

A Mass-Independent Visibility Suppression in the Bose–Marletto–Vedral Experiment from Quantum-Geometric Duality

M. Sperzel*

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Abstract

The Bose–Marletto–Vedral (BMV) and QGEM proposals seek to witness the quantum nature of gravity by detecting entanglement generated between two massive particles each placed in spatial superposition. Standard quantum-gravity treatments predict the entanglement amplitude reaches the witness threshold after a time $\tau_{\text{BMV}} \sim \hbar d^3 / (Gm^2 \Delta^2)$, where d is the inter-particle separation and Δ the superposition arm. Quantum-Geometric Duality (QGD) – through the constrained Feynman–Vernon influence functional – predicts a competing per-particle decoherence rate $\Gamma = Gm^2 / (\hbar \Delta)$ that suppresses the visibility of the off-diagonal density-matrix element by $V(\tau_{\text{BMV}}) = \exp(-\mathcal{O}(1)(d/\Delta)^3)$ at the witness criterion. The suppression depends only on the geometric ratio Δ/d and on the arm-to-axis angle θ : it is mass-independent and parameter-free, in sharp contrast with continuous-spontaneous-localisation models and the Diósi–Penrose collapse rate. We derive the suppression for the full one-parameter family of arm orientations, including the parallel-axis and perpendicular-arm (QGEM “diamond”) layouts and the exact-phase corrections to both. The angular dependence is a Legendre $P_2(\cos \theta)$: the leading dipole contribution to the witness phase vanishes identically at the magic angle $\cos^2 \theta_m = 1/3$, where the quadrupole takes over and the visibility scales as a *quintic* $\exp[-(9\pi/7)(d/\Delta)^5]$ rather than a cubic. Combined with the factor-of-two prefactor difference between the cardinal cubic geometries and the sign-flipped sub-leading correction between them, an apparatus that can rotate its arms relative to the inter-particle axis sweeps three qualitatively distinct QGD signatures with the same hardware. Observation of the standard BMV signal at QGEM-class parameters would falsify QGD’s constrained-IF picture; observation of mass-independent visibility suppression following the cubic law in the cardinal geometries and the quintic law at the magic angle would distinguish QGD from all extant collapse models in a single experiment.

1 Introduction

The question of whether gravity is quantum has remained, despite a century of effort, without a direct experimental answer. Tests with matter-wave interferometers and levitated mirrors [12, 13] have pushed the regime of demonstrated superposition into the mesoscopic but not yet into the regime where gravitational self-energy effects are accessible. The Bose–Marletto–Vedral (BMV) proposal [1, 2] sharpened the question by identifying a tabletop test: place two massive

*Corresponding author.

particles each in spatial superposition, near enough that their mutual gravitational interaction can entangle their internal degrees of freedom, and detect that entanglement with a witness. By the LOCC argument, mediation of entanglement by the gravitational field implies that the field cannot be modelled as classical [2, 4]; this conclusion has been further scrutinised by analysing the causality constraints that the same setup places on the gravitational field [15]. The QGEM team [3] have developed a concrete experimental design using nanodiamonds with embedded NV centres in Stern–Gerlach interferometers, with target masses of $\sim 10^{-17}$ kg and superposition arms of ~ 100 μm .

A standard quantum-gravity treatment of the BMV setup predicts that the entanglement reaches the witness threshold after a time

$$\tau_{\text{BMV}} \sim \frac{\hbar d^3}{Gm^2 \Delta^2}, \quad (1)$$

where d is the inter-particle separation and Δ the superposition arm. Throughout this time, the gravitational field is treated as if it neither decoheres the source masses nor itself fluctuates appreciably – gravity is a passive mediator of phase.

Quantum-Geometric Duality (QGD) takes a different view of the gravitational field in this scenario. In the constrained Feynman–Vernon influence functional formulation [17], the linearised Wheeler–DeWitt constraint forces the field to remain in a dressed-coherent state for each matter configuration, and the result is a non-vanishing decoherence rate at first order in G :

$$\Gamma = \frac{Gm^2}{\hbar \Delta} \text{ per particle}, \quad (2)$$

the Diósi–Penrose rate [5, 7] with $d \rightarrow \Delta$ (the relevant length scale for a single mass in superposition is its own arm separation, not the inter-particle distance). A companion paper [16] establishes the same rate from the Hamiltonian-constraint perspective in linearised gravity.

