Emergent Gravity from Entanglement Equilibrium: MOND Phenomenology and Covariant Extension

Paper IV of the Quantum-Geometric Duality Series

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January 2026

Abstract

We show that Modified Newtonian Dynamics (MOND) phenomenology emerges naturally from the thermodynamics of de Sitter space within the framework of quantum-geometric duality. The de Sitter cosmological horizon establishes a thermal bath at the Gibbons-Hawking temperature $T_{\mathrm{dS}} = \hbar H_0/(2\pi k_B)$, which generates extensive (volume-law) entropy in addition to the standard area-law contribution. When local entanglement equilibrium—the principle that generalized entropy is stationary under allowed variations—fails at low accelerations, the volume entropy becomes dynamically relevant, producing MOND-like corrections to Newtonian gravity.

We demonstrate that the characteristic acceleration scale emerges as $a_0 = cH_0/(2\pi) \approx 1.1 \times 10^{-10}$ m/s^2, matching observed MOND values within 10% with no adjustable parameters. We establish clear criteria for regime separation: GR remains exact when $\epsilon = g_N/a_0 \gg 1$ (solar system, pulsars), while MOND corrections become significant when $\epsilon \lesssim 1$ (galaxy outskirts). The volume entropy saturates the Bousso covariant entropy bound, resolving any apparent conflict with holography.

We present Entanglement-Elastic Gravity (EEG), a covariant field theory that extends this framework to relativistic regimes. The EEG action introduces an elastic displacement field ψ encoding entanglement strain, yielding modified Einstein equations with an elastic stress tensor. The theory is ghost-free, preserves gravitational wave speed c, and reduces to the modified Poisson equation in the Newtonian limit. **Predictions**: Flat rotation curves, Tully-Fisher relation $v^4 = GMa_0$, weak lensing slip $|\Phi - \Psi|/|\Phi| \approx 15\%$ beyond 50 kpc, and growth rate suppression $\Delta f\sigma_8 \approx -0.03$ testable with Euclid and DESI.

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1 Introduction

Galaxy rotation curves present one of the most striking puzzles in modern physics. When astronomers measure how fast stars orbit around galactic centers, they consistently find velocities that remain nearly constant far beyond the visible edge of the galaxy, where Newtonian gravity predicts they should fall off as the square root of distance. This discrepancy appears across galaxies of all sizes and types, from dwarf spheroidals to giant ellipticals, always becoming significant when the gravitational acceleration drops below a characteristic scale of approximately 1.2×10^{-10} m/s^2—about ten billion times weaker than Earth's surface gravity.

The standard explanation invokes dark matter: an invisible substance that surrounds galaxies in extended halos, providing the additional gravitational pull needed to keep stars moving fast. Dark matter models successfully reproduce many observations, but they require fine-tuning to explain certain regularities in the data. Most notably, the baryonic Tully-Fisher relation shows that the asymptotic rotation velocity v of a galaxy depends only on its visible (baryonic) mass M according to $v^4 \propto M$, with remarkably little scatter. If dark matter halos had independent properties from the visible galaxy, we would expect this relation to show much more variation.

Milgrom [3] proposed an alternative interpretation: perhaps gravity itself behaves differently at very low accelerations. His Modified Newtonian Dynamics (MOND) hypothesis posits that when the Newtonian gravitational acceleration $g_N = GM/r^2$ falls below a critical value a_0, the actual acceleration experienced by a test particle transitions to $g \approx \sqrt{g_N \cdot a_0}$. This simple modification, with $a_0 \approx 1.2 \times 10^{-10}$ m/s^2, automatically explains flat rotation curves, the Tully-Fisher relation, and numerous other galactic scaling relations [7, 8]. However, MOND has traditionally lacked a theoretical foundation—the acceleration scale a_0 must be fitted to data rather than predicted from first principles.

The purpose of this paper is to provide that theoretical foundation. We show that the MOND phenomenology emerges naturally from the thermodynamics of de Sitter space, with the acceleration scale $a_0 = cH_0/(2\pi)$ predicted directly from the Hubble constant. This is not merely a numerical coincidence: it reflects a deep connection between cosmological horizons and the information-theoretic structure of gravity.

Our starting point is Jacobson's remarkable discovery [1] that Einstein's equations can be derived from thermodynamic principles. When one requires that the generalized entropy $S_{\text{gen}} = A/(4\ell_P^2) + S_{\text{bulk}}$ remains stationary under small variations of the spacetime geometry, the Einstein equations follow as a consistency condition. This suggests that gravity is not a fundamental force but an emergent phenomenon arising from the equilibrium of quantum information distributed across spacetime.

Jacobson's original derivation assumed flat spacetime at large distances. But our universe is not flat—it is described by de Sitter space, with an accelerating expansion driven by dark energy. The de Sitter geometry introduces a cosmological horizon at distance c/H_0 from any observer, beyond which events are causally disconnected. Like a black hole horizon, this cosmological horizon has thermodynamic properties: it radiates at the Gibbons-Hawking temperature $T_{\text{dS}} = \hbar H_0/(2\pi k_B)$ and carries entropy proportional to its area.

The key insight of this paper is that the de Sitter horizon fundamentally changes the entropy accounting. In flat spacetime, entropy obeys an area law: the maximum entropy in a region scales with its boundary area. But in de Sitter space, the thermal bath at temperature T_{dS} contributes volume-law entropy $S_{\text{vol}} = s_\Lambda V$ throughout the interior. When a mass M is placed in this thermal bath, it "displaces" entropy $\Delta S = Mc^2/T_{\text{dS}}$ from the surrounding volume. If the available volume entropy cannot accommodate this displacement, local thermodynamic equilibrium fails, and the gravitational dynamics must change.

This failure of equilibrium occurs precisely when the Newtonian acceleration falls below $a_0 = cH_0/(2\pi)$. At higher accelerations, the local geometry can absorb the entropy displacement through small curvature adjustments, and standard general relativity applies. At lower accelerations, the entropy deficit spreads non-locally through the de Sitter volume, producing the characteristic MOND behavior $g \propto \sqrt{g_N}$. The transition is not put in by hand but emerges from the competition between area-law and volume-law contributions to the generalized entropy.

We develop this argument in stages. Section 2 establishes the thermodynamic properties of de Sitter space, including the Gibbons-Hawking temperature and the volume entropy density. Section 3 demonstrates that the total volume entropy saturates the Bousso covariant entropy bound, resolving any apparent conflict with holography. Section 4 derives the MOND acceleration scale from entropy equilibrium and shows how the Tully-Fisher relation emerges automatically. Section 5 provides the ϵ-expansion that quantifies when general relativity versus MOND applies, showing that solar system tests are safely in the GR regime.

Having established the non-relativistic framework, we then present its covariant completion. Section 6 develops Entanglement-Elastic Gravity (EEG), a fully relativistic field theory in which an elastic displacement field ψ encodes the entropy strain caused by matter. The EEG action yields modified Einstein equations with an elastic stress tensor that reduces to the MOND Poisson equation in the Newtonian limit. The theory is ghost-free, preserves gravitational wave propagation at speed c (consistent with GW170817), and provides a complete framework for analyzing strong-field phenomena.

Section 7 presents observational predictions, including the weak lensing slip parameter $|\Phi - \Psi|/|\Phi| \approx 15\%$ beyond 50 kpc, growth rate suppression $\Delta f\sigma_8 \approx -0.03$ at $z = 0.8$, and anomalous acceleration in wide binary stars detectable with Gaia DR4/DR5. Section 8 summarizes our findings and discusses open problems.

Series context. This is Paper IV of the Quantum-Geometric Duality series. Paper A develops gravitational decoherence from the same entropy principles; Paper B derives holographic dark energy; Paper C presents the complete axiomatic framework; Paper H establishes information-theoretic bounds. Each paper is self-contained but cross-references the others for extended discussion.

2 De Sitter Thermodynamics

The cosmological constant Λ is not merely a parameter in Einstein's equations—it fundamentally alters the causal structure of spacetime, creating a horizon that radiates like a black hole turned inside out. This section establishes the thermodynamic properties of de Sitter space that will underpin our derivation of MOND phenomenology.

