From Modular Flow to Fringe Visibility: A Missing Dynamical Step

Marc Sperzel*

Independent Researcher
MSci Physics, King's College London

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Abstract

Identifying modular flow with physical time does not, by itself, determine the decay of interferometric visibility. We examine the additional step needed to infer a gravitational decoherence rate from a modular energy scale. Explicit Type-I controls have nonzero quantum Fisher information with constant or recurrent visibility, including an exactly solvable bounded-below oscillator. They separate generator normalization, distinguishability speed, and the branch-conditioned channel measured by an interferometer. A quantum speed limit bounds orthogonalization frequency from above and supplies no positive lower bound on visibility decay. KMS detailed balance likewise leaves the coupling spectrum undetermined. Consequently an exponential law at the gravitational self-energy rate requires a specified state, interaction, readout, and controlled dynamical regime. These controls identify a gap in a proposed derivation; they neither construct the gravitational observer algebra nor exclude a gravitational mechanism that supplies the missing dynamics.

1 Introduction: the observable and the proposed implication

A branch-energy difference can rotate an interference fringe without reducing its contrast. The distinction matters when a gravitational energy scale is converted into a proposed decoherence rate,

$$\Gamma = CE_G/\hbar, \qquad V(t) = e^{-\Gamma t}. \tag{1}$$

Here E_G denotes the gravitational energy of the specified density difference; its geometry and regularization must be supplied independently. The proposed modular-time route to Eq. (1) in Quantum-Geometric Correspondence [1] identifies a physical generator and then invokes branch distinguishability. We examine the second implication. The thermal-time hypothesis [2] motivates the generator identification; it does not substitute for the calculation of an interference observable.

*ORCID: 0009-0000-6252-3155. Email: me@marcsperzel.com

Consider a path qubit and an environment with initial state $|+\rangle\langle+| \otimes \rho_E$, where $|+\rangle = (|L\rangle + |R\rangle)/\sqrt{2}$. In a Type-I description let

$$U = \sum_{j=L,R} |j\rangle\langle j| \otimes U_j(t), \qquad U_j(t) = e^{-ih_j t}, \quad h_j = H_j/\hbar. \tag{2}$$

Tracing the environmental block gives

$$\rho_{LR}(t) = \frac{z(t)}{2}, \qquad z(t) = \mathrm{Tr}[U_L \rho_E U_R^\dagger], \qquad V(t) = |z(t)|. \tag{3}$$

The normalization assumes balanced path populations. A binary fringe measurement at phase ϕ, with apparatus contrast c, has probability

$$p_+(\phi) = \frac{1 + c\,\mathrm{Re}[ze^{i\phi}]}{2}. \tag{4}$$

The probability oscillation has amplitude $c|z|/2$ about $1/2$; its normalized fringe visibility $(p_{\max} - p_{\min})/(p_{\max} + p_{\min})$ is $c|z|$. A change at one phase setting is therefore not by itself evidence of visibility loss. Environmental records enter through the overlap, as in ordinary decoherence theory [3].

For initially correlated dressings $|\Psi\rangle = (|L\rangle|e_L\rangle + |R\rangle|e_R\rangle)/\sqrt{2}$, the corresponding quantity is $z(t) = \langle e_R|U_R^\dagger U_L|e_L\rangle$. Its initial modulus may be less than unity without any subsequent decay. Such correlated data do not define a universal reduced channel for independently chosen path inputs.

Remark 1.1 (Scope). The tensor trace above is a control calculation. A gravitational Type-II or Type-III observer algebra requires its own prepared state and relational interference observable; no tensor factorization is assumed for that algebra. The counterexamples below test a generic inference from information geometry to decay, not the full gravitational construction.

2 Solvable controls

Proposition 2.1 (Information speed does not fix visibility)**.** *A nonzero quantum Fisher information for time encoding does not imply a positive exponential visibility-decay rate.*

Proof. For a pure state evolving under $h = H/\hbar$, the time-encoding quantum Fisher information is $F_Q = 4\,\mathrm{Var}(h)$ [4]. Take $h = \mathrm{diag}(0, \omega)$ and $\rho_E = |+\rangle\langle+|$. Then $F_Q = \omega^2$. Common evolution $h_L = h_R = h$ gives

$$z(t) = \mathrm{Tr}[U\rho_E U^\dagger] = 1. \tag{5}$$

To test relative encoding as well, take $h_L = 0$ and $h_R = h$. Directly,

$$z(t) = \frac{1 + e^{i\omega t}}{2}, \qquad V(t) = |\cos(\omega t/2)|. \tag{6}$$

The relative family has $F_Q = \omega^2$ but recurrent visibility. Neither case has a positive constant exponential decay rate. □

Generator normalization adds no decay in these controls. For any finite Hermitian h, choose a faithful reference state $\rho_{\mathrm{ref}} = e^{-\beta_* h}/Z$, where β_* has units of time. Its density-matrix modular

Hamiltonian obeys

$$K = -\log \rho_{\mathrm{ref}} = \beta_* h + \log Z \mathbb{1}. \tag{7}$$

Writing $\beta_* = 2\pi t_*$ and $\widetilde{H} = t_* H/\hbar$ gives $K = 2\pi \widetilde{H} + \log Z$. This fixes the conversion between dimensionless modular and physical generators, with the conventional possible reversal of flow parameter, but does not specify U_L, U_R, or a readout channel. Equation (7) is an elementary Gibbs identity, not a realization of a gravitational modular theorem. In particular, the reference state and a prepared interferometric state need not coincide.

