}$ is of order unity, and the present epoch is defined as the time when dark energy becomes dynamically important. The naturalness is circular.

3.3 The saturation parameter and the equation of state

The equation of state $w = p_{\rm DE}/\rho_{\rm DE}$ governs the dark energy dynamics. For an event-horizon cutoff, the standard derivation (Appendix A) gives

$$w = -\frac{1}{3} - \frac{2\sqrt{\Omega_{\rm DE}}}{3\xi}, \tag{13}$$

where ξ is the saturation parameter of the ansatz $\rho_{\rm DE} = 3\xi^2 M_P^2/R_h^2$. Li's original HDE proposal [10] took $\xi = 1$, which with $\Omega_{\rm DE} = 0.689$ gives $w_0 = -0.88$, in mild tension with observations; the Hubble-scale cutoff gives the degenerate $w = 0$ [16].

Theorem 3.1 (Saturation parameter and equation of state at the attractor)**.** *Imposing the de Sitter condition $HR_h = c$ of Section 2 fixes*

$$\xi = \sqrt{\Omega_{DE}} \approx 0.83, \tag{14}$$

and the equation of state (13) reduces to $w = -1$ exactly at all epochs.

Proof. The holographic relation $\Omega_{\rm DE} = \xi^2 c^2/(H^2 R_h^2)$ with $HR_h = c$ gives (14), a consistency condition of the attractor, not a fitted parameter. Substituting into (13),

$$w = -\frac{1}{3} - \frac{2\sqrt{\Omega_{\rm DE}}}{3\sqrt{\Omega_{\rm DE}}} = -\frac{1}{3} - \frac{2}{3} = -1. \tag{15}$$

□

When the de Sitter saturation condition $\xi = \sqrt{\Omega_{\rm DE}}$ holds, holographic dark energy has the equation of state of a cosmological constant. Current observations give $w_0 = -1.03 \pm 0.03$, consistent with this value.

The attractor is dynamically stable: a perturbation $\delta\Omega_m$ about the de Sitter fixed point evolves as $\delta\Omega_m \propto e^{\lambda N}$ with $N = \ln a$ and $\lambda = -3\Omega_{\rm DE} < 0$, decaying over roughly half an e-fold, so initial conditions in the basin converge to the de Sitter endpoint.

The dark energy formula $\rho_{\rm DE} = \alpha c^2 H^2/G$ follows from dimensional analysis given the holographic ansatz; the coefficient $\alpha = 0.082$ is fitted to observations; the de Sitter condition determines $\xi = \sqrt{\Omega_{\rm DE}}$ as a consistency requirement, yielding $w = -1$. The framework is internally consistent but does not explain the value of α or supply a microscopic mechanism for dark energy.

4 The Coincidence Problem

The coincidence problem asks why the dark energy and matter densities are comparable today, when their ratio has varied enormously over cosmic history. In ΛCDM, Λ has a fixed density while $\rho_m \propto a^{-3}$, so $\rho_\Lambda/\rho_m \propto a^3$ grows by a factor $\sim 10^{30}$ from matter-radiation equality to the distant future; observing it at order unity is either explained or accepted as coincidence.

4.1 Tracking during matter domination

The dark energy density $\rho_{\rm DE} = \alpha c^2 H^2/G$ tracks the expansion rate rather than staying constant. During matter domination $H^2 \propto \rho_m$, so $\rho_{\rm DE} \propto \rho_m$ and

$$\frac{\rho_{\rm DE}}{\rho_m} = \frac{\Omega_{\rm DE}}{1-\Omega_{\rm DE}} = \frac{8\pi\alpha/3}{1-8\pi\alpha/3}, \tag{16}$$

constant during matter domination. Dark energy and matter keep a fixed proportion, removing the timing coincidence of ΛCDM.

At the transition to dark-energy domination, the matter density dilutes below $\rho_{\rm DE}$, the Hubble parameter asymptotes to $H_\infty = \sqrt{8\pi G\rho_{{\rm DE},\infty}/(3c^2)}$, and $\rho_{\rm DE}/\rho_m$ grows as the universe approaches de Sitter space. This transition occurs when $\Omega_m \sim \Omega_{\rm DE}$ by definition, an epoch set by α rather than fine-tuned.

