The Horizon Clock: A Single de Sitter Frequency Behind the MOND Scale, Cosmic Decoherence, and the Chaos Bound

Quantum-Geometric Correspondence

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July 2, 2026

Abstract

The MOND acceleration scale $a_0 \approx cH_0/2\pi$ and the cosmic gravitational decoherence rate $\Gamma_{\rm cosmic} \approx g(1)H/4\pi$ are usually treated as unrelated numerical coincidences with the Hubble rate. They are two readings of one quantity: the Gibbons–Hawking temperature of the cosmological horizon written as a frequency, the *horizon clock* $\omega_\Lambda \equiv k_{\rm B}T_{\rm dS}/\hbar = H/2\pi$, equal to the rate of Tomita–Takesaki modular flow of the de Sitter vacuum. The MOND scale is this frequency carried by c (an acceleration), $a_0 = c\,\omega_\Lambda = \kappa_{\rm dS}/2\pi$ with $\kappa_{\rm dS} = cH$ the horizon surface gravity; the cosmic decoherence rate is the same frequency carried by a pure number, $\Gamma_{\rm cosmic} = \frac{1}{2}g(1)\,\omega_\Lambda$. The two are therefore locked at a parameter-free ratio, $a_0/(c\,\Gamma_{\rm cosmic}) = 2/g(1) = 5.03$, in which every cosmological input cancels. The single non-trivial number $g(1) = 1-(2/\pi)\,\mathrm{Si}(1) = 0.398$ is the sub-horizon fraction of the gravitational self-energy (Dirichlet) integral evaluated at one Hubble radius; the same ratio counts the modular "ticks" the universe takes to decohere. We place these relations in a *horizon clock family* and separate what is established in the literature from what is new—the lock between cosmic decoherence and a_0, and its consequences. Current high-redshift rotation curves favour the constant-a_0 branch, with which the framework is consistent.

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1 Introduction: two horizon-rate coincidences

Two numbers in contemporary gravitational physics sit close to the Hubble rate. Milgrom's acceleration scale

$$a_0 \;\approx\; 1.2 \times 10^{-10}\ \mathrm{m\,s^{-2}} \;\approx\; \frac{cH_0}{2\pi}, \tag{1}$$

the acceleration below which galactic rotation curves depart from Newton, is the empirical core of MOND and the scale a dark-matter halo must reproduce [9, 10]. The rate at which horizon-scale configurations of the cosmic wavefunction lose coherence through their own gravity,

$$\Gamma_{\mathrm{cosmic}} \;\approx\; \frac{g(1)\,H}{4\pi} \;\approx\; \frac{H}{10\pi} \;\approx\; 7 \times 10^{-20}\ \mathrm{Hz}, \tag{2}$$

is derived within Quantum-Geometric Correspondence (QGC) on a de Sitter background [1], the cosmic-scale instance of the Diósi–Penrose gravitational decoherence rate [26, 27]. One belongs to galactic astronomy, the other to the foundations of quantum mechanics; this paper shows they are two readings of a single clock.

The cosmological horizon has the Gibbons–Hawking temperature [16]

$$T_{\mathrm{dS}} \;=\; \frac{\hbar H}{2\pi k_{\mathrm{B}}}, \tag{3}$$

and any temperature defines a frequency $k_{\mathrm{B}}T/\hbar$. For the horizon,

$$\boxed{\omega_\Lambda \;\equiv\; \frac{k_{\mathrm{B}}T_{\mathrm{dS}}}{\hbar} \;=\; \frac{H}{2\pi}} \tag{4}$$

the *horizon clock*. With no assumption beyond the two results above,

$$a_0 \;=\; c\,\omega_\Lambda, \qquad\qquad \Gamma_{\mathrm{cosmic}} \;=\; \tfrac{1}{2}\,g(1)\,\omega_\Lambda\ : \tag{5}$$

the MOND scale is the horizon clock carried by c (a frequency becomes an acceleration), the cosmic decoherence rate the same clock carried by a pure number. In the QGC reading, the horizon's single thermodynamic frequency projects onto an acceleration scale (how gravity behaves) and onto a rate (how fast gravity decoheres); two coincidences with H_0 are one fact about the horizon, seen twice.

Remark 1.1 (Two strengths of claim)*.* The relation between a_0 and Γ_{cosmic} is a *lock*: both equal the same ω_Λ, so their ratio $2/g(1)$ is parameter-free and fixed. The chaos bound (Sec. 7) is *family membership*: it shares the horizon's 2π but is not pinned to a_0 or Γ_{cosmic} by a parameter-free relation. The lock is the load-bearing result; the family is the wider organizing picture.