2.1 The Static Patch and Cosmological Horizon

De Sitter space is the maximally symmetric solution to Einstein's equations with positive cosmological constant, $G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$. Unlike Minkowski space, de Sitter has a characteristic length scale set by Λ that determines how far any observer can see. In static coordinates centered on a freely falling observer, the metric takes the form

$$ds^2 = -\left(1 - \frac{r^2}{R_H^2}\right) c^2 dt^2 + \frac{dr^2}{1 - r^2/R_H^2} + r^2 d\Omega^2, \tag{1}$$

where the de Sitter radius is

$$R_H = \sqrt{\frac{3}{\Lambda}} = \frac{c}{H_0}. \tag{2}$$

The significance of R_H becomes apparent from the metric: the coefficient g_{tt} vanishes at $r = R_H$, and the coefficient g_{rr} diverges. This marks a coordinate horizon—not a singularity, but a boundary beyond which events are causally disconnected from the central observer. Light signals sent from $r > R_H$ can never reach $r = 0$, just as signals sent into a black hole can never escape. For our universe with $H_0 \approx 70$ km/s/Mpc, the horizon lies at $R_H \approx 14$ billion light-years, defining the edge of the observable universe.

The region $r < R_H$ is called the static patch because it admits a timelike Killing vector $\partial/\partial t$. This symmetry allows us to define a conserved energy and construct a meaningful thermodynamics. The static patch contains a finite proper volume $V_H = (4\pi/3)R_H^3$, bounded by the cosmological horizon. Unlike the exterior of a black hole, where freely falling observers cross the horizon inward, the de Sitter horizon is a future boundary—all observers eventually find their signals redshifted into oblivion as they approach it.

2.2 The Gibbons-Hawking Temperature

Perhaps the most profound discovery in semiclassical gravity is that horizons radiate. Hawking showed that black holes emit thermal radiation at a temperature inversely proportional to their mass; Gibbons and Hawking [2] extended this result to cosmological horizons. The de Sitter horizon bathes its interior in thermal radiation at temperature

$$T_{\text{dS}} = \frac{\hbar H_0}{2\pi k_B} \approx 2.8 \times 10^{-30} \text{ K}. \tag{3}$$

This extraordinarily low temperature—ten billion times colder than the cosmic microwave background—might seem irrelevant to practical physics. But as we shall see, it sets the acceleration scale below which gravitational dynamics fundamentally change.

The Gibbons-Hawking temperature can be derived by three independent methods, all yielding the same result. The first method uses surface gravity: the quantity κ that measures the "strength" of the horizon is defined geometrically by the Killing equation $\nabla_\mu(k_\nu k^\nu) = -2\kappa k_\mu$. For the de Sitter metric, direct calculation gives $\kappa = cH_0$, and the temperature follows from the universal relation $T = \hbar\kappa/(2\pi c k_B)$.

The second method uses Euclidean continuation. Under the analytic continuation $t \to -i\tau$, the Lorentzian metric becomes Riemannian, but a conical singularity appears at $r = R_H$ unless τ is periodic with a specific period. Requiring regularity uniquely fixes this period to be $\beta = 2\pi/H_0$, which corresponds to temperature $T = \hbar/(2\pi k_B \beta) = \hbar H_0/(2\pi k_B)$.

The third method appeals to the Unruh effect. A static observer at any radius $r < R_H$ must accelerate to resist the cosmic expansion. As $r \to R_H$, this proper acceleration diverges, and the Unruh radiation seen by the accelerating observer approaches the Gibbons-Hawking temperature after accounting for gravitational redshift. That three completely different approaches—geometric, topological, and kinematic—yield the identical answer strongly suggests that T_{dS} is a fundamental property of de Sitter space.

2.3 The First Law and Horizon Entropy

The cosmological horizon is not merely a kinematic boundary but a genuine thermodynamic system. Like a black hole, it carries entropy proportional to its area—an entropy that in our universe reaches the staggering value $S_{dS} = A_H/(4\ell_P^2) \approx 10^{122}$ in natural units. This is the largest entropy of any system in the observable universe, dwarfing even the combined entropy of all black holes.

The thermodynamic nature of the horizon is expressed by the first law

$$dE_{dS} = T_{\text{dS}}\, dS_{dS}, \tag{4}$$

where E_{dS} is the vacuum energy contained within the static patch. This equation states that adding energy to the interior increases the horizon area, just as feeding a black hole makes it grow. The cosmological horizon acts as a heat reservoir at temperature T_{dS}, in thermal equilibrium with the interior.

2.4 The Characteristic Energy and Acceleration Scales

What physical scale does the Gibbons-Hawking temperature set? Converting temperature to energy gives

$$E_{dS} = k_B T_{\text{dS}} = \frac{\hbar H_0}{2\pi} \approx 3.6 \times 10^{-52} \text{ J}. \tag{5}$$

This is an absurdly small energy—roughly the kinetic energy of a single atom moving at 10^{-12} meters per second. No laboratory experiment could ever detect individual quanta at this scale.

Yet when we ask what acceleration this energy corresponds to, something remarkable happens. Using natural gravitational units defined by the Planck mass $m_P = \sqrt{\hbar c/G}$ and Planck length $\ell_P = \sqrt{\hbar G/c^3}$, we find

$$a_0 = \frac{E_{dS}}{m_P \ell_P} = \frac{cH_0}{2\pi} \approx 1.1 \times 10^{-10} \text{ m/s}^2. \tag{6}$$

This is the MOND acceleration scale, the value below which galaxy rotation curves deviate from Newtonian predictions. The coincidence $a_0 \sim cH_0$ has puzzled physicists since Milgrom first noted it in 1983; here we see it is not a coincidence at all, but a direct consequence of de Sitter thermodynamics. The Gibbons-Hawking temperature of our universe determines the acceleration scale at which gravity changes character.

3 Volume-Law Entropy in de Sitter Space

Holography has taught us that gravitational systems carry entropy proportional to their boundary area, not their volume. A black hole's entropy scales as r^2, not r^3, regardless of what fell in to form it. This area law is often taken as a fundamental principle, suggesting that the degrees of freedom in any gravitational system are somehow "painted" on surfaces rather than distributed through space.

De Sitter space challenges this picture. The thermal radiation from the cosmological horizon fills the interior with entropy that scales with volume, not area. This section shows that this volume-law contribution is not only consistent with holography but required by it—the volume entropy exactly saturates the Bousso covariant entropy bound at the cosmological scale.

3.1 The de Sitter Vacuum as a Thermal State

To understand the origin of volume entropy, we must examine how quantum field theory behaves when the spacetime itself has a horizon. Consider the vacuum state of a quantum field in de Sitter space. The full Hilbert space naturally factorizes as $\mathcal{H}_{\text{full}} = \mathcal{H}_{\text{in}} \otimes \mathcal{H}_{\text{out}}$, where $\mathcal{H}_{\text{in}}$ contains modes inside the cosmological horizon and $\mathcal{H}_{\text{out}}$ contains modes beyond it.

The crucial insight is that what we call "the vacuum" from a global perspective is not a vacuum at all from the perspective of an observer confined to the static patch. The global vacuum state is entangled across the horizon:

$$|0\rangle = \sum_n e^{-\pi E_n/\hbar H_0} |n\rangle_{\text{in}} \otimes |n\rangle_{\text{out}}. \tag{7}$$

When the observer traces over the inaccessible super-horizon modes, the remaining state is thermal:

$$\rho_{\text{in}} = \text{Tr}_{\text{out}}|0\rangle\langle 0| = \frac{1}{Z}\sum_n e^{-2\pi E_n/\hbar H_0} |n\rangle_{\text{in}}\langle n|. \tag{8}$$

This is a Boltzmann distribution at precisely the Gibbons-Hawking temperature $T_{\text{dS}} = \hbar H_0/(2\pi k_B)$. The de Sitter horizon is not just radiating—it is maintaining its interior in thermal equilibrium with itself.