4.2 Relocation of the coincidence to α

The timing aspect is removed: in ΛCDM the epoch when $\rho_\Lambda \sim \rho_m$ is special, whereas here no time during matter domination is distinguished, since the ratio is constant. The coincidence is relocated from cosmic time to the parameter α: the puzzle becomes why $\alpha \approx 0.08$.

This is a rephrasing, not a solution. The bounds within which α produces an accelerating universe span a factor of two, and the naturalness argument is circular: $\alpha = 3\Omega_{\rm DE}/(8\pi)$ is of order 0.1 whenever $\Omega_{\rm DE} \sim 1$, and the present is defined as the epoch when $\Omega_{\rm DE} \sim 1$. The magnitude of α encodes the same mystery as the cosmological constant: why $H_0 \sim 10^{-18}$ s^{-1} rather than $H_P \sim 10^{44}$ s^{-1}, why $\rho_{\rm DE}^{1/4} \sim 10^{-3}$ eV rather than the Planck energy. The 10^{120} discrepancy reappears in disguised form. We have reparameterized the problem—from why Λ is small to why αH^2 is small—but the deep mystery remains. The coincidence problem is transformed, not solved.

5 Predictions and Falsifiability

The framework makes definite statements about cosmological observables that, at present precision, do not distinguish it from ΛCDM.

5.1 The core prediction and its falsifiers

Dark energy arises from the holographic bound on the future event horizon, giving $\rho_{\rm DE} = \alpha c^2 H^2/G$ with $\alpha = 0.082 \pm 0.001$ and a de Sitter attractor with $w = -1$. Combined Planck, baryon acoustic oscillation, and Type Ia supernova data give $w_0 = -1.03 \pm 0.03$, consistent with the prediction; the DESI hints of time-varying $w(z)$ remain at low significance and subject to systematics.

Three classes of measurement would falsify the framework.

1. *Equation of state.* A $w(z)$ evolution incompatible with a constant near -1 at greater than 3σ would rule out the holographic ansatz as formulated, requiring either $\xi \neq \sqrt{\Omega_{\rm DE}}$ against the consistency analysis or a failure of the horizon relation at some epoch. Euclid and the Vera Rubin Observatory's LSST will constrain $w(z)$ at the sub-percent level across redshift bins.

2. *Modified gravity.* HDE operates within general relativity with $G_{\text{eff}} = G_N$ exactly; a measured $G_{\text{eff}}(z)/G_N \neq 1$ at the percent level—via lensing statistics or the integrated Sachs-Wolfe effect—would rule it out. $f(R)$ and scalar-tensor theories generically predict $G_{\text{eff}} \neq G_N$, a clean discriminant.

3. *Early dark energy.* The CMB constrains $\Omega_{\text{DE}}(z = 1100) \lesssim 10^{-4}$, whereas the tracking value during matter domination is ≈ 0.7. The tracking analysis of Section 4 applies only during matter domination; the radiation era and earlier follow a different scaling, requiring a full solution of the Friedmann equations across all epochs, deferred to future work. Early dark energy beyond the CMB bounds would require re-examining whether the framework accommodates it.

Table 1 separates what is derived as a consistency condition, what follows from general relativity, and what is fitted to observations.

Observable	**Value**	**Status**
Equation of state $w(z = 0)$	-1	De Sitter attractor consistency
Saturation parameter ξ	$\sqrt{\Omega_{\text{DE}}} = 0.83$	Consistency condition
Holographic coefficient α	0.082 ± 0.001	Fitted to Ω_{DE}
Effective gravitational constant G_{eff}	$= G_N$	Follows from GR
ISW effect	Matches ΛCDM	Follows from $w = -1$

Table 1. *Summary of HDE predictions. The equation of state follows from the de Sitter attractor consistency condition; the saturation parameter follows from the same condition; the coefficient α is fitted; the gravitational constant follows from operating within general relativity; ISW suppression is predicted but within current cosmic variance.*

5.2 CMB consistency and late-time expansion

With $w = -1$ from the de Sitter attractor, the framework is observationally indistinguishable from a cosmological constant at the level of the equation of state, consistent with $w_0 = -1.03 \pm 0.03$.