2 The horizon clock and the surface gravity

2.1 The horizon clock as modular flow

The frequency ω_Λ is the rate of the Tomita–Takesaki modular flow of the de Sitter vacuum, the Connes–Rovelli *thermal time* of the universe [19]. For a KMS state at temperature T, the modular automorphism group runs in a parameter s related to physical time by $t = (\hbar/k_{\mathrm{B}}T)\,s = s/\omega_\Lambda$, so the modular clock ticks at $k_{\mathrm{B}}T/\hbar$. Explicit constructions of the de Sitter modular Hamiltonian establish that, in the large-diamond limit, modular flow becomes static-patch time translation at this rate [20, 21]. The 2π in $\omega_\Lambda = H/2\pi$ is the thermal (KMS) period, the 2π of every Unruh, Hawking, and Gibbons–Hawking temperature.

2.2 The MOND scale as horizon surface gravity

Any causal horizon with surface gravity κ has temperature $T = \hbar\kappa/(2\pi c k_{\mathrm{B}})$, so its acceleration scale is

$$a_\star \;\equiv\; c\,\frac{k_{\mathrm{B}}T}{\hbar} \;=\; \frac{\kappa}{2\pi}. \tag{6}$$

The de Sitter horizon has $\kappa_{\mathrm{dS}} = cH$, giving

$$\boxed{a_0 \;=\; \frac{\kappa_{\mathrm{dS}}}{2\pi} \;=\; \frac{cH}{2\pi}} \tag{7}$$

the MOND acceleration as the de Sitter horizon's surface gravity divided by 2π. A Rindler horizon of acceleration a gives $a_\star = a/2\pi$, the inverse of the Unruh relation. The de Sitter thermal mass $\Delta m_{\mathrm{dS}} = k_{\mathrm{B}}T_{\mathrm{dS}}/c^2 = \hbar H/(2\pi c^2)$ has Compton wavelength equal to the horizon *circumference*, $\lambda_C(\Delta m_{\mathrm{dS}}) = \hbar/(\Delta m_{\mathrm{dS}}c) = 2\pi R_{\mathrm{H}}$, so

$$a_0 \;=\; \frac{c^2}{2\pi R_{\mathrm{H}}} \;=\; \frac{c^2}{\lambda_C(\Delta m_{\mathrm{dS}})}: \tag{8}$$

the MOND scale is the de Sitter thermal mass delocalised once around the horizon.

Remark 2.1 (Prior appearances). The identification $a_0 = c\,k_{\mathrm{B}}T_{\mathrm{dS}}/\hbar = cH/2\pi$ is present in Klinkhamer and Kopp [12], reached from a holographic minimum-temperature argument; the qualitative $a_0 \sim cH$ goes back to Milgrom's vacuum-effect proposal [11] and appears in Verlinde's emergent-gravity program [13]. The new content is the lock to cosmic decoherence.

3 The lock between a_0 and the cosmic decoherence rate

3.1 The parameter-free ratio

The cosmic decoherence rate of QGC on de Sitter is [1]

$$\Gamma_{\mathrm{cosmic}} \;=\; \frac{g(1)\,H}{4\pi}, \qquad g(x) \;=\; 1 - \frac{2}{\pi}\,\mathrm{Si}(x), \qquad g(1) = 0.3977, \tag{9}$$

with Si the sine integral and the form-factor argument evaluated at the horizon scale (Sec. 5). Writing $H/4\pi = \frac{1}{2}\,\omega_\Lambda$,

$$\Gamma_{\mathrm{cosmic}} \;=\; \tfrac{1}{2}\,g(1)\,\omega_\Lambda, \tag{10}$$

the same clock ω_Λ as $a_0 = c\omega_\Lambda$, multiplied by $g(1)/2 = 0.199$. Dividing the two relations cancels every cosmological input (H, G, c, $\hbar$).

Proposition 3.1 (The lock)**.** *The MOND scale and the cosmic decoherence rate satisfy the parameter-free ratio*

$$\frac{a_0}{c\,\Gamma_{\text{cosmic}}} \;=\; \frac{2}{g(1)} \;=\; \frac{2}{1-(2/\pi)\,\mathrm{Si}(1)} \;=\; 5.029. \tag{11}$$

A framework explaining the MOND scale and cosmic decoherence with unrelated physics has two independent knobs here; QGC has one. Measuring a_0 fixes Γ_{cosmic} and conversely.