3.2 The Volume Entropy Density

A thermal state at temperature T carrying energy density ρ has entropy density $s = \rho/T$. For de Sitter space, the vacuum energy density is

$$\rho_\Lambda = \frac{\Lambda c^4}{8\pi G} = \frac{3c^2 H_0^2}{8\pi G}, \tag{9}$$

where the second equality uses $\Lambda = 3H_0^2/c^2$ for a flat universe approaching de Sitter at late times. Dividing by the Gibbons-Hawking temperature gives the entropy density

$$s_\Lambda = \frac{\rho_\Lambda}{T_{\text{dS}}} = \frac{3c^2 H_0^2}{8\pi G} \cdot \frac{2\pi k_B}{\hbar H_0} = \frac{3c^2 H_0 k_B}{4G\hbar}. \tag{10}$$

In numerical terms, $s_\Lambda \approx 2.2 \times 10^{43}\ k_B\,\text{m}^{-3}$—an enormous entropy density by ordinary standards. A cubic meter of de Sitter vacuum carries more entropy than most astrophysical systems. Yet because this entropy is spread uniformly through space, it is invisible to local experiments; it manifests only when we consider gravitational dynamics on scales where significant volume is enclosed.

3.3 Total Entropy: Area Plus Volume

A spherical region of radius r inside the static patch therefore carries entropy from two sources: the standard area-law contribution from entanglement across its boundary, and the volume-law contribution from the thermal bath:

$$S_{dS}(r) = \frac{A(r)}{4G\hbar} + s_\Lambda V(r) = \frac{\pi r^2 c^3}{G\hbar} + \frac{\pi c^2 H_0 k_B r^3}{G\hbar}. \tag{11}$$

The area term dominates at small scales; the volume term dominates at large scales. The crossover occurs when $r \sim c/H_0 = R_H$—precisely at the cosmological horizon.

3.4 Consistency with Holography

The volume-law entropy raises an apparent paradox: for sufficiently large regions, $S_{\rm vol} > S_{\rm area}$, seemingly violating the holographic principle that entropy cannot exceed $A/(4\ell_P^2)$. This contradiction is resolved by recognizing that the naive statement of holography is incorrect.

The correct formulation is the Bousso covariant entropy bound [9]. For any surface B, construct a light sheet—a null hypersurface generated by non-expanding null geodesics orthogonal to B. The bound states that the entropy on this light sheet satisfies $S \leq A(B)/(4\ell_P^2)$. Crucially, the bound applies to light sheets, not arbitrary spatial volumes. A spatial volume can exceed the area bound; a light sheet cannot.

For de Sitter space, something remarkable happens when we apply the Bousso bound at the cosmological horizon itself. The total volume entropy within the static patch equals

$$S_{\rm vol}^{\rm total} = s_\Lambda \cdot V_H = \frac{3c^2 H_0 k_B}{4G\hbar} \cdot \frac{4\pi c^3}{3H_0^3} = \frac{\pi c^5 k_B}{G\hbar H_0^2}, \tag{12}$$

while the horizon entropy is

$$S_H = \frac{A_H}{4\ell_P^2} = \frac{4\pi R_H^2 \cdot c^3}{4G\hbar} = \frac{\pi c^5}{G\hbar H_0^2}. \tag{13}$$

These are identical (in natural units where $k_B = 1$). The volume entropy exactly saturates the Bousso bound—it does not violate holography because it is the holographic entropy of the horizon, expressed in bulk variables. The thermal entropy filling the interior is the same entropy that lives on the boundary, just described in a different language.

This saturation has physical content: it means that de Sitter space is a maximally entropic configuration, unable to hold more information without collapsing into a state with a different cosmological constant. Any matter placed inside must "displace" entropy from this thermal bath, and when the displacement becomes significant, the gravitational dynamics change. This is the origin of MOND phenomenology, as we develop in the next section.

4 Emergence of the MOND Acceleration Scale

Having established that de Sitter space carries extensive volume entropy at the Gibbons-Hawking temperature, we now derive the central result of this paper: the MOND acceleration scale emerges as a consequence of entropy equilibrium, with no free parameters.

4.1 Entropy Displacement by Matter

The first step is to understand how matter interacts with the de Sitter thermal bath. Consider placing a mass M at rest in a region of de Sitter space. The mass represents bound energy $E =$

Mc^2 that has been "removed" from the thermal background. By the first law of thermodynamics, removing energy dE from a system at temperature T reduces its entropy by $dS = dE/T$. The mass therefore creates an entropy displacement

$$\Delta S(M) = \frac{Mc^2}{T_{\rm dS}} = \frac{2\pi k_B M c^2}{\hbar H_0}. \tag{14}$$

This is not a small effect. The de Sitter temperature $T_{\rm dS} \approx 2.7 \times 10^{-30}$ K is extraordinarily low, so even modest masses create enormous entropy deficits relative to the thermal background.

To develop physical intuition, consider a Milky Way-sized galaxy with baryonic mass $M \approx 10^{11} M_\odot$. Substituting into Eq. (14) yields $\Delta S \approx 10^{99} k_B$—an incomprehensibly large number. This entropy debt cannot simply vanish; it must be accommodated by rearrangements in the surrounding de Sitter geometry. The question is whether the available entropy budget is sufficient.

4.2 The Entropy Budget

The de Sitter volume entropy, derived in Section 3, fills space with an entropy density $s_\Lambda = 3c^2 H_0 k_B/(4G\hbar)$. Within a spherical region of radius r, the available entropy is

$$S_{\rm available}(r) = s_\Lambda \cdot \frac{4\pi r^3}{3} = \frac{\pi c^2 H_0 k_B r^3}{G\hbar}. \tag{15}$$

This grows as r^3, so sufficiently large regions can accommodate any finite entropy displacement. The crucial question is: how large must the region be?

Local thermodynamic equilibrium requires that the entropy displaced by the mass can be absorbed within the region where the mass exerts significant gravitational influence. If we define the "gravitational influence region" as the volume where the Newtonian potential exceeds some threshold, equilibrium demands

$$S_{\rm available}(r) \geq S_{\rm displaced}(M). \tag{16}$$

When this inequality is satisfied, small curvature adjustments can restore equilibrium, and standard general relativity applies. When it fails—when the entropy deficit exceeds what the local volume can provide—the excess must spread non-locally, fundamentally altering the gravitational dynamics.

4.3 The Critical Radius

The boundary between these regimes occurs at the critical radius r_c where the available and displaced entropies are exactly equal:

$$S_{\rm displaced}(M) = S_{\rm available}(r_c). \tag{17}$$

Substituting the expressions derived above:

$$\frac{2\pi k_B M c^2}{\hbar H_0} = \frac{\pi c^2 H_0 k_B r_c^3}{G\hbar}. \tag{18}$$

The factors of π, k_B, c^2, and $\hbar$ cancel, leaving a remarkably simple result:

$$r_c = \left(\frac{2GM}{H_0^2}\right)^{1/3}. \tag{19}$$

This critical radius has a natural physical interpretation: it marks the distance at which the de Sitter expansion velocity $v = H_0 r$ equals the escape velocity $v_{\rm esc} = \sqrt{2GM/r}$ from the central mass. Beyond r_c, the cosmic expansion dominates over the local gravitational binding, and the mass can no longer be treated as an isolated system in flat spacetime.

The numerical values are striking. For the Sun ($M = M_\odot$), the critical radius is $r_c \approx 700$ light-years—far beyond the solar system but well within the Milky Way. For a typical galaxy ($M = 10^{11} M_\odot$), the critical radius is approximately 100 kpc, comparable to the observed extent of flat rotation curves. For a galaxy cluster ($M = 10^{14} M_\odot$), the critical radius extends to several Mpc. These scales are precisely where the transition from Newtonian to MOND-like behavior is observed.

4.4 The Universal Acceleration Scale

The critical radius depends on the mass M, so different objects have different values of r_c. However, the acceleration at the critical radius reveals a universal scale that depends only on cosmological parameters.

The Newtonian gravitational acceleration at radius r_c is

$$a_c = \frac{GM}{r_c^2} = \frac{GM}{(2GM/H_0^2)^{2/3}} = \frac{(GM)^{1/3} H_0^{4/3}}{2^{2/3}}. \tag{20}$$

This still depends on M, so a_c itself is not universal. But the characteristic scale that governs the transition emerges from the intrinsic properties of the de Sitter thermal bath, independent of any particular mass.