The integrated Sachs-Wolfe (ISW) effect matches ΛCDM. For $w = -1$, gravitational potentials decay at the standard rate, so at multipoles $\ell < 30$, where the ISW contributes roughly 10% of the temperature power, the framework predicts no deviation from ΛCDM; any difference lies within cosmic variance ($\sigma \sim 30\%$ at $\ell = 2$). The ISW-galaxy cross-correlation likewise matches ΛCDM, with current measurements consistent at the $\sim 2\sigma$ level; CMB-S4 combined with Euclid and the Vera Rubin Observatory will tighten this but are not expected to distinguish the two given the identical $w = -1$.

The model with $\alpha = 0.082$ is consistent with current CMB constraints from DESI 2024 baryon acoustic oscillations and Planck. The modified-gravity parameters from the ISW-lensing bispectrum, $\mu_0 = 0.05 \pm 0.22$ and $\Sigma_0 = 0.008 \pm 0.045$, are consistent with general relativity. The primary acoustic peaks ($30 < \ell < 1000$) show no deviation from ΛCDM, since they probe recombination, where dark energy contributed negligibly.

5.3 Distinguishing power

At present precision the framework predicts nearly identically to ΛCDM: both give $G_{\text{eff}} = G_N$ and w within current uncertainties. The conceptual difference—relating dark energy to horizon physics rather than treating Λ as fundamental—has limited observational consequence at current sensitivity. Dark energy perturbations do not help: for $w = -1$ from the de Sitter attractor, the perturbation analysis matches ΛCDM at leading order, and subleading structure-growth effects, not yet computed quantitatively, are likely below near-future sensitivity. The framework's value is conceptual organization, a thermodynamic language connecting dark energy to horizon physics. It remains falsifiable in principle—w significantly different from -1, or $G_{\rm eff} \neq G_N$, would rule it out—and will face stringent tests as precision improves.

6 Comparison with Alternative Approaches

We compare the framework with the main alternatives, stating what each achieves and what it leaves unexplained.

6.1 ΛCDM and Li's HDE

ΛCDM treats Λ as a fundamental parameter: a single constant in the Einstein equations explains cosmic acceleration, the age of the universe, and structure growth, fitting the data with high precision. It explains nothing about the origin or magnitude of Λ, which is fine-tuned at 10^{-120} relative to naive quantum field theory. The holographic framework replaces the unexplained Λ with the unexplained dimensionless $\alpha \sim 0.08$; neither explains the magnitude, the holographic framework reorganizing the mystery into different variables.

Li's HDE [10] posits $\rho_{\rm DE} = 3\xi^2 M_P^2/L^2$ with L the future event horizon and ξ a free parameter; $\xi = 1$ gives $w_0 \approx -0.88$, in mild tension with observations. Here $\xi = \sqrt{\Omega_{\rm DE}}$ follows as a consistency condition of the de Sitter attractor, removing the freedom and yielding $w = -1$ exactly. Both frameworks share the limitation that the holographic ansatz is not derived from first principles.

6.2 Quintessence and modified gravity

Quintessence [7] drives acceleration with a scalar field ϕ and potential $V(\phi)$, giving dynamical $w(t)$. The potential is arbitrary and can produce essentially any $w(z)$, so the approach is underconstrained: it trades the parameter Λ for the function $V(\phi)$. HDE with the de Sitter condition has a single parameter α and predicts $w = -1$, whereas quintessence generically predicts $w \neq -1$; precision $w(z)$ measurements may discriminate.