The ratio admits three readings, all equal to $\omega_\Lambda/\Gamma_{\text{cosmic}}$:

1. **The lock (acceleration vs. rate):** $a_0 = 5.03\,c\,\Gamma_{\text{cosmic}}$.
2. **Modular ticks to decohere:** the universe decoheres in $\omega_\Lambda/\Gamma_{\text{cosmic}} = 2/g(1) = 5.03$ ticks of its own thermal-time clock. Using the closed-form Bures–Fisher trajectory of QGC pure dephasing [4], $d_B^2(t) = 2 - \sqrt{2(1+e^{-\Gamma t})}$, at $\Gamma t = 2/g(1)$ the cosmic state has covered 99.2% of its geometric distance from coherent to maximally mixed.
3. **Hubble units:** $\Gamma_{\text{cosmic}}^{-1} = (4\pi/g(1))H^{-1} = 31.6\,H^{-1} \approx 10\pi\,H^{-1}$, i.e. decoherence after $\sim 10\pi$ e-folds—which is approximately five modular ticks, each tick being 2π e-folds.

Remark 3.1 (Hubble-tension slack). The lock is exact at a fixed cosmology. In the source papers a_0 is quoted with $H_0 = 70$ and Γ_{cosmic} with $H_0 = 67.4$ (Planck); the only thing breaking the exact ratio is this $70/67.4 \approx 1.039$, the Hubble tension itself. There is no other slack.

4 The two $\pi/2$'s: speed limit versus form factor

Two of this framework's results carry a factor $2/\pi$: the Margolus–Levitin speed limit of the canonical core paper, $\Gamma_{\text{ML}} = 2E_G/(\pi\hbar)$ [2, 22], and the form-factor normalisation in Eq. (9). They share an origin but are distinct objects, differing in exactly one way, identified below.

4.1 The spatial $\pi/2$: an analytic, deformable constant

The QGC decoherence functional for a two-branch source is a mode integral whose integrand factorises into a space part and a time part [2, 3]:

$$\Gamma(t) \;\propto\; \int \frac{d^3k}{k^4\,\omega_k}\; \underbrace{(1-\cos\mathbf{k}\cdot\mathbf{d})}_{\text{space}}\; \underbrace{(1-\cos\omega_k t)}_{\text{time}}\,. \tag{12}$$

The 3D angular average of the space factor is the spherical Bessel function $1 - j_0(kd) = 1 - \sin(kd)/(kd)$, and its radial integral is the Dirichlet integral

$$\int_0^\infty j_0(kd)\,dk \;=\; \frac{\pi}{2d}, \tag{13}$$

whose half-value $\pi/2$ is the *spatial* $\pi/2$. It sits inside E_G: $E_G = (2GM^2/\pi d)\int_0^\infty j_0 = (2GM^2/\pi d)(\pi/2) = GM^2/d$. This $\pi/2$ is an *analytic* constant and it is *deformable*: an infrared cutoff $k > k_H = 1/R_{\text{H}}$ replaces it by $\pi/2 - \mathrm{Si}(x)$, the form factor $g(x)$ of Sec. 5.

4.2 The temporal $\pi/2$: a geometric, rigid constant

The time factor governs orthogonalisation. The two-branch overlap amplitude is $\propto \cos(E_G t/2\hbar)$, orthogonal at the Fubini–Study quarter-turn $\theta = \pi/2$; the Margolus–Levitin inequality $1 - \cos x \leq$

$(2/\pi)(x + \sin x)$ is tight at $x = \pi$ (the two-level orthogonalisation point), fixing the temporal $2/\pi$. This $\pi/2$ is a *geometric* constant—the angle from a ray to an orthogonal ray—and it is *rigid*: an infrared cutoff rescales the rate but never the orthogonality angle.

4.3 Distinctness via opposite cutoff response

Proposition 4.1 (The two $\pi/2$'s are distinct)**.** *As functions of the cutoff scale, the spatial weight obeys $\partial_\Lambda P_{\text{space}} \neq 0$ (it slides from $\pi/2$ to 0 as g), while the orthogonalisation angle obeys $\partial_\Lambda P_{\text{time}} = 0$, with $P_{\text{space}}(\infty) = P_{\text{time}} = \pi/2$. A function with nonzero derivative and one with zero derivative in the same variable are not the same function; the two $\pi/2$'s, equal only at zero cutoff, are distinct objects [6].*

The shared value is the coincidence that the Dirichlet integral and the Hilbert-space right angle are both $\pi/2$. The horizon edits the spatial $\pi/2$ (turning E_G into $g(x)E_G$) but cannot touch the temporal $\pi/2$ (the orthogonality angle is geometric), so the cosmic rate is $\Gamma_{\text{cosmic}} = \frac{1}{2}g(1)\omega_\Lambda$: the horizon deforms the energy, not the rate-per-energy. This asymmetry—one constant analytic and horizon-edited, one geometric and untouchable—is the structural origin of the $g(1)$ in the lock $2/g(1)$.