The purpose of this paper is to combine these two ingredients – the standard BMV phase formula and the QGD per-particle decoherence rate – and ask: what does QGD predict for the visibility of the BMV witness?

The answer is sharp. At the BMV witness criterion ($\phi_{\text{BMV}} = \pi/2$, when the entanglement is half-formed), the off-diagonal element of the two-particle reduced density matrix is suppressed by a visibility factor

$$V(\tau_{\text{BMV}}) = \exp\left[-\mathcal{O}(1)(d/\Delta)^3\right]. \quad (3)$$

The cube law arises algebraically: substituting $\Gamma \propto 1/\Delta$ into $V = \exp(-2\Gamma\tau_{\text{BMV}})$ with $\tau_{\text{BMV}} \propto d^3/\Delta^2$ yields the exponent $\propto d^3/\Delta^3$. The mass dependence of Γ and τ_{BMV} cancels exactly, so the suppression depends only on the geometric ratio Δ/d . The numerical prefactor depends only on the arm geometry: $\pi/2$ for parallel-axis arms and π for perpendicular (QGEM “diamond”) arms.

Three features make Eq. (3) an unusually clean experimental discriminator. First, the mass-independence is unique among gravitational decoherence proposals: the Diósi–Penrose rate [5, 7, 6], continuous spontaneous localisation [8], and Penrose’s reduction-time hypothesis [7] all carry explicit mass scaling. Second, the prediction is parameter-free within QGD’s constrained influence functional – there is no analogue of the CSL collapse rate λ to fit. Third, the angular dependence of the leading BMV phase is a Legendre $P_2(\cos\theta)$ in the arm-to-axis angle, with the

parallel-axis ($\theta = 0$) and perpendicular ($\theta = \pi/2$) geometries differing by a factor of two in the leading exponent and by the sign of the next-to-leading correction. Between these two cardinal orientations sits the magic angle $\cos^2 \theta_m = 1/3$ where the dipole term vanishes identically and a higher-multipole quintic law takes over. An experiment that can rotate its arms relative to the inter-particle axis therefore sweeps three qualitatively distinct QGD signatures with the same hardware: cubic suppression with prefactor $\pi/2$ at $\theta = 0$, cubic suppression with prefactor π at $\theta = \pi/2$, and quintic suppression with prefactor $9\pi/7$ at $\theta = \theta_m$.

Section 2 sets up the two-mass dynamics in QGD’s constrained-IF picture. Section 3 derives the leading-order cube law for both cardinal arm geometries. Section 4 computes the exact-phase corrections, identifies a sign-flip in the sub-leading term between them, and notes that the perpendicular formula has no singularity at $\Delta \rightarrow d$ where the parallel formula breaks down. Section 5 extends the analysis to arbitrary arm angle θ , identifies the magic angle, and derives both the leading-order quintic visibility and its exact-phase form. Section 6 gives explicit numerical predictions for QGEM’s proposed parameters in all three geometries and compares with competing theories. Section 7 discusses limitations, falsification routes, and the place of this prediction among other near-term tests.

2 The Two-Mass Dynamics in QGD’s Constrained Influence Functional

2.1 State and field

Two equal masses m , labelled A and B , are each prepared in a balanced spatial superposition,

$$|\psi_A\rangle = \frac{1}{\sqrt{2}}(|L\rangle_A + |R\rangle_A), \quad |\psi_B\rangle = \frac{1}{\sqrt{2}}(|L\rangle_B + |R\rangle_B), \quad (4)$$

with arm separation Δ and centre-to-centre separation d between the two particles. Two arm geometries are of experimental interest:

- *Parallel-axis* – the two arms of each particle lie along the inter-particle axis; the four branch distances are $r_{LL} = r_{RR} = d$ and $r_{LR} = d - \Delta$, $r_{RL} = d + \Delta$ (so the minimum branch distance is $d - \Delta$, requiring $\Delta < d$).
- *Perpendicular-arm* (QGEM “diamond”) – both arms of each particle lie in a plane transverse to the inter-particle axis, displaced by $\pm\Delta/2$ from the central line; the branch distances are $r_{LL} = r_{RR} = d$ and $r_{LR} = r_{RL} = \sqrt{d^2 + \Delta^2}$ (no constraint between Δ and d).