Modified gravity [8] alters the Einstein equations rather than adding a dark energy component. In $f(R)$ gravity the Einstein-Hilbert action becomes a function of the Ricci scalar, introducing degrees of freedom that produce acceleration without dark energy but generically give $G_{\rm eff} \neq G_N$, varying with redshift. HDE, within general relativity, predicts $G_{\rm eff} = G_N$ exactly; $G_{\rm eff}(z)/G_N \neq 1$ at the percent level would falsify HDE and support modified gravity, while $G_{\rm eff} = G_N$ to high precision would disfavor most modified-gravity scenarios.

6.3 Standing critiques

Three critiques stand. The event-horizon cutoff is teleological, depending on the future evolution; we read this not as future causation but as the global character of solutions to Einstein's equations. No microscopic mechanism is provided: the holographic ansatz is postulated, and although string theory furnishes examples of holography (AdS/CFT), no derivation of holographic dark energy from string theory exists. The framework does not explain why quantum field theory vacuum energy, $\rho_{\rm vac} \sim 10^{113}$ J/m^3, fails to gravitate down to the observed $\rho_{\rm DE} \sim 10^{-9}$ J/m^3; it takes $\rho_{\rm DE}$ as given and provides a consistency structure around it.

The framework connects dark energy to horizon physics, ameliorates (without solving) the coincidence problem, and makes falsifiable predictions ($w = -1$, $G_{\rm eff} = G_N$). It does not solve the cosmological constant problem, supply a microscopic mechanism, or yield predictions currently distinguishable from ΛCDM. Its value is conceptual and organizational.

7 Conclusions

The framework rests on two assumptions—the generalized second law extended to cosmological horizons, and the holographic entropy bound—under which de Sitter space is the unique late-time attractor and $HR_h = c$ holds as a geometric identity. This condition fixes $\xi = \sqrt{\Omega_{\rm DE}}$ as a consistency requirement, not a fitted parameter. The dark energy density is $\rho_{\rm DE} = \alpha c^2 H^2/G$ with $\alpha = 0.082 \pm 0.001$ fitted to the observed fraction; the event-horizon cutoff with the de Sitter saturation condition gives $w = -1$ exactly, consistent with $w_0 = -1.03 \pm 0.03$.

What the framework achieves: it connects dark energy to horizon thermodynamics, relating cosmic acceleration to the principles underlying black hole entropy; it ameliorates the coincidence problem by keeping $\rho_{\rm DE}/\rho_m$ stable during matter domination, removing the timing coincidence of ΛCDM; and it is falsifiable—a measurement of $w \neq -1$ at high significance, or $G_{\rm eff} \neq G_N$, would rule it out.

What it does not achieve: the cosmological constant problem remains unsolved. Why the Hubble scale today is 60 orders of magnitude below the Planck scale—equivalently, why the observed dark energy density is 120 orders below quantum field theory expectations—is reparameterized, not explained. The coefficient $\alpha \approx 0.08$ encodes this mystery in dimensionless form without illuminating its origin. The coincidence problem is ameliorated but not solved: the magnitude of α, which sets when acceleration begins, remains unexplained. No microscopic mechanism is provided; the framework offers consistency conditions, not a derivation from quantum gravity. At present precision it predicts identically to ΛCDM, with no empirical advantage.

The framework is an organizational tool rather than a fundamental explanation. Euclid and the Vera Rubin Observatory will constrain $w(z)$ at the sub-percent level; a confirmed deviation from $w = -1$ would falsify the event-horizon HDE framework as formulated here, and the CMB predictions provide further tests as precision improves. The deeper questions—why the cosmological constant is so small, why quantum field theory vacuum energy apparently fails to gravitate, what microscopic mechanism underlies dark energy—lie beyond the present work and will require progress on quantum gravity. The holographic framework is a phenomenological bridge between observations and theoretical principles whose foundations rest on ground not yet understood.

Appendices

A Why the Hubble Cutoff Fails

The infrared cutoff determines the equation of state. The Hubble radius $r_H = c/H$ fails against observations; the future event horizon succeeds. This appendix gives both calculations.

With the Hubble cutoff, holographic dark energy has tracking equation of state $w = 0$: dark energy dilutes as $\rho_{\rm DE} \propto a^{-3}$, like matter, producing no acceleration. Current data give $w_0 = -1.03 \pm 0.03$, ruling out $w = 0$ at more than 30σ.