The physically meaningful statement of the canonical core paper [2] is that the Diósi–Penrose rate sits *at* the Margolus–Levitin scale, of order $E_G/\hbar$, not the 10^{35} below it of perturbative QFT. The exact factor $\Gamma_{\text{DP}}/\Gamma_{\text{ML}} = (E_G/\hbar)/(2E_G/\pi\hbar) = \pi/2$ is a definitional restatement—the ratio of the $1/e$ decay time to the orthogonalisation time—and carries no information about whether the two $\pi/2$'s are the same object. Proposition 4.1 settles that they are not.

4.4 Lab and cosmos as probes of the two siblings

A BMV/QGEM experiment [24, 25] entangles two masses through their mutual gravity; the two-qubit concurrence is $C(t) = |\sin(\Phi/2)|$ with entangling phase $\Phi = \Delta E_{\text{BMV}}\, t/\hbar$, maximal at $\Phi = \pi$. The *same* mutual energy ΔE_{BMV} that entangles the pair also mutually decoheres it (the entanglement–decoherence duality [5]), so the maximal-entanglement time and the mutual-decoherence time obey, *for any geometry*,

$$\frac{\tau_{\text{BMV}}(\text{max})}{\tau_{\text{dec}}^{\text{mutual}}} = \frac{\pi\hbar/\Delta E_{\text{BMV}}}{2\hbar/\Delta E_{\text{BMV}}} = \frac{\pi}{2}, \tag{14}$$

the Fubini–Study quarter-turn—identical to the Margolus–Levitin orthogonalisation time, and independent of the masses (checked over 10^{-15}–10^{-13} kg) and of the source geometry. This is the *temporal*, rigid $\pi/2$: the lab is Minkowski ($x = Hd/c \approx 0$, $g \to 1$), so the spatial Dirichlet $\pi/2$ is undeformed and only the temporal one is exposed. A lab entanglement experiment measures the temporal sibling (the quarter-turn, geometry-fixed and horizon-untouchable); the de Sitter form factor $g(1) = 1 - (2/\pi)\,\text{Si}(1)$ measures the spatial sibling (the Dirichlet integral, truncated by the horizon). Lab and cosmos share the temporal $\pi/2$ (geometric, the same everywhere); the entire difference is the spatial form factor ($g = 1$ in the lab, 0.398 at the horizon), which is why the cosmic suppression in $\Gamma_{\text{cosmic}} = \frac{1}{2}g(1)\omega_\Lambda$ is carried by $g(1)$ alone.

A consequence, derived elsewhere: while the *mutual* channel obeys the clean $\pi/2$, the single-particle *self*-decoherence is a different, much larger energy ($E_G^{\text{self}} \sim Gm^2/R$, set by the particle radius R, versus $\Delta E_{\text{BMV}} \sim 2Gm^2\Delta x^2/d^3$). Their ratio $E_G^{\text{self}}/\Delta E_{\text{BMV}} \sim 0.6\, d^3/(R\Delta x^2) \sim 10^{2\text{–}3}$