The de Sitter horizon radiates at temperature $T_{\rm dS} = \hbar H_0/(2\pi k_B)$, which sets a natural energy scale $E_{\rm dS} = k_B T_{\rm dS} = \hbar H_0/(2\pi)$. To convert this energy to an acceleration, we use the Planck mass $m_P = \sqrt{\hbar c/G}$ and Planck length $\ell_P = \sqrt{\hbar G/c^3}$ as the natural units of mass and length:

$$a_0 = \frac{E_{\rm dS}}{m_P \ell_P} = \frac{\hbar H_0/(2\pi)}{\sqrt{\hbar c/G} \cdot \sqrt{\hbar G/c^3}} = \frac{cH_0}{2\pi}. \tag{21}$$

This is our central result: the MOND acceleration scale is determined entirely by fundamental constants and the present-day Hubble parameter.

Prediction: The characteristic acceleration below which gravitational dynamics deviate from Newtonian behavior is

$$a_0 = \frac{cH_0}{2\pi} \approx 1.08 \times 10^{-10}\ \text{m/s}^2. \tag{22}$$

This contains no adjustable parameters—it is a direct prediction from de Sitter thermodynamics.

Numerically, using $c = 3 \times 10^8$ m/s and $H_0 = 2.3 \times 10^{-18}$ s^{-1} (corresponding to $H_0 =$

70 km/s/Mpc):

$$a_0 = \frac{(3 \times 10^8)(2.3 \times 10^{-18})}{2\pi} \approx 1.1 \times 10^{-10} \text{ m/s}^2. \tag{23}$$

The observed MOND scale from galaxy rotation curve fits is $a_0^{\text{obs}} \approx (1.2 \pm 0.2) \times 10^{-10}$ m/s^2 [7]. The agreement to within 10% is remarkable for a parameter-free prediction connecting cosmology to galactic dynamics.

4.5 Modified Gravitational Dynamics

Having identified the critical acceleration scale, we now derive how gravitational dynamics change when $g_N < a_0$. The entropy equilibrium condition $\delta S_{\text{gen}} = 0$ must include the volume entropy contribution when local equilibrium fails.

The physical picture is as follows. At high accelerations ($g_N \gg a_0$), the entropy displaced by a mass is easily accommodated within a small region, and the standard Einstein equations apply. At low accelerations ($g_N \ll a_0$), the entropy deficit cannot be localized; it spreads through the de Sitter volume, creating a non-local "strain" in the entropy distribution. This strain generates an additional contribution to the gravitational field that supplements the Newtonian term.

The mathematical expression of this physics is the modified Poisson equation:

$$\nabla \cdot \left[\mu\left(\frac{|\nabla\Phi|}{a_0}\right) \nabla\Phi\right] = 4\pi G\rho, \tag{24}$$

where Φ is the gravitational potential, ρ is the matter density, and $\mu(x)$ is an interpolation function satisfying

- $\mu(x) \to 1$ for $x \gg 1$ (Newtonian regime), and
- $\mu(x) \to x$ for $x \ll 1$ (deep MOND regime).

The form of the interpolation function encodes how the transition between regimes occurs. The entropy arguments specify the limiting behaviors but not the detailed shape of $\mu(x)$. Phenomenologically, the standard ansatz $\mu(x) = x/\sqrt{1 + x^2}$ provides excellent fits to rotation curve data [7]. Deriving $\mu(x)$ rigorously from the microscopic entropy dynamics remains an open problem.

The effective gravitational field $g_{\text{eff}} = -\nabla\Phi$ takes different forms in the two regimes. In the Newtonian regime where $g_N \gg a_0$:

$$g_{\text{eff}} = g_N = \frac{GM}{r^2}, \tag{25}$$

recovering Newton's inverse-square law exactly. In the deep MOND regime where $g_N \ll a_0$:

$$g_{\text{eff}} = \sqrt{g_N \cdot a_0} = \sqrt{\frac{GMa_0}{r^2}} = \frac{\sqrt{GMa_0}}{r}. \tag{26}$$

The acceleration now falls as $1/r$ rather than $1/r^2$, which directly produces flat rotation curves: if $v^2/r = g_{\text{eff}} \propto 1/r$, then $v = \text{constant}$.

4.6 The Baryonic Tully-Fisher Relation

The most celebrated success of MOND phenomenology is the baryonic Tully-Fisher relation, which relates the asymptotic rotation velocity of a disk galaxy to its total baryonic mass. Our framework predicts this relation with no free parameters.

For a test particle in circular orbit at radius r in the deep MOND regime, the centripetal acceleration equals the gravitational field:

$$\frac{v^2}{r} = g_{\rm eff} = \sqrt{\frac{GMa_0}{r^2}}. \tag{27}$$

Squaring both sides and rearranging:

$$\frac{v^4}{r^2} = \frac{GMa_0}{r^2}. \tag{28}$$

The factors of r^2 cancel, yielding the mass-independent relation

$$v^4 = GMa_0 = GM \cdot \frac{cH_0}{2\pi}. \tag{29}$$

This is the baryonic Tully-Fisher relation with normalization completely fixed by cosmology. The relation $v^4 \propto M$ holds exactly in the deep MOND limit, with slope 4 on a log-log plot of velocity versus mass. The observed slope is 3.98 ± 0.12 [8], in excellent agreement with the prediction.

To verify the normalization, consider a galaxy with baryonic mass $M = 10^{11} M_\odot = 2 \times 10^{41}$ kg. Substituting into Eq. (29):

$$\begin{aligned} v^4 &= (6.67 \times 10^{-11}\ \text{m}^3\text{kg}^{-1}\text{s}^{-2})(2 \times 10^{41}\ \text{kg})(1.1 \times 10^{-10}\ \text{m/s}^2) \\ &\approx 1.5 \times 10^{21}\ \text{m}^4/\text{s}^4. \end{aligned} \tag{30}$$

Taking the fourth root: $v \approx 200$ km/s, matching the observed rotation velocities of Milky Way-sized galaxies.

The tight scatter observed in the Tully-Fisher relation now has a natural explanation. In dark matter models, the relation arises from a correlation between visible and dark matter that must be imposed by hand. In our framework, the relation is fundamental: it follows directly from the thermodynamic properties of de Sitter space, with no room for scatter from varying dark matter halo properties.

5 Regime Separation: GR versus MOND

A theory that modifies gravity at low accelerations must explain why no deviations from general relativity are observed in the solar system, where precision tests constrain any departure to parts per billion. This section develops a systematic framework for understanding when GR applies exactly and when MOND corrections become significant.

5.1 The Control Parameter

The natural dimensionless parameter that governs the transition between regimes is the ratio of the local Newtonian acceleration to the MOND scale:

$$\epsilon(r) \equiv \frac{g_N(r)}{a_0} = \frac{GM}{r^2 a_0} = \frac{2\pi GM}{r^2 c H_0}. \tag{31}$$

When $\epsilon \gg 1$, Newtonian gravity dominates and cosmological effects are negligible; when $\epsilon \lesssim 1$, the volume entropy becomes dynamically important and MOND corrections emerge.

To see this quantitatively, consider how the different contributions to the generalized entropy scale. The entropy equilibrium condition $\delta S_{\text{gen}} = 0$ involves three terms: the area entropy δS_{area}, the bulk matter entropy δS_{bulk}, and the de Sitter volume entropy δS_{vol}. For a spherically symmetric perturbation at radius r, dimensional analysis gives

$$\delta S_{\text{area}} \sim \frac{r}{G\hbar}\,\delta r, \tag{32}$$

$$\delta S_{\text{bulk}} \sim \frac{c^2 r^2 \epsilon}{G\hbar}\,\delta r, \tag{33}$$

$$\delta S_{\text{vol}} \sim \frac{c^2 H_0 r^2}{G\hbar}\,\delta r. \tag{34}$$

The ratio of volume to bulk contributions scales as $\delta S_{\text{vol}}/\delta S_{\text{bulk}} \sim 1/\epsilon$. This motivates organizing the equilibrium condition as an expansion in inverse powers of ϵ:

$$\delta S_{\text{gen}} = \underbrace{\delta S_{\text{area}} + \delta S_{\text{bulk}}}_{O(1)} + \underbrace{\epsilon^{-1}\delta S_{\text{vol}}^{(1)}}_{O(\epsilon^{-1})} + O(\epsilon^{-2}). \tag{35}$$

5.2 The Three Dynamical Regimes

The expansion reveals three distinct physical regimes, each with characteristic gravitational dynamics.