The combined matter state has four computational-basis amplitudes,

$$|\Psi^{\text{matter}}\rangle = \frac{1}{2} \sum_{i,j \in \{L,R\}} |i\rangle_A |j\rangle_B, \quad (5)$$

each of which sources its own gravitational configuration. In the constrained Feynman–Vernon picture [17], integrating out the linearised gravitational degrees of freedom subject to the Wheeler–DeWitt constraint replaces this state with a dressed total state

$$|\Psi(t)\rangle = \frac{1}{2} \sum_{i,j} |i\rangle_A |j\rangle_B e^{-i\phi_{ij}(t)} |\alpha_{ij}\rangle, \quad (6)$$

where $|\alpha_{ij}\rangle$ is the constraint-enforced dressed coherent state of the gravitational field for matter configuration (i, j) and the phase $\phi_{ij}(t)$ is the Newtonian gravitational interaction phase for that configuration:

$$\phi_{ij}(t) = -\frac{E_{ij}t}{\hbar}, \quad E_{ij} = -\frac{Gm^2}{r_{ij}}. \quad (7)$$

2.2 Reduced matter density matrix

Tracing over the gravitational field,

$$\rho_{ij,kl}^{\text{matter}}(t) = \frac{1}{4} \langle \alpha_{kl} | \alpha_{ij} \rangle e^{-i(\phi_{ij}-\phi_{kl})t/\hbar}. \quad (8)$$

Two distinct objects appear:

- the *phase* $\phi_{ij} - \phi_{kl}$ is the standard BMV entanglement phase, generating coherent two-particle correlations as t grows;
- the *dressed-state overlap* $\langle \alpha_{kl} | \alpha_{ij} \rangle$ is the QGD-specific decoherence factor. In the conventional unconstrained Feynman–Vernon analysis this overlap equals unity (the gravitational field is pure noise that adds no off-diagonal damping at first order in G). In the constrained analysis it is a non-trivial Gaussian.

2.3 The QGD decoherence factor

Two coherent states satisfy $|\langle \alpha_a | \alpha_b \rangle| = \exp(-\|\delta\alpha_{ab}\|^2/2)$. In the constrained-IF picture the squared overlap exponent for two configurations differing by displacement of a single mass m over a distance Δ saturates [17] at

$$\|\delta\alpha\|_{\text{sat}}^2 = \xi \ln(d/\ell_P), \quad \xi = \frac{Gm^2}{\hbar c}, \quad (9)$$

with ℓ_P the Planck length. This logarithmic saturation describes the equilibrium of the constraint-dressed field after sound-crossing time d/c . Beyond saturation, the constraint enforces a continued linear growth of the overlap exponent at the per-particle rate

$$\Gamma = \frac{Gm^2}{\hbar \Delta} \quad (10)$$

for each particle whose configuration differs between the two branches. This linear growth is the post-saturation decoherence rate; it reduces to the Diósi–Penrose rate [5, 7] for a single mass with the single-particle separation reinterpreted as the arm separation Δ (the relevant length scale for one mass in superposition). The companion paper [16] derives the same rate from the linearised Hamiltonian-constraint perspective.

Throughout this work we are interested in the regime $\tau \gg d/c$ where saturation has been reached and the linear-rate formula applies; for QGEM-class parameters $d/c \sim 10^{-12}$ s while $\tau_{\text{BMV}} \sim 1$ s, so the saturated regime is overwhelmingly the relevant one.

2.4 The competition

The BMV proposal asks the experimenter to wait until the off-diagonal phase $\phi_{ij} - \phi_{kl}$ has grown large enough to be witnessed. QGD adds a second clock: the dressed-state overlap $|\langle \alpha_{kl} | \alpha_{ij} \rangle|$

decays exponentially during this wait, at a rate set by 2Γ for the channels in which both particles change configuration (e.g., $LL \leftrightarrow RR$). The remainder of the paper computes the visibility of the off-diagonal element at the time the BMV phase witness criterion is reached, in the saturated regime.