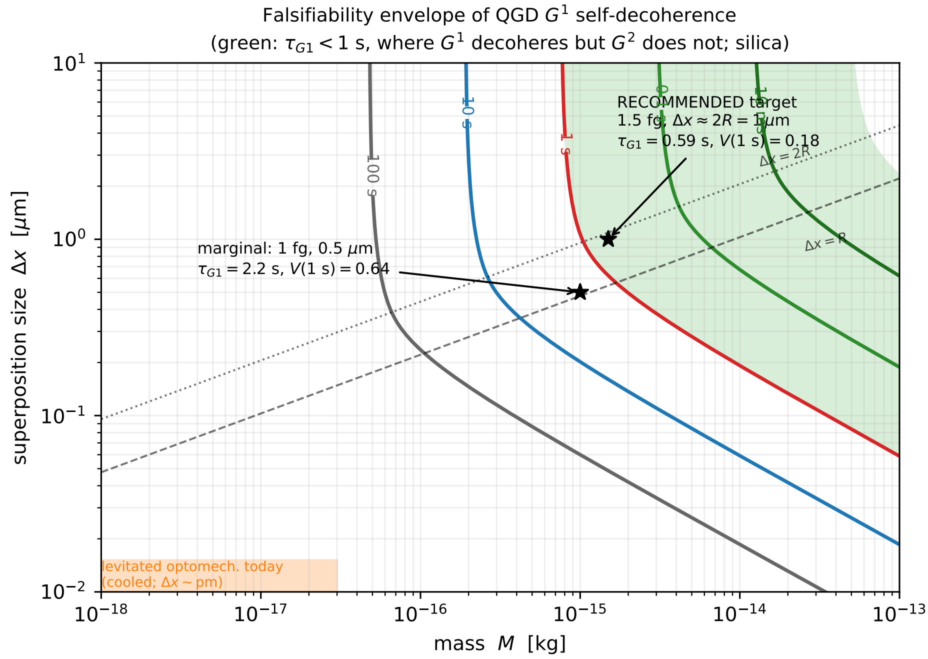

Figure 1. *Falsifiability envelope of QGC G^1 self-decoherence (silica). Contours of $\tau_{G1} = \hbar/E_G(M, \Delta x)$; the green region ($\tau_{G1} < 1$ s) is where G^1 decoheres a superposition within a 1 s hold while G^2 ($\tau_{G2} \sim 10^{18}$ yr) does not. Dashed/dotted lines mark $\Delta x = R$ and $\Delta x = 2R$ (saturation). Stars: a recommended target (1.5 fg, $\Delta x \approx 2R = 1\,\mu m$, $V(1\,s) = 0.18$) and a marginal one (1 fg, $0.5\,\mu m$, $V = 0.64$). The orange band indicates today's levitated-optomechanics regime (cooled, $\Delta x \sim$pm), far below the discriminating zone.*

for realistic parameters means that, *if* QGC's G^1 self-decoherence is real, each superposition self-decoheres 10^2–$10^3\times$ faster than the BMV entanglement builds (and geometrically unavoidably so, since $\tau_{\rm dec}^{\rm self} > \tau_{\rm BMV}$ would need $R > \Delta x$). BMV is thus, within QGC, also a G^1-vs-G^2 discriminator: a positive entanglement result would constrain the very G^1 self-decoherence scaling the framework rests on. Details and caveats (cutoff choice, pointer basis) are in the BMV note [6].

The same self-decoherence gives a near-term single-particle target (Fig. 1). For a uniform sphere the exact self-energy is $E_G(\Delta x) = (GM^2/R)[\frac{1}{2}s^2 - \frac{3}{16}s^3 + \frac{1}{160}s^5]$ for $s = \Delta x/R \le 2$, saturating at $1.2\,GM^2/R$ for $\Delta x \gtrsim 2R$. The decoherence time $\tau_{G1} = \hbar/E_G \sim M^{-5/3}$ falls below a second only for $M \gtrsim M_\star \approx 7 \times 10^{-16}$ kg (a femtogram, $R \sim 0.4\,\mu$m). A silica sphere of $M = 1.5$ fg in a $\Delta x \approx 1\,\mu$m superposition held for 1 s gives $\tau_{G1} = 0.59$ s and visibility $V = 0.18$, versus $V \simeq 1$ for G^2—an unmistakable discriminant, requiring cryogenic extreme-high vacuum ($\sim 10^{-15}$ mbar) to suppress gas collisions below Γ_{G1}. The first $\gtrsim$femtogram solid-object superposition held for ~ 1 s thus tests the G^1 scaling directly [6].

5 $g(1)$ as a sub-horizon energy fraction

Everything in Eq. (11) is rational geometry except $g(1)$. The gravitational self-energy of the two-branch source is the Dirichlet integral above; on de Sitter the modes with $k < k_H = 1/R_{\rm H}$ are super-horizon and cannot drive decoherence within a Hubble time, so the surviving energy is the same integral cut at k_H:

$$E_G^{\mathrm{dS}}(x) \propto \int_{k_H}^{\infty} \frac{\sin(kd)}{kd}\, dk = \frac{1}{d}\Big(\frac{\pi}{2} - \mathrm{Si}(x)\Big), \qquad x = k_H d = \frac{Hd}{c}. \tag{15}$$

The form factor is the ratio of surviving to total energy,

$$g(x) = \frac{E_G^{\mathrm{dS}}(x)}{E_G^{\mathrm{Mink}}} = \frac{\frac{\pi}{2} - \mathrm{Si}(x)}{\frac{\pi}{2}} = 1 - \frac{2}{\pi}\,\mathrm{Si}(x), \tag{16}$$

so the $2/\pi$ is the inverse Dirichlet integral, $\mathrm{Si}(x)$ is the super-horizon energy removed, and $x = 1$ is forced because the horizon mode has $d = R_{\mathrm{H}}$. At $x = 1$ the super-horizon piece $\mathrm{Si}(1) = 0.946$ is 60.2% of $\pi/2$, leaving