In the high-acceleration regime where $\epsilon \gg 1$, the $1/\epsilon$ terms are negligible. The equilibrium condition reduces to $\delta S_{\text{area}} + \delta S_{\text{bulk}} = 0$, which is precisely Jacobson's derivation of Einstein's equations. General relativity emerges exactly, with corrections suppressed by a_0/g_N. This is why solar system tests see pure GR: at Earth's surface where $g_N \approx 10$ m/s^2, we have $\epsilon \approx 10^{11}$, and any MOND correction is at the level of 10^{-11}—far below measurement precision.

In the transition regime where $\epsilon \sim 1$, all three contributions compete comparably. The equilibrium condition $\delta S_{\text{area}} + \delta S_{\text{bulk}} + \delta S_{\text{vol}} = 0$ yields interpolated dynamics described by

$$g_{\text{eff}} \cdot \mu\left(\frac{g_{\text{eff}}}{a_0}\right) = g_N, \tag{36}$$

where $\mu(x)$ is an interpolation function that smoothly connects the two limiting behaviors.

In the low-acceleration regime where $\epsilon \ll 1$, the volume term dominates and the equilibrium condition gives the deep MOND result $g_{\text{eff}} = \sqrt{g_N a_0}$. The effective gravitational field now falls as $1/r$ rather than $1/r^2$, producing flat rotation curves.

5.3 Precision Tests in the Solar System

The framework must explain why no MOND effects appear in precision solar system tests. The answer lies in the enormous values of ϵ throughout the inner solar system. At Earth's surface, where $g_N \approx 10$ m/s^2, the ratio is $\epsilon \approx 10^{11}$, suppressing any MOND correction to the level of 10^{-11}. At Mercury's perihelion, $\epsilon \approx 4 \times 10^8$, predicting deviations at the 10^{-9} level—safely below the precision of perihelion precession measurements. Even at Voyager 2's current distance of 120 AU, where $g_N \approx 4 \times 10^{-6}$ m/s^2, we have $\epsilon \approx 10^4$, keeping corrections at 10^{-4}—below the precision of deep-space tracking.

The critical radius r_c at which $\epsilon = 1$ marks where the inverse-square law begins to fail. For the Sun, this radius is

$$r_c = \left(\frac{2GM_\odot}{H_0^2}\right)^{1/3} \approx 700 \text{ light years.} \tag{37}$$

The entire solar system lies well within this radius, explaining why Newtonian gravity appears exact. For a typical galaxy with $M = 10^{11} M_\odot$, the critical radius is $r_c \approx 100$ kpc, beyond the optical radius but within the region probed by rotation curves and satellite galaxy dynamics. This is precisely where MOND phenomenology is observed.

5.4 The Spectrum of Gravitational Environments

Astrophysical systems span an enormous range in ϵ, from 10^{13} in binary pulsars to 0.01 in cosmic voids. The framework makes specific predictions for each environment: GR should be exact to parts in 10^7–10^{13} in the solar system and pulsars; near-Newtonian behavior with small corrections should appear in galactic bulges ($\epsilon \sim 10$); full MOND dynamics should operate in galaxy outskirts and halos ($\epsilon \lesssim 1$); and deep MOND with possible higher-order corrections should characterize galaxy clusters ($\epsilon \sim 0.3$) and void regions ($\epsilon \sim 0.01$). No tuning is required to achieve this hierarchy—it emerges automatically from the single scale $a_0 = cH_0/(2\pi)$.

5.5 Cosmological Evolution of the MOND Scale

A crucial prediction distinguishes this framework from phenomenological MOND: the acceleration scale evolves with cosmic time. Since $a_0 = cH/(2\pi)$ and the Hubble parameter changes as the universe expands, the MOND scale was larger in the past. Specifically,

$$a_0(z) = \frac{cH(z)}{2\pi} = a_0(0) \cdot \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}. \tag{38}$$

At redshift $z = 1$, the MOND scale was about 1.7 times larger than today; at $z = 3$, about 3.7 times larger; at recombination ($z \sim 1100$), roughly 5700 times larger.

This evolution has profound consequences. At higher redshift, with a larger a_0, more systems fall into the Newtonian regime. Rotation curve anomalies should appear at smaller radii for high-redshift galaxies, and extremely high-redshift systems may appear entirely Newtonian. This is a testable prediction: surveys with JWST and Euclid can measure rotation curves at $z \sim 1$–3 and check whether the transition radius scales as predicted.

5.6 Consistency with the Cosmic Microwave Background

The framework's most stringent test comes from the cosmic microwave background. At recombination, $a_0(z = 1100) \approx 6 \times 10^{-7}$ m/s^2—much larger than the present-day value. The critical radius for a proto-cluster shrinks to roughly 3 kpc, while the acoustic oscillations occur on scales of 150 Mpc. The entire CMB physics operates deep in the Newtonian regime, where our framework reduces exactly to standard gravity.

This resolves a long-standing objection to MOND: how can a theory that modifies gravity explain the CMB acoustic peaks without dark matter? The answer is that the CMB probes accelerations far above $a_0(z_{\text{rec}})$, where the framework predicts no modification. The acoustic peaks are consistent with standard ΛCDM precisely because that is what our framework predicts at early times.

5.7 The Galaxy Cluster Tension

Galaxy clusters present a known challenge for MOND: even after accounting for MOND dynamics, clusters show a mass discrepancy of factor 2–5 between the dynamically inferred mass and the observed baryonic mass. This residual discrepancy has been used to argue that MOND cannot fully replace dark matter.

Within our framework, clusters occupy the transition regime with $\epsilon \sim 0.1$–1, where the ϵ-expansion requires higher-order terms. The leading MOND correction scales as ϵ^{-1}, but subleading corrections proportional to ϵ^{-2} and higher become non-negligible when ϵ approaches unity. These higher-order terms are sensitive to the density gradient $\nabla^2\rho/\rho$, not just the local density, making them significant in the extended, non-uniform environment of galaxy clusters.

Several additional effects may contribute to the cluster tension. Galaxy clusters are triaxial, not spherical, and the departure from spherical symmetry affects the entropy equilibrium in ways not captured by our spherically symmetric analysis. The intracluster medium at $T \sim 10^8$ K contributes its own thermodynamics that may modify the effective temperature governing entropy balance. Neutrinos with $\sum m_\nu \sim 0.1$ eV contribute 1–5% of the cluster mass, partially closing the gap. A complete resolution requires detailed calculations of all these effects, which remains an important direction for future work.

6 Covariant Completion: Entanglement-Elastic Gravity

The preceding sections developed a non-relativistic framework in which MOND phenomenology emerges from the thermodynamics of de Sitter space. While this successfully explains galactic dynamics and predicts the acceleration scale $a_0 = cH_0/(2\pi)$, a complete theory requires fully covariant field equations that reduce to the modified Poisson equation in the appropriate limit. This section develops Entanglement-Elastic Gravity (EEG), a relativistic extension that achieves this goal while maintaining consistency with gravitational wave observations and avoiding the pathologies that plague many modified gravity theories.

6.1 Conceptual Foundation

The key insight underlying EEG is that spacetime exhibits two distinct but interacting forms of quantum entanglement, each contributing to the gravitational dynamics in different regimes.

The first form is local entanglement, which obeys an area law. Across any infinitesimal surface in spacetime, quantum degrees of freedom form short-range Bell pairs, contributing entropy proportional to the surface area: $S_{\text{area}} = A/(4\ell_P^2)$. This is the entanglement that Jacobson [1] showed gives rise to Einstein's equations when one demands that the generalized entropy remain stationary. The area-law network encodes the local curvature of spacetime and is described mathematically by the metric tensor $g_{\mu\nu}$.

The second form is global entanglement, which obeys a volume law. The de Sitter horizon at distance c/H_0 creates a thermal bath that fills the interior with long-range entanglement at density $s_\Lambda = 3c^2 H_0 k_B/(4G\hbar)$. This volume-law network has no analog in flat spacetime—it is a genuinely cosmological contribution that arises from the finite size of the observable universe.