The future event horizon

$$R_h(t) = a(t) \int_t^\infty \frac{c\,dt'}{a(t')} \tag{17}$$

is the maximum distance from which signals can ever reach the observer, encoding the entire future evolution. Differentiating the integral gives $dR_h/dt = HR_h - c$. With $\rho_{\rm DE} = 3\xi^2 M_P^2/R_h^2$, the dark energy fraction is

$$\Omega_{\rm DE} = \frac{\xi^2}{H^2 R_h^2}, \tag{18}$$

and the continuity equation with the R_h evolution gives

$$w = -\frac{1}{3} - \frac{2\sqrt{\Omega_{\rm DE}}}{3\xi}. \tag{19}$$

Li's $\xi = 1$ gives $w_0 = -1/3 - 2\sqrt{0.689}/3 \approx -0.88$ at $\Omega_{\rm DE} = 0.689$, in mild tension with observations.

The de Sitter attractor condition $HR_h = c$ is consistent with the generalized second law with saturation, $dS_E/dt \to 0$ (Section 2), as a consistency condition rather than a first-principles derivation. With $HR_h = c$, the relation $\Omega_{\rm DE} = \xi^2 c^2/(HR_h)^2$ gives $\xi = \sqrt{\Omega_{\rm DE}} \approx 0.83$, and the equation of state becomes $w = -1/3 - 2/3 = -1$ exactly, indistinguishable from a cosmological constant.

The normalized Hubble parameter $E(z) = H(z)/H_0$ separates the three cases at $z = 1$: ΛCDM with $\Omega_m = 0.311$ gives $E(1) = \sqrt{0.311 \times 8 + 0.689} \approx 1.78$; Hubble-cutoff HDE with $w = 0$ gives $E(1) = (1+z)^{3/2} \approx 2.83$, a 60% deviation that is ruled out; event-horizon HDE with de Sitter saturation gives $w = -1$ and $E(1) \approx 1.78$, identical to ΛCDM within observational uncertainties. The framework is thus degenerate with ΛCDM at the level of the background expansion history.

The Hubble cutoff fails empirically and the event horizon succeeds, but the preference for the event horizon is not derived from first principles—a limitation that future work may address.

B Derivation of the Saturation Parameter

This appendix derives the saturation parameter $\xi = \sqrt{\Omega_{\rm DE}}$ by two equivalent routes—temperature equality and GSL saturation—both giving the de Sitter condition $HR_h = c$.

The Hubble horizon at $r_H = c/H$ carries the Gibbons-Hawking temperature [13]

$$T_H = \frac{\hbar H}{2\pi k_B}, \tag{20}$$

and the event horizon at R_h carries

$$T_E = \frac{\hbar c}{2\pi k_B R_h}, \tag{21}$$

each of order 10^{-30} K today.

Temperature equality $T_H = T_E$ gives

$$HR_h = c, \tag{22}$$

which holds identically in de Sitter space, where constant H and $R_h = c/H$ make $HR_h = c$ automatic. The horizons are not in causal contact; the equality is a geometric identity of the de Sitter solution, not thermal equilibration.

The same condition emerges from the generalized second law. The event horizon entropy is proportional to its area:

$$S_E = \frac{\pi c^3 R_h^2}{G\hbar}. \tag{23}$$

Differentiating in time with $dR_h/dt = HR_h - c$,

$$\frac{dS_E}{dt} = \frac{2\pi c^3 R_h}{G\hbar}(HR_h - c). \tag{24}$$

The GSL requires the generalized entropy to approach saturation at late times, so the horizon entropy reaches a maximum when

$$\frac{dS_E}{dt} \to 0 \implies HR_h \to c, \tag{25}$$

the de Sitter condition, consistent with entropy maximization. The two routes are equivalent geometric identities of de Sitter space; as Section 2 states, this shows the de Sitter attractor is thermodynamically sensible, not that thermodynamics alone derives it.