3 The Cube Law: Leading-Order Derivation for Both Geometries

3.1 Parallel-axis arms

For arms aligned with the inter-particle axis, the Newtonian interaction energies of the four configurations are $E_{LL} = E_{RR} = -Gm^2/d$ and $E_{LR} = -Gm^2/(d - \Delta)$, $E_{RL} = -Gm^2/(d + \Delta)$. The off-diagonal phase between LL and RR is $\phi_{LL} - \phi_{RR} = 0$ (both diagonal channels share the same energy); the relevant phase for the witness is the one between the two ‘‘crossed’’ channels and the diagonal:

$$\Delta\phi_{\text{par}}^{\text{lead}}(t) = \frac{Gm^2t}{\hbar} \left[\frac{2}{d} - \frac{1}{d - \Delta} - \frac{1}{d + \Delta} \right] = -\frac{Gm^2t}{\hbar} \frac{2\Delta^2}{d(d^2 - \Delta^2)}. \quad (11)$$

The leading-order expansion in Δ/d gives the standard BMV formula:

$$\Delta\phi_{\text{par}}^{\text{lead}}(t) \simeq \frac{2Gm^2\Delta^2t}{\hbar d^3}. \quad (12)$$

The BMV witness criterion is reached when this phase equals $\pi/2$:

$$\tau_{\text{BMV}}^{\text{par,lead}} = \frac{\pi\hbar d^3}{4Gm^2\Delta^2}. \quad (13)$$

The QGD visibility at this time, using the per-particle rate $\Gamma = Gm^2/(\hbar\Delta)$ from Eq. (10), is the dressed-state-overlap modulus

$$V_{\text{lead}}^{\text{par}}(\tau_{\text{BMV}}^{\text{par,lead}}) = \exp(-2\Gamma\tau_{\text{BMV}}^{\text{par,lead}}) = \exp\left(-\frac{\pi}{2}(d/\Delta)^3\right). \quad (14)$$

The factor of two in 2Γ counts the two particles that each contribute coherence loss to the off-diagonal element. The mass dependence cancels: $\Gamma \propto m^2$ and $\tau_{\text{BMV}} \propto 1/m^2$, so the exponent is geometric.

3.2 Perpendicular-arm (QGEM ‘‘diamond’’) geometry

For arms transverse to the inter-particle axis, the four configurations have $E_{LL} = E_{RR} = -Gm^2/d$ and $E_{LR} = E_{RL} = -Gm^2/\sqrt{d^2 + \Delta^2}$. The witness phase difference is

$$\Delta\phi_{\text{perp}}(t) = \frac{2Gm^2t}{\hbar} \left[\frac{1}{d} - \frac{1}{\sqrt{d^2 + \Delta^2}} \right]. \quad (15)$$

At leading order in Δ/d ,

$$\Delta\phi_{\text{perp}}^{\text{lead}}(t) \simeq \frac{Gm^2\Delta^2t}{\hbar d^3}, \quad (16)$$

which is exactly half the parallel-axis leading-order formula (12). The phase therefore takes twice as long to reach the $\pi/2$ witness criterion:

$$\tau_{\text{BMV}}^{\text{perp,lead}} = \frac{\pi \hbar d^3}{2 G m^2 \Delta^2} = 2 \tau_{\text{BMV}}^{\text{par,lead}}. \quad (17)$$

Substituting into the visibility formula gives

$$V_{\text{lead}}^{\text{perp}}(\tau_{\text{BMV}}^{\text{perp,lead}}) = \exp\left(-\pi (d/\Delta)^3\right), \quad (18)$$

twice the suppression coefficient of the parallel-axis case at the same Δ/d .

3.3 The cube law

Both leading-order results take the form

$$V_{\text{lead}}^{(g)} = \exp\left(-c_g (d/\Delta)^3\right), \quad c_{\text{par}} = \frac{\pi}{2}, \quad c_{\text{perp}} = \pi, \quad (19)$$

where the prefactor c_g depends only on the arm geometry g . Observation that the suppression follows a cube law in d/Δ would already be a strong qualitative signature of the underlying mechanism (per-particle rate $\Gamma \propto 1/\Delta$ competing against $\tau_{\text{BMV}} \propto d^3/\Delta^2$). Three features deserve emphasis:

1. *Mass independence.* The mass enters $\Gamma \propto m^2$ and $\tau_{\text{BMV}} \propto 1/m^2$ symmetrically; the exponent in V is purely geometric. This contrasts sharply with the Diósi–Penrose collapse rate [5, 7] (which scales as m^2), continuous spontaneous localisation [8] (which scales as the squared local mass density and thus is parameter-tuned), and Penrose’s reduction-time formula (also $\propto m^2$).
2. *Parameter freedom.* The prefactor c_g is fixed by the geometry alone; QGD has no analogue of the CSL collapse rate λ to fit. Up to the small ambiguity of choosing the witness criterion at $\phi = \pi/2$ versus $\phi = \pi$ (which doubles c_g), the prediction is rigid.
3. *Geometry sensitivity.* Choosing parallel versus perpendicular arms doubles the exponent at the same Δ/d . An experiment that can run with both arm orientations therefore provides an internal cross-check of the mechanism.

The next section examines the corrections that arise when the leading-order BMV phase is not adequate – specifically, when Δ is comparable to d rather than much smaller, which is precisely the QGEM operating regime.

4 Exact-Phase Corrections and the Sign-Flip Between Geometries

The leading-order cube law of Section 3 assumes $\Delta \ll d$. QGEM-class proposals operate at $\Delta \sim d$ (sometimes $\Delta > d$ in the parallel-axis layout, where the formula breaks down), so the sub-leading corrections matter for any quantitative comparison. The corrections arise from using the exact branch energies in Eqs. (11) and (15) rather than their leading-order expansions. Both

geometries develop sub-leading (Δ/d) corrections to the visibility exponent, but with *opposite signs*.

4.1 Parallel-axis exact-phase visibility

Setting the exact phase (11) equal to $\pi/2$ gives

$$\tau_{\text{BMV}}^{\text{par,exact}} = \frac{\pi \hbar d (d^2 - \Delta^2)}{4 G m^2 \Delta^2} = \tau_{\text{BMV}}^{\text{par,lead}} (1 - (\Delta/d)^2), \quad (20)$$

i.e. the exact phase reaches the witness threshold *faster* than the leading-order formula predicts (because the singularity at $\Delta \rightarrow d$ enhances the rate of phase accumulation). QGD therefore has *less* time to decohere the superposition, and the visibility prediction is correspondingly more permissive:

$$V_{\text{exact}}^{\text{par}}(d/\Delta) = \exp\left[-\frac{\pi}{2} \left((d/\Delta)^3 - (d/\Delta)\right)\right]. \quad (21)$$

The ratio to the leading-order prediction is

$$\frac{V_{\text{exact}}^{\text{par}}}{V_{\text{lead}}^{\text{par}}} = \exp\left(+\frac{\pi}{2} (d/\Delta)\right), \quad (22)$$

which at QGEM-relevant $d/\Delta \sim 2$ is $\exp(\pi) \approx 23$ – the leading-order formula *over-suppresses* the QGD visibility prediction by more than an order of magnitude in this regime.

Equation (21) reduces to the leading cube law in the wide-arm limit $\Delta/d \rightarrow 0$ and diverges to $V \rightarrow 1$ as $\Delta \rightarrow d^-$, where the parallel-axis arms touch the partner particle and the linearised Newtonian formula breaks down. The expression is well-defined only for $\Delta < d$.

4.2 Perpendicular-arm exact-phase visibility

Setting Eq. (15) equal to $\pi/2$ and writing $u \equiv \Delta/d$, one finds

$$\tau_{\text{BMV}}^{\text{perp,exact}} = \frac{\pi \hbar d}{4 G m^2} \frac{\sqrt{1+u^2}}{\sqrt{1+u^2}-1}. \quad (23)$$

After rationalising and substituting into the visibility formula (see Appendix A),

$$V_{\text{exact}}^{\text{perp}}(u) = \exp\left[-\frac{\pi}{2} \frac{1+u^2+\sqrt{1+u^2}}{u^3}\right]. \quad (24)$$

Two structural differences from the parallel-axis case:

- *Sign of the sub-leading correction.* Expanding Eq. (24) in u ,

$$-\ln V_{\text{exact}}^{\text{perp}} = \frac{\pi}{u^3} + \frac{3\pi}{4} u^{-1} + \mathcal{O}(u), \quad (25)$$

which differs from the parallel-axis exact result $-\ln V_{\text{exact}}^{\text{par}} = (\pi/2) u^{-3} - (\pi/2) u^{-1} + \dots$ by the *sign* of the u^{-1} correction: parallel-axis is more permissive than its leading-order limit, perpendicular is more suppressive.