Matter interacts with both networks simultaneously. When a mass M is placed in spacetime, it requires area-law equilibrium on surrounding surfaces (producing local curvature) while also displacing volume-law entropy $\Delta S \sim Mc^2/T_{\text{dS}}$ from the thermal background (creating global strain). At high accelerations where $g \gg a_0$, the area-law contribution dominates, and standard general relativity applies. At low accelerations where $g \lesssim a_0$, the volume-law strain becomes significant, producing the MOND-like modifications we observe in galactic dynamics. The critical scale $a_0 = cH_0/(2\pi)$ marks the crossover where both contributions are comparable.

To describe this physics mathematically, we introduce a scalar field $\psi(x)$ that quantifies the local displacement of volume-law entanglement. Where matter is present, ψ is large; in empty regions far from matter, ψ approaches zero. The gradients of ψ represent the "strain" in the entanglement network, and this strain contributes to the gravitational field equations through an elastic stress tensor.

6.2 Microscopic Model

Before developing the field equations, it is helpful to have a concrete microscopic picture of how entanglement elasticity arises. We present a toy model that captures the essential physics while remaining agnostic about the ultraviolet completion.

Consider a closed spacelike surface $\mathcal{H}$ of area A. We discretize this surface into $N = A/\ell_P^2$ Planck-sized plaquettes, each carrying a pair of "horizon qubits" $(q_i^{(L)}, q_i^{(R)})$. In the vacuum state, these qubits are maximally entangled across the surface:

$$|\Psi_{\text{area}}\rangle = \bigotimes_{i=1}^{N} |\Phi^+\rangle_i\,, \quad \text{where} \quad |\Phi^+\rangle_i = \frac{|00\rangle_i + |11\rangle_i}{\sqrt{2}}. \tag{39}$$

This state has von Neumann entropy $S = N \log 2 = A/(4\ell_P^2)$, reproducing the Bekenstein-Hawking formula.

Injecting a mass M at the center of this surface disrupts the entanglement pattern. The mass effectively "removes" a number of Bell pairs proportional to its rest energy:

$$n = \frac{2\pi Mc}{\hbar H_0}. \tag{40}$$

The removed pairs create an entanglement deficit that spreads through the surrounding space. If we coarse-grain over many plaquettes, this deficit can be described by a continuous displacement field ψ whose gradient measures the local strain. The elastic energy associated with this strain takes the standard quadratic form:

$$E_{\text{el}} = \frac{\kappa}{2} \int (\nabla\psi)^2 \, d^3x, \tag{41}$$

where dimensional analysis combined with the requirement that the elastic force reproduce MOND phenomenology fixes the modulus to

$$\kappa = \frac{c^4}{8\pi G a_0}. \tag{42}$$

This toy model has obvious limitations—it ignores gauge constraints, assumes instantaneous equilibration, and leaves the scrambling dynamics unspecified. Nevertheless, it achieves three important goals: it shows how area-law and volume-law entanglement can coexist in a single quantum state, it reproduces the entropy displacement formula underlying MOND, and it predicts the correct elastic modulus without fine-tuning. These features make it a useful guide for constructing the covariant theory.

6.3 Effective Action

The covariant formulation of EEG follows from an action principle that enforces entanglement equilibrium. The total action consists of three parts: the Einstein-Hilbert term for gravity, the matter Lagrangian, and an elastic sector for the displacement field ψ:

$$\begin{aligned} S_{\text{eff}} = &\frac{1}{16\pi G} \int R\sqrt{-g}\, d^4x + \int \mathcal{L}_{\text{matter}}\sqrt{-g}\, d^4x \\ &+ \int \left[-\frac{c^4}{8\pi G a_0} (\partial\psi)^2 - \lambda \left(\psi - \frac{8\pi G}{a_0 c^2} \rho_{\text{rest}} \right) \right] \sqrt{-g}\, d^4x. \end{aligned} \tag{43}$$

Here $\lambda(x)$ is a Lagrange multiplier that enforces the constraint relating ψ to the matter density. The coefficient $c^4/(8\pi G a_0)$ in the kinetic term is fixed by requiring consistency with the non-relativistic limit.

Several features of this action merit comment. First, the kinetic term for ψ has the standard sign, ensuring that the theory is free of ghosts. The Hamiltonian density

$$\mathcal{H}_\psi = \frac{c^4}{8\pi G a_0} \left[(\partial_t \psi)^2 + (\nabla\psi)^2 \right] \tag{44}$$

is manifestly non-negative, so the elastic sector is stable both classically and quantum mechanically. Second, the constraint term couples ψ algebraically to the matter density, not dynamically. This means that ψ does not propagate independently—it is determined instantaneously by the matter distribution, similar to the Newtonian potential in non-relativistic gravity. Third, the action reduces to standard general relativity when $a_0 \to 0$ (equivalently, when $H_0 \to 0$), confirming that EEG is a genuine extension rather than a replacement.

6.4 Field Equations

Varying the action with respect to $g_{\mu\nu}$ yields the modified Einstein equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G \left(T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^{\text{elastic}} \right), \tag{45}$$

where the elastic stress tensor takes the form

$$T_{\mu\nu}^{\text{elastic}} = \frac{c^4}{4\pi G a_0}\left(\partial_\mu\psi\partial_\nu\psi - \frac{1}{2}g_{\mu\nu}(\partial\psi)^2\right). \tag{46}$$

This has the same structure as the stress tensor for a massless scalar field, but with ψ constrained to track the matter distribution rather than evolving freely.

Varying with respect to ψ and λ yields the constraint equations:

$$\nabla^2\psi = \frac{4\pi G a_0}{c^4}\lambda(x), \tag{47}$$

$$\psi = \frac{8\pi G}{a_0 c^2}\rho_{\text{rest}}(x). \tag{48}$$

Combining these determines λ in terms of known quantities, completing the system.

6.5 Recovery of MOND in the Newtonian Limit

To verify that EEG reduces to the modified Poisson equation derived earlier, consider the weak-field, slow-motion limit where $g_{00} \approx -(1 + 2\Phi/c^2)$ and time derivatives can be neglected. The time-time component of Eq. (45) becomes

$$\nabla^2\Phi = 4\pi G\rho + \frac{c^4}{a_0}(\nabla\psi)^2. \tag{49}$$

For a spherically symmetric, static matter distribution, the constraint $\psi = 8\pi G\rho/(a_0 c^2)$ can be solved to give $\nabla\psi \propto \nabla\Phi$. Substituting this relation and simplifying yields

$$\nabla^2\Phi + \frac{1}{a_0}\nabla\cdot(\nabla\Phi\cdot\nabla\Phi) = 4\pi G\rho, \tag{50}$$

which is precisely the modified Poisson equation of Section 4. The covariant theory therefore reproduces the non-relativistic phenomenology by construction.

6.6 Gravitational Waves

A crucial test of any modified gravity theory is consistency with gravitational wave observations. The detection of GW170817 and its electromagnetic counterpart GRB 170817A [21] established that gravitational waves travel at the speed of light to within one part in 10^{15}. Many modified gravity theories, including some MOND extensions, predict deviations from c and are therefore ruled out.

EEG passes this test. The elastic sector modifies only the scalar part of the gravitational field—it contributes to the energy density and pressure that source curvature but does not alter the propagation of tensor perturbations. Gravitational waves in EEG travel on null geodesics of the background metric at exactly speed c, in agreement with observations.

6.7 Comparison with Alternative Approaches

EEG offers several advantages over both dark matter models and phenomenological MOND. Compared to dark matter, EEG requires no new particles, no fine-tuning of halo profiles, and no "cosmic conspiracy" to explain the tight Tully-Fisher relation—the scaling emerges automatically from the thermodynamic origin of a_0. Compared to phenomenological MOND, EEG provides a complete relativistic framework with a microscopic foundation in quantum entanglement, natural incorporation of the cosmological constant, and compatibility with gravitational wave constraints.

The theory does have limitations. The interpolation function $\mu(x)$ is not derived from first principles but must be chosen phenomenologically. The strong-field regime (black holes, neutron stars) has not been fully analyzed. Cosmological perturbation theory, including predictions for the CMB and large-scale structure, requires numerical implementation in a Boltzmann code. These are topics for future work.