$$\boxed{g(1) = \frac{\frac{\pi}{2} - \mathrm{Si}(1)}{\frac{\pi}{2}} = 0.398,} \tag{17}$$

the sub-horizon fraction of the gravitational self-energy at $d = R_{\mathrm{H}}$. The merge number then decomposes transparently,

$$\frac{2}{g(1)} = \underbrace{\frac{4\pi}{2\pi}}_{\substack{\text{horizon area}\\ \text{/KMS period}}} \Big/ \underbrace{g(1)}_{\substack{\text{sub-horizon energy}\\ \text{fraction at } R_{\mathrm{H}}}} = \frac{\pi}{\frac{\pi}{2} - \mathrm{Si}(1)} = 5.029: \tag{18}$$

the universe decoheres in ~ 5 modular ticks because the horizon area carries a factor 2 over the thermal period and only $\sim 40\%$ of horizon-scale gravitational binding survives the super-horizon cut. The value is transcendental, not a hidden integer; the closeness $g(1) \approx 2/5$ (to 0.6%) is a coincidence.

Remark 5.1 (Validity range). The closed form $g(x) = 1 - (2/\pi)\,\mathrm{Si}(x)$ is non-monotonic and becomes negative for $x \gtrsim 1.9$, where Si overshoots $\pi/2$. A negative rate is unphysical: the cut-Dirichlet construction is valid only for $x \le 1$ (sub-horizon to horizon). The framework uses only $g(1)$, which is safely positive; the super-horizon tail requires the infrared-complete de Sitter mode sum, not this formula.

6 Redshift rigidity and the choice of horizon

6.1 The lock is redshift-rigid

Both a_0 and Γ_{cosmic} are the one clock $\omega_\Lambda(z)$, and the decoherence form factor is pinned at $x = HR_{\mathrm{H}}/c = 1$ at every epoch (for a spatially flat universe the apparent-horizon radius is $R_{\mathrm{H}} = c/H$ exactly at all times [17]). Provided a_0 and Γ_{cosmic} are referred to the same horizon H, every power of $H(z)$ cancels:

$$\frac{a_0(z)}{c\,\Gamma_{\mathrm{cosmic}}(z)} = \frac{2}{g(1)} \quad \text{for all } z. \tag{19}$$

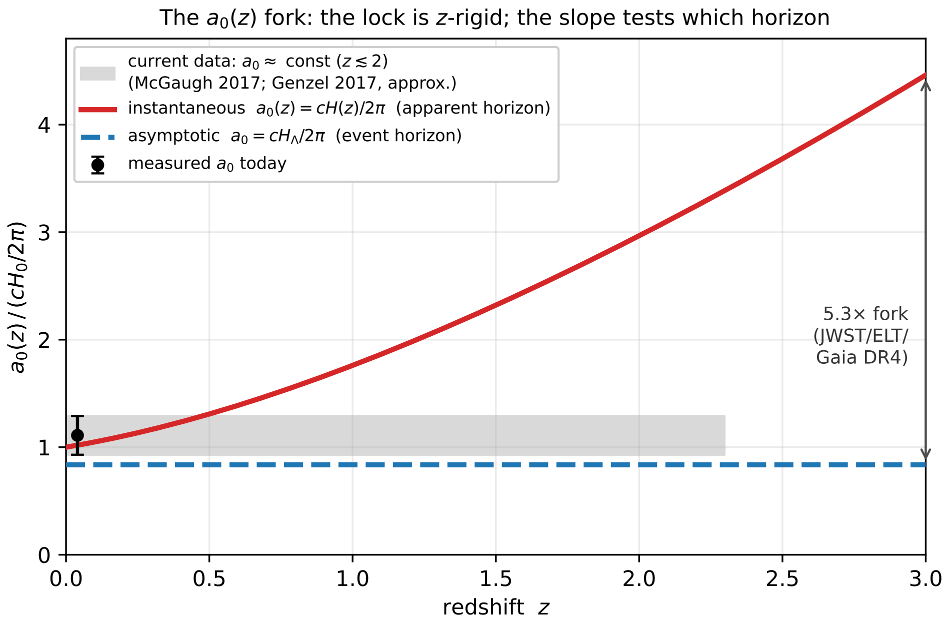

Figure 2. *The $a_0(z)$ fork. The lock $a_0/(c\Gamma_{\rm cosmic}) = 2/g(1)$ is redshift-rigid, but the shared horizon may be the* instantaneous *apparent horizon (red, $a_0 \propto H(z)$) or the* asymptotic *event horizon (blue dashed, constant $a_0 = cH_\Lambda/2\pi$). The measured a_0 today (black point) lies above both $z = 0$ values, so the normalisation is $O(1)$ for either branch; the discriminant is the* slope. *Current high-redshift rotation curves favour a constant a_0 (grey band, schematic; [15, 14]), which the rising instantaneous curve exits by $z \sim 1.5$. JWST/ELT and Gaia DR4 resolve the $\sim 5\times$ fork at $z = 3$. ($\Omega_m = 0.30$, $\Omega_\Lambda = 0.70$.)*