The holographic ansatz relates the dark energy fraction to the saturation parameter,

$$\Omega_{\mathrm{DE}} = \frac{\xi^2 c^2}{(HR_h)^2}, \tag{26}$$

and imposing $HR_h = c$ gives

$$\xi = \sqrt{\Omega_{\mathrm{DE}}} \approx 0.83 \tag{27}$$

at $\Omega_{\mathrm{DE}} = 0.689$, a consistency condition rather than a fitted parameter. Substituting into $w = -1/3 - 2\sqrt{\Omega_{\mathrm{DE}}}/(3\xi)$,

$$w = -\frac{1}{3} - \frac{2}{3} = -1 \tag{28}$$

exactly at all epochs, indistinguishable from a cosmological constant and arising as a consistency requirement rather than imposed by hand. In Li's original proposal ξ was a free parameter, with $\xi = 1$ giving $w_0 \approx -0.88$; the de Sitter attractor condition removes this freedom and improves agreement with the equation-of-state constraints.

C Numerical Values and Error Estimates

This appendix records the numerical values, propagating uncertainties from observational inputs to derived quantities.

The Planck 2018 analysis gives $\Omega_{\rm DE} = 0.689 \pm 0.006$, $\Omega_m = 0.311 \pm 0.006$, and $H_0 = (67.4 \pm 0.5)$ km/s/Mpc, satisfying flatness $\Omega_m + \Omega_{\rm DE} = 1$ within uncertainty.

The holographic coefficient is $\alpha = 3\Omega_{\rm DE}/(8\pi)$, so

$$\alpha = \frac{3 \times 0.689}{8\pi} = 0.0823, \tag{29}$$

with $\delta\alpha/\alpha = \delta\Omega_{\rm DE}/\Omega_{\rm DE} = 0.006/0.689 \approx 0.9\%$, giving $\alpha = 0.082 \pm 0.001$.

The formula $\rho_{\rm DE} = \alpha c^2 H^2/G$ ties dark energy to the Hubble parameter; the intrinsic equation of state follows from solving the Friedmann equations self-consistently. With the Hubble cutoff,

$$H^2 = \frac{8\pi G}{3c^2}\rho_m + \frac{8\pi\alpha}{3}H^2. \tag{30}$$

Solving for H^2,

$$H^2 = \frac{8\pi G\rho_m}{3c^2(1 - 8\pi\alpha/3)}. \tag{31}$$

During matter domination $\rho_m \propto a^{-3}$, so $H^2 \propto a^{-3}$ and $\dot{H}/H^2 = -3/2$. From

$$w = -1 - \frac{2\dot{H}}{3H^2}, \tag{32}$$

substituting $\dot{H}/H^2 = -3/2$,

$$w_{\rm intrinsic} = -1 - \frac{2 \times (-3/2)}{3} = 0. \tag{33}$$

With the Hubble cutoff the intrinsic equation of state is $w = 0$: dark energy dilutes like matter, producing no acceleration. The tracking keeps $\rho_{\rm DE}/\rho_m$ constant during matter domination but fails against observations, which require $w \approx -1$.

In the asymptotic regime, as $\rho_m \to 0$ the Hubble parameter approaches H_∞ set by $\rho_{\rm DE} = \alpha c^2 H_\infty^2/G$; then $\dot{H} \to 0$ and $w \to -1$, the de Sitter value. The present epoch is intermediate, where tracking dominates.

The tracking prediction $w = 0$ against $w_0 \approx -1.03 \pm 0.03$ rules out the Hubble cutoff. The event-horizon cutoff (Appendix A) yields $w = -1$ at all epochs once the de Sitter consistency condition is imposed.

Current data combining Planck, baryon acoustic oscillations, and Type Ia supernovae give $w_0 = -1.03 \pm 0.03$, consistent with $w = -1$; the CPL parameterization $w(a) = w_0 + w_a(1 - a)$ gives $w_a = -0.1 \pm 0.3$, consistent with no evolution. The DESI hints of time-varying dark energy remain at low significance. All observations are consistent with $w = -1$ to within a few percent, the value the framework predicts.

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