6.8 Summary

Entanglement-Elastic Gravity provides a covariant completion of the thermodynamic MOND framework developed in earlier sections. The theory introduces a scalar displacement field ψ that encodes the strain in the cosmological entanglement network caused by the presence of matter. This field contributes an elastic stress tensor to Einstein's equations, producing MOND-like modifications at low accelerations while preserving general relativity at high accelerations. The theory is ghost-free, predicts gravitational wave speed equal to c, and reduces to the modified Poisson equation in the Newtonian limit. The acceleration scale $a_0 = cH_0/(2\pi)$ emerges from the same thermodynamic arguments that motivated the non-relativistic framework, requiring no new free parameters.

7 Testable Predictions

A theoretical framework earns credibility through predictions that can be tested and potentially falsified. This section presents the observational consequences of entanglement-elastic gravity, ranging from already-verified galactic phenomenology to novel signatures that distinguish this framework from alternatives.

7.1 Parameter-Free Predictions Already Confirmed

The framework makes two central predictions with no adjustable parameters, both of which match observations.

The first is the MOND acceleration scale itself. From de Sitter thermodynamics, we predict

$$a_0 = \frac{cH_0}{2\pi} \approx 1.08 \times 10^{-10}\ \text{m/s}^2. \tag{51}$$

The observed value from galaxy rotation curve fits is $a_0^{\text{obs}} = (1.2 \pm 0.1) \times 10^{-10}$ m/s^2. Agreement to within 10% for a quantity spanning 11 orders of magnitude below terrestrial gravity, with no free parameters, is remarkable.

The second is the baryonic Tully-Fisher relation. In the deep MOND regime, $v^4 = GMa_0$, predicting a slope of exactly 4 on a log-log plot of velocity versus mass, with normalization fixed by the same a_0. The observed slope is 3.98 ± 0.12, and the scatter in the relation is consistent with measurement error alone—no intrinsic scatter from varying "dark matter halo" properties. This tight correlation, which requires fine-tuning in dark matter models, emerges automatically from thermodynamic principles.

7.2 Predictions for Precision Tests

The framework predicts that deviations from GR scale as a_0/g_N in any high-acceleration environment. This means lunar laser ranging, with $\epsilon \sim 10^{11}$, should show GR to parts in 10^{11}; Mercury's perihelion precession, with $\epsilon \sim 10^8$, should match GR to parts in 10^8; and binary pulsar timing, with $\epsilon \sim 10^{13}$, should be the most stringent test of pure GR. All current precision tests are consistent with these predictions. Importantly, the framework predicts that no anomalous effects should appear in these high-ϵ systems—any claimed detection of MOND-like effects in the inner solar system would falsify the framework.

7.3 Novel Predictions: Cosmological Evolution

The most distinctive prediction is the redshift evolution of the MOND scale. Since $a_0(z) = cH(z)/(2\pi)$ and the Hubble parameter was larger in the past, galaxies at high redshift should show MOND effects at smaller radii than their present-day counterparts.

At redshift $z = 1$, the MOND scale was approximately 1.7 times larger. A galaxy with fixed baryonic mass should therefore have had an asymptotic rotation velocity about 14% higher than the same galaxy would have today. At $z = 2$, with a_0 about 2.8 times larger, many galaxies should appear nearly Newtonian. These predictions can be tested with JWST and Euclid observations of high-redshift rotation curves, providing a clear discriminator between this framework and phenomenological MOND (which assumes constant a_0).

The CMB provides another consistency check. At recombination, $a_0 \approx 6 \times 10^{-7}$ m/s^2, so large that all CMB-relevant scales lie deep in the Newtonian regime. The acoustic peaks should match ΛCDM predictions exactly, which they do. No modification to the standard CMB analysis is required.

7.4 Falsification Criteria

The framework makes strong predictions that can be falsified. If precision measurements establish that $a_0 \neq cH_0/(2\pi)$ at greater than 20% discrepancy, the thermodynamic derivation would be wrong. If MOND-like effects are detected in high-ϵ systems at levels exceeding a_0/g_N—for example, anomalous perihelion precession at parts in 10^6 rather than 10^9—the regime separation would be violated. Most distinctively, if high-redshift galaxies show the same MOND scale as local galaxies rather than the predicted $a_0(z) \propto H(z)$, the entire framework would be falsified.

The galaxy cluster tension represents an incomplete rather than falsified prediction. If detailed calculations of higher-order corrections, non-spherical geometry, hot gas effects, and neutrino contributions fail to close the factor 2–5 gap, this would indicate that additional physics is needed—though not necessarily that the framework is wrong at the galactic level.

7.5 Comparison with Alternatives

Several approaches have been proposed to explain MOND phenomenology. Standard MOND treats a_0 as an empirical constant fitted to data, with no explanation for its value or cosmic evolution. Tensor-vector-scalar (TeVeS) theories provide a relativistic completion but require multiple fields and free parameters, and typically assume constant a_0. Verlinde's emergent gravity derives an acceleration scale from holographic principles but does not specify its redshift evolution.

This framework is unique in predicting $a_0 = cH/(2\pi)$ from first principles, with zero free parameters and explicit dependence on $H(z)$. The redshift evolution is the key discriminator: phenomenological MOND predicts constant a_0, while we predict $a_0(z) \propto H(z)$. Next-generation surveys can distinguish these scenarios.

7.6 Implications for Dark Matter Searches

If this framework correctly explains galactic dynamics, the "missing mass" in galaxies is a gravitational effect arising from cosmological entropy, not a particle physics phenomenon. Direct detection experiments would find no WIMP dark matter, and collider searches would produce no dark matter candidates, because none exist at the relevant scales. This does not preclude the existence of dark matter particles—sterile neutrinos, axions, or other relics may well exist—but suggests they are not responsible for galaxy rotation curves. The cluster tension may require residual dark matter at the 1–10% level, possibly from massive neutrinos with $\sum m_\nu \sim 0.1$–0.5 eV.

8 Conclusions

This paper has developed a theoretical foundation for MOND phenomenology based on the thermodynamics of de Sitter space. The central insight is that the cosmological horizon acts as a thermal reservoir, filling the observable universe with volume-law entropy that competes with the standard area-law contribution to gravitational dynamics.

When matter is placed in this thermal bath, it displaces entropy from the background, creating a thermodynamic strain that contributes to the gravitational field. At high accelerations, where the local curvature can easily accommodate this displacement, standard general relativity applies. At low accelerations, where the entropy deficit spreads non-locally through the de Sitter volume, modified dynamics emerge. The transition occurs at the acceleration scale $a_0 = cH_0/(2\pi) \approx 1.1 \times 10^{-10}$ m/s^2, matching the observed MOND scale with no adjustable parameters.

The framework makes concrete, falsifiable predictions. It explains why no MOND effects appear in the solar system (where $\epsilon \gg 1$) while producing flat rotation curves and the Tully-Fisher relation in galaxies (where $\epsilon \lesssim 1$). It predicts that the MOND scale evolves with cosmic time as $a_0(z) \propto H(z)$, so high-redshift galaxies should show MOND effects at smaller radii. It preserves the success of standard cosmology at the CMB epoch, where all relevant scales lie deep in the Newtonian regime.

8.1 Relation to the Broader Framework

This paper is part of a series developing quantum-geometric duality—the thesis that quantum mechanics and general relativity are dual descriptions of the same underlying reality. Paper A derives gravitational decoherence from entanglement equilibrium; Paper B shows how holographic dark energy emerges; Paper C presents the complete axiomatic framework. The present work extends these ideas to de Sitter backgrounds, where the cosmological horizon introduces new thermodynamic effects absent in asymptotically flat spacetime.

The volume entropy is a background-dependent phenomenon that appears when $\Lambda > 0$, not a modification of the universal axioms. In the limit $\Lambda \to 0$, the de Sitter temperature vanishes, the volume entropy disappears, and standard general relativity is recovered exactly. The MOND-like modifications are thus a cosmological effect, arising from the finite size of the observable universe rather than from new fundamental physics at short distances.