6.2 Which horizon sets the shared rate

The lock fixes the *ratio*; it does not fix *which* horizon sets the shared H (Fig. 2). Two readings compete:

- the **instantaneous** apparent-horizon rate $H(z)$, giving an evolving $a_0(z) = cH(z)/2\pi$ (a factor ~ 3 larger at $z = 2$);
- the **asymptotic** event-horizon rate $H_\Lambda = H_0\sqrt{\Omega_\Lambda}$, giving a constant a_0.

The decoherence sector does not force the instantaneous branch, and the data point the other way: high-redshift rotation curves [15, 14] find a_0 consistent with constant over $z = 0$–2 and exclude strong evolution. The de Sitter decoherence calculation is most naturally referred to the asymptotic (event-horizon) rate H_Λ; then both legs are constant, the lock holds trivially, and the framework is consistent with the data. Milgrom [14] frames the same $a_H = cH(t)$ versus $a_\Lambda = c\sqrt{\Lambda/3}$ choice as open. The robust prediction is the *ratio* $2/g(1)$, not a particular $a_0(z)$ law; the instantaneous branch remains a live, mildly disfavoured possibility that JWST/ELT and Gaia DR4 can settle.

7 The horizon clock family

Every causal horizon carries one clock $\omega = \kappa/2\pi c$, surfacing three ways.

reading	de Sitter	black hole
acceleration scale $a_\star = c\omega = \kappa/2\pi$	$a_0 = cH/2\pi$ (MOND)	$c^4/8\pi GM$
chaos / Lyapunov rate $\lambda = 2\pi\omega = \kappa/c$	H (de Sitter Lyapunov)	$2\pi k_B T_H/\hbar$ (MSS bound)
decoherence rate $\sim O(1)\,\omega$	$\frac{1}{2}g(1)\omega_\Lambda = \Gamma_{\rm cosmic}$	$\sim \omega_H$ (Hawking/scrambling)

Two rows are textbook: the de Sitter Lyapunov exponent is exactly H [18], and the black-hole row is the Maldacena–Shenker–Stanford chaos bound $\lambda \le 2\pi k_B T/\hbar$, saturated at horizons [23]. The new content is the third row and its lock to the first: the same clock ω that sets the chaos bound (via 2π) and the acceleration scale (via c) also sets the decoherence rate (via an $O(1)$ number). Black-hole chaos, the MOND scale, and cosmic decoherence are three projections of one horizon clock.

7.1 Intensive and extensive readings: dark energy as the fourth face

The readings above are *intensive*—properties of a single horizon mode of frequency ω_Λ. The horizon carries $S_{\rm dS} = \pi c^5/(\hbar G H^2)$ such modes, and summing over them yields the framework's two large-scale readings. The total horizon thermal energy is exactly

$$S_{\rm dS}\, k_B T_{\rm dS} \;=\; \frac{c^5}{2GH} \;=\; \rho_{\rm crit}\, c^2\, V_H, \tag{20}$$

the critical energy of the Hubble volume (holographic equipartition, the de Sitter case of Padmanabhan's relation [8]). Holographic dark energy [7] writes $\rho_{\rm DE} = \alpha c^2 H^2/G$ with $\alpha = 3\Omega_{\rm DE}/8\pi$; combining with Eq. (20) gives $\rho_{\rm DE} = \Omega_{\rm DE}\,(S_{\rm dS} k_B T_{\rm dS})/(c^2 V_H)$, so the dark-energy density is the fraction $\Omega_{\rm DE}$ of the horizon's thermal energy density, with the pure–de Sitter coefficient $\alpha_\infty = 3/8\pi = 0.119$ fixed by the clock. The cosmic decoherence rate is likewise an extensive sum: $\Gamma_{\rm cosmic} = S_{\rm dS} \times$ (per-mode rate) $= g(1)H/4\pi$ [1]. Thus

	intensive (per mode, ω_Λ)	extensive ($\times\, S_{\rm dS}$)
energy / acceleration	$a_0 = c\omega_\Lambda$ (MOND)	$S_{\rm dS} k_B T_{\rm dS} = c^5/2GH$ (dark energy)
rate	$\lambda = 2\pi\omega_\Lambda = H$ (chaos)	$g(1)H/4\pi = \Gamma_{\rm cosmic}$ (decoherence)

Dark energy and cosmic decoherence are siblings—both *extensive*, both carrying the holographic mode count $S_{\rm dS}$—while MOND and chaos are the *intensive*, per-mode readings. The horizon is one clock with $S_{\rm dS}$ hands: read per hand it gives the acceleration scale and the chaos bound; read over all hands, the dark-energy density and the cosmic decoherence rate.