2.1 A bounded-below oscillator

Consider a single mode with $[a, a^\dagger] = 1$ and

$$h_\pm = \omega a^\dagger a \pm g(a + a^\dagger) = \omega(a^\dagger \pm g/\omega)(a \pm g/\omega) - g^2/\omega, \tag{8}$$

where $\omega > 0$ and g is real. Both generators are bounded below. For the bare thermal state $\rho_E = (1 - e^{-b})e^{-b a^\dagger a}$, $b > 0$, the conditional displacement difference has modulus

$$|\delta\alpha(t)|^2 = \left| \frac{2g}{\omega}(1 - e^{-i\omega t}) \right|^2 = 16 \frac{g^2}{\omega^2} \sin^2(\omega t/2). \tag{9}$$

The thermal displacement characteristic function is $\mathrm{Tr}[\rho_E D(\alpha)] = \exp[-|\alpha|^2 \coth(b/2)/2]$. Normal ordering D and summing its diagonal matrix elements gives

$$(1 - r)e^{-x/2} \sum_{n \geq 0} r^n L_n(x) = \exp\left[-\frac{x(1+r)}{2(1-r)} \right], \qquad r = e^{-b}, \quad x = |\alpha|^2, \tag{10}$$

where L_n is a Laguerre polynomial. Thus Eq. (3) gives

$$V_b(t) = \exp\left[-8 \frac{g^2}{\omega^2} \sin^2(\omega t/2) \coth(b/2) \right]. \tag{11}$$

The vacuum limit is $b \to \infty$. Short-time loss is quadratic; a full period restores unit visibility. Changing the initial temperature changes the depth. For $g/\omega = 1/4$, the vacuum half-period visibility is $e^{-1/2} \simeq 0.607$, followed by $V(2\pi/\omega) = 1$. Independent matrix exponentiation with increasing occupation cutoffs recovers Eq. (11) on a fixed finite time interval.

This is an oscillator control, not a model of the canonical gravitational clock: a mode count and a lower spectral bound do not identify two physical algebras. Finite matrices also do not satisfy exact canonical commutation relations. The analytic recurrence belongs to the untruncated single-mode model; it is not a truncation artifact.

For initial product data, branch-conditioned inverses $U_L^\dagger$ and $U_R^\dagger$ erase the records and restore coherence. Recombining path labels alone does not erase an inaccessible environmental record. This distinction must be retained in any claimed preparation–hold–recombination prediction.

3 Additional dynamical inputs

An exponential can be supplied by the path dephasing generator

$$\mathcal{L}_\gamma(\rho) = \frac{\gamma}{2}(\sigma_z\rho\sigma_z - \rho), \qquad \dot{\rho}_{LR} = -\gamma\rho_{LR}. \tag{12}$$

For every $\gamma \geq 0$, its finite-time Kraus operators are

$$A_0 = \sqrt{\frac{1+e^{-\gamma t}}{2}}\,\mathbb{1}, \qquad A_1 = \sqrt{\frac{1-e^{-\gamma t}}{2}}\,\sigma_z. \tag{13}$$

They give a completely positive map with $\sum_j A_j^\dagger A_j = \mathbb{1}$. Complete positivity permits all these rates; selecting $\gamma = E_G/\hbar$ is additional physics. Adding this dissipator changes the dynamics rather than deriving it from a modular identity. Dephasing need not exchange energy, so absence of energy dissipation does not establish absence of phase noise.

3.1 Orthogonalization and spectral information

For time-independent unitary dynamics with a lower spectral bound, the Margolus–Levitin theorem [5] gives

$$\tau_\perp \geq \frac{\pi\hbar}{2E}, \qquad \frac{1}{\tau_\perp} \leq \frac{2E}{\pi\hbar}, \qquad E = \langle H\rangle - E_0. \tag{14}$$

It bounds orthogonalization frequency from above, not visibility decay from below. Moreover, identifying E with a gravitational density-difference energy requires a state-dependent argument. The proposed interval $C \in [2/\pi, 1]$ in Eq. (1) therefore remains a phenomenological hypothesis; Eq. (14) does not establish it.

A KMS condition similarly leaves the coupling spectral density open. To exhibit the freedom, let β_* be a fixed inverse-temperature time and consider the positive unsymmetrized spectra

$$S_\Omega(\omega) = \frac{\eta\omega e^{-(\omega/\Omega)^2}}{1-e^{-\beta_*\omega}}, \quad \Omega > 0, \quad \eta > 0. \tag{15}$$

Their continuous zero-frequency limit and detailed balance are

$$S_\Omega(0) = \eta/\beta_*, \qquad S_\Omega(-\omega) = e^{-\beta_*\omega}S_\Omega(\omega). \tag{16}$$

Every cutoff has the same temperature and zero-frequency level, but a different spectrum. These are admissible oscillator-bath correlation spectra; they are not derived gravitational spectra. A measured knee requires the physical coupling and frequency conversion as well as detailed balance.

4 Discussion and conclusion

The controls establish a limited negative result: neither an information speed nor a modular-generator normalization suffices to infer exponential interference loss. They do not exclude a gravitational theory whose specified interaction and relational measurement supply that loss. An irreversible continuum or coarse-grained regime is compatible with the finite-model recurrences, but its range of validity and coefficient require a calculation.

A gravitational derivation must specify the prepared state, the accessible observable algebra, and the branch-conditioned evolution throughout recombination. It must then calculate the interference probability and justify any approximation producing Eq. (1). Static overlap, phase accumulation, and persistent loss of accessible contrast are different outcomes of that calculation. Until the operational step is supplied, $\Gamma = CE_G/\hbar$ is a testable rate hypothesis rather than a consequence of modular flow alone. Observing that rate would test the specified family; it would not uniquely identify its proposed microscopic explanation.

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