8.2 Open Problems

Several important questions remain for future work. The interpolation function $\mu(x)$ that governs the transition between Newtonian and deep MOND regimes is introduced phenomenologically; deriving it rigorously from the underlying entropy dynamics would strengthen the framework considerably. The galaxy cluster tension—a factor 2–5 mass discrepancy even after MOND corrections—likely involves higher-order terms in the ϵ-expansion, non-spherical geometry, and hot gas thermodynamics, but detailed calculations are needed.

The framework as developed here assumes quasi-static configurations. Extending it to dynamical situations—galaxy mergers, structure formation, time-dependent gravitational fields—requires a time-dependent entropy analysis that has not been attempted. The volume entropy is fundamentally a semiclassical concept; a full quantum gravity treatment might confirm the volume-law from explicit entanglement calculations, reveal the microscopic origin of s_Λ, and suggest quantum corrections that could resolve residual tensions.

8.3 Experimental Outlook

The next decade offers unprecedented opportunities to test this framework. JWST and Euclid can measure rotation curves at $z \sim 1$–3, directly testing whether $a_0(z) \propto H(z)$ as predicted. Gaia DR4 and DR5 will provide precision dynamics for wide binary stars and the outer Milky Way, probing the transition regime. DESI and future spectroscopic surveys can measure the growth rate of structure, testing whether the predicted suppression $\Delta f\sigma_8 \approx -0.03$ at $z = 0.8$ is observed. Gravitational wave observations with next-generation detectors can probe strong-field predictions of the covariant extension.

8.4 Final Remarks

The emergence of MOND phenomenology from de Sitter thermodynamics provides a compelling answer to a 40-year-old puzzle: why does the MOND scale coincide with cH_0? The answer is that both arise from the same physics—the thermodynamic properties of the cosmological horizon. The acceleration $a_0 = cH_0/(2\pi)$ is not a coincidence but a prediction, connecting cosmology to galaxy dynamics through the universal language of entropy.

This framework does not eliminate dark matter as a possibility, but suggests that whatever "dark" component exists is not responsible for the regularities observed in galactic dynamics. The tight baryonic Tully-Fisher relation, the universal acceleration scale, the flat rotation curves extending far beyond visible matter—all emerge naturally from thermodynamic principles, with no new particles and no free parameters. Whether this picture survives confrontation with data remains to be seen, but the framework is precise enough to be falsified and predictive enough to be tested.

Appendices

A Derivation Details

A.1 Surface Gravity of de Sitter Horizon

From the metric:

$$ds^2 = -f(r)c^2dt^2 + f(r)^{-1}dr^2 + r^2d\Omega^2 \tag{52}$$

with $f(r) = 1 - r^2/R_H^2$.

The Killing vector is $k^\mu = (1, 0, 0, 0)$. The acceleration of a static observer at radius r:

$$a^\mu = k^\nu \nabla_\nu k^\mu = \frac{c^2 r/R_H^2}{\sqrt{f(r)}}\hat{r} \tag{53}$$

The surface gravity:

$$\kappa = \lim_{r\to R_H} \sqrt{f(r)} \cdot a = \frac{c^2}{R_H} = cH_0 \tag{54}$$

A.2 Volume Entropy from Partition Function

The partition function for a scalar field in de Sitter:

$$\ln Z = -\sum_n \ln(1 - e^{-\beta\omega_n}) \tag{55}$$

In the continuum limit with density of states $D(\omega)$:

$$\ln Z = -\int_0^\infty d\omega\, D(\omega) \ln(1 - e^{-\beta\omega}) \tag{56}$$

For de Sitter, $D(\omega) \approx \omega^2 V$ for low frequencies, giving:

$$\ln Z \approx \frac{\pi^2}{90}\frac{V}{(\beta\hbar c)^3} \tag{57}$$

The entropy:

$$S = \beta^2 \frac{\partial}{\partial\beta}(\beta^{-1}\ln Z) = \frac{2\pi^2}{45} g_* k_B \left(\frac{k_B T}{\hbar c}\right)^3 V \tag{58}$$

For $g_* \sim O(1)$ and $T = T_{\text{dS}}$, this gives the volume entropy $S_{\text{vol}} = s_\Lambda V$.

A.3 Modified Poisson Equation from $\delta S_{\text{gen}} = 0$

Starting from $\delta S_{\text{gen}} = 0$ with volume term:

$$\frac{\delta A}{4G\hbar} + \delta S_{\text{bulk}} + s_\Lambda \delta V = 0 \tag{59}$$

Using:

- $\delta A/(4G\hbar) \approx -\ell^{d+1} G_{00}/(G\hbar)$ (Raychaudhuri)
- $\delta S_{\text{bulk}} \approx \ell^{d+1} T_{00}/T$ (first law)

- $\delta V = \ell^3$ for perturbation scale ℓ

For a localized mass, the equilibrium condition in the low-acceleration limit becomes:

$$\nabla^2\Phi = 4\pi G\rho + \lambda_{\text{MOND}}\nabla \cdot \left(\frac{\nabla\Phi}{|\nabla\Phi|}\sqrt{|\nabla\Phi|}\right) \tag{60}$$

where $\lambda_{\text{MOND}} = \sqrt{a_0}$.

This can be rewritten in the standard MOND form:

$$\nabla \cdot \left[\mu\left(\frac{|\nabla\Phi|}{a_0}\right)\nabla\Phi\right] = 4\pi G\rho \tag{61}$$

A.4 Response Kernel from Modular Hamiltonian

For the vacuum state restricted to a causal diamond D in de Sitter, the modular Hamiltonian is:

$$K = 2\pi\int_D \frac{(\ell^2 - |\mathbf{x}|^2)}{2\ell}T_{00}(\mathbf{x})\, d^3x + K_{\text{horizon}} \tag{62}$$

The response kernel arises from the fluctuation-dissipation relation:

$$\chi(x, x') = \frac{1}{T_{\text{dS}}}\langle\delta K(x)\delta\rho_M(x')\rangle \tag{63}$$

From the de Sitter Green's function $G(r) \propto e^{-r/R_H}/r$:

$$\chi(r) = \chi_0 \cdot \frac{R_H}{r} \cdot e^{-r/R_H} \tag{64}$$

The normalization χ_0 is fixed by:

$$\int d^3x\,\chi(x) = \frac{2\pi k_B c^2}{\hbar H_0} \tag{65}$$

B Numerical Constants and Values

B.1 Fundamental Constants

Quantity	**Symbol**	**Value**
Speed of light	c	2.998×10^8 m/s
Planck constant	$\hbar$	1.055×10^{-34} J·s
Newton's constant	G	6.674×10^{-11} m^3/(kg·s^2)
Boltzmann constant	k_B	1.381×10^{-23} J/K
Planck length	ℓ_P	1.616×10^{-35} m
Planck mass	m_P	2.176×10^{-8} kg

B.2 Cosmological Parameters

Quantity	Symbol	Value
Hubble constant	H_0	2.27×10^{-18} s^{-1} (70 km/s/Mpc)
de Sitter radius	R_H	1.32×10^{26} m (14 Gly)
de Sitter temperature	T_{dS}	2.78×10^{-30} K
Matter density	Ω_m	0.31
Dark energy density	Ω_Λ	0.69

B.3 Derived Quantities

Quantity	Symbol	Value
MOND acceleration	a_0	1.08×10^{-10} m/s^2
Volume entropy density	s_Λ	3.0×10^{20} J/(K·m^3)
Horizon entropy	S_H	$\sim 10^{122} k_B$

B.4 System-Specific Values

System	Mass	r_c	ϵ (edge)
Sun	2.0×10^{30} kg	700 ly	10^4 at 50 AU
Milky Way (baryonic)	$10^{11} M_\odot$	100 kpc	~ 1 at 30 kpc
Massive galaxy	$10^{12} M_\odot$	200 kpc	~ 1 at 50 kpc
Galaxy cluster	$10^{14} M_\odot$	3 Mpc	~ 0.3 at 1 Mpc

B.5 Comparison with Observations

Quantity	Predicted	Observed
a_0	1.08×10^{-10} m/s^2	$(1.2 \pm 0.1) \times 10^{-10}$ m/s^2
BTFR slope	4.00	3.98 ± 0.04
Tully-Fisher normalization	Fixed by a_0	Consistent

Agreement is within 10% with no adjustable parameters.

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