Remark 7.1 (No solution to the cosmological constant problem). The present $\Omega_{\rm DE}$ (equivalently, why we live near the matter/dark-energy transition) is not explained here, just as the decoherence reading does not explain $g(1)$. Only the structure and the asymptotic $\alpha_\infty = 3/8\pi$ are clock-fixed; the latter is the framework's GSL/event-horizon attractor (the de Sitter limit $\Omega_{\rm DE} \to 1$, $HR_h \to 1$ [7]), not merely a limit. The near-coincidence $\alpha \approx 1/4\pi$ at $\Omega_{\rm DE} = 2/3$ is *not* a fixed point: $\Omega_{\rm DE} = 2/3$ is no dynamical landmark (it gives $q = -1/2$; acceleration onset is $q = 0$ at $\Omega_{\rm DE} = 1/3$, de Sitter is $q = -1$ at $\Omega_{\rm DE} = 1$), and $1/4\pi = \frac{2}{3}\cdot 3/8\pi$ carries no independent meaning.

8 Established, new, and falsifiable content

8.1 Established (cited, not claimed)

The identification $a_0 = ck_\mathrm{B}T_\mathrm{dS}/\hbar = cH/2\pi$ [12]; $a_0 \sim cH$ as a de Sitter/vacuum scale [11, 13]; the apparent-horizon temperature $\hbar H/2\pi k_\mathrm{B}$ valid at all flat-FRW epochs [17]; the de Sitter modular-flow rate $H/2\pi$ [19, 20]; the chaos bound [23]; the de Sitter Lyapunov exponent [18].

8.2 New here

(i) The lock between the cosmic decoherence rate and the MOND scale as one ω_Λ, Eq. (11), with its three equal readings; (ii) the identity $a_0 = c^2/2\pi R_\mathrm{H}$ (Compton wavelength of the de Sitter thermal mass equals the horizon circumference); (iii) the reading of $g(1)$ as the sub-horizon fraction of the gravitational self-energy, and the resulting decomposition of $2/g(1)$; (iv) showing the two $\pi/2$'s are distinct (speed limit vs. form factor) via their opposite cutoff response, and the reading of a lab BMV experiment and the cosmic form factor as measurements of the temporal and spatial siblings respectively; (v) the horizon clock family with decoherence as a reading, and its intensive/extensive structure placing dark energy (via holographic equipartition) and cosmic decoherence as the two extensive readings. No prior work we are aware of links a gravitational or cosmological decoherence rate to a_0, nor reads the holographic dark-energy coefficient as the extensive face of the same clock. We do *not* claim to solve the cosmological constant problem: the present Ω_DE is unexplained, only the structure and $\alpha_\infty = 3/8\pi$ are clock-fixed.

8.3 Falsifiable content

The robust prediction is the locked ratio $a_0/(c\Gamma_\mathrm{cosmic}) = 2/g(1)$: the MOND scale and the cosmic decoherence normalisation are not independent. The cosmic decoherence rate itself ($\sim 10^{-19}$ Hz) is not directly observable; the observable leg is a_0 and its possible redshift evolution. A measured $a_0(z)$ tracking the instantaneous $H(z)$ would favour the apparent-horizon branch; the present constraint of approximate constancy [15, 14] favours the event-horizon branch, with which the framework is consistent. The framework would be stressed—though not the lock itself—if a_0 were found to depend on a scale unrelated to any horizon.

9 Conclusion

The acceleration scale at which galaxies stop obeying Newton, and the rate at which the universe as a whole goes classical, are the same de Sitter horizon frequency $\omega_\Lambda = k_\mathrm{B}T_\mathrm{dS}/\hbar = H/2\pi$, read once as an acceleration and once as a rate, and locked at $a_0/(c\Gamma_\mathrm{cosmic}) = 2/g(1) = 5.03$. The one non-trivial number in that ratio is the fraction of horizon-scale gravitational binding that survives the super-horizon cut. Whether the shared horizon is the instantaneous apparent horizon or the asymptotic event horizon is an open, data-driven question; the ratio is not. In the language of Quantum-Geometric Correspondence, the cosmological horizon has a single clock, and dark-sector phenomenology and cosmic decoherence are two of its hands.

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