- *No singularity at $\Delta = d$.* The perpendicular formula (24) is regular for all positive u , since the perpendicular arms never coincide with the partner particle. At $u = 1$ the suppression

is $V_{\text{exact}}^{\text{perp}}(1) = \exp(-\frac{\pi}{2}(2 + \sqrt{2})) \approx 4.7 \times 10^{-3}$, still well within the noise but far from the $V \rightarrow 1$ blow-up of the parallel-axis formula.

4.3 Discrimination-feasible windows

Defining the experimentally interesting window as $0.05 < V < 0.95$ (the regime where QGD can be distinguished from standard quantum gravity at modest signal-to-noise), the leading cube laws give

$$\begin{aligned} \text{parallel-axis } (V_{\text{lead}}^{\text{par}}) : \quad \Delta/d &\in [0.807, 3.13], \\ \text{perpendicular } (V_{\text{lead}}^{\text{perp}}) : \quad \Delta/d &\in [1.015, 3.94]. \end{aligned} \tag{26}$$

The perpendicular window is shifted to larger Δ/d by a factor ~ 1.26 (the cube root of the prefactor ratio 2). The parallel-axis window straddles the singularity at $\Delta = d$; the realistic parallel-axis discrimination range is therefore $\Delta/d \in [0.807, \sim 0.99]$ – a narrow but accessible regime. The perpendicular window is wider and entirely free of singularities.

4.4 Implication for QGEM design

At the QGEM design point $\Delta = d/2$ (so $u = 0.5$, $d/\Delta = 2$), the four visibility predictions are:

$$\begin{aligned} V_{\text{lead}}^{\text{par}}(2) &= \exp(-4\pi) \approx 3.5 \times 10^{-6}, \\ V_{\text{exact}}^{\text{par}}(2) &= \exp(-3\pi) \approx 8.0 \times 10^{-5}, \\ V_{\text{lead}}^{\text{perp}}(2) &= \exp(-8\pi) \approx 1.3 \times 10^{-11}, \\ V_{\text{exact}}^{\text{perp}}(0.5) &= \exp(-29.76) \approx 1.2 \times 10^{-13}. \end{aligned} \tag{27}$$

All four values are well below detection threshold: at the nominal QGEM operating point the QGD prediction is essentially zero residual visibility. To leave detectable residual visibility, the experiment must move to $\Delta/d \rightarrow 1$ from below in the parallel-axis layout (within $\sim 15\%$ of the singularity) or to $\Delta/d \sim 2.5$ in the perpendicular layout.

Experimental design takeaway: the choice of arm orientation alters the QGD-vs-standard-QG discrimination by approximately five orders of magnitude in V at the QGEM design point. This sensitivity is not present in any standard quantum-gravity treatment of BMV and is therefore a directly testable prediction. The QGEM design literature should specify the intended arm orientation – QGD-vs-QG discrimination depends on it.

5 Oblique-Arm Geometry and the Magic Angle

Sections 3 and 4 treated the two limiting arm orientations $\theta = 0$ (parallel-axis) and $\theta = \pi/2$ (perpendicular). Any physical BMV apparatus can in principle orient its arms at an arbitrary angle θ to the inter-particle axis, with each particle's two arms displaced along the common unit vector $\hat{n} = (\cos \theta, \sin \theta, 0)$. The full one-parameter family interpolating between Eqs. (14) and (18) contains an isolated angle at which the leading-order BMV phase vanishes identically; the QGD prediction at that angle therefore probes a higher multipole of the source distribution and is qualitatively distinct from the cubic suppression at the two limits.

5.1 Branch distances and witness phase at arbitrary θ

For arms tilted by θ relative to the inter-particle axis, the four branch separations are

$$r_{LL} = r_{RR} = d, \quad r_{\pm}(\theta) = \sqrt{d^2 \pm 2d\Delta \cos\theta + \Delta^2}. \quad (28)$$

At $\theta = 0$ these reduce to $r_{\pm} = d \pm \Delta$ (the parallel-axis case of Section 3); at $\theta = \pi/2$ they collapse to $r_+ = r_- = \sqrt{d^2 + \Delta^2}$ (the perpendicular case). The witness phase magnitude is

$$|\Delta\phi(\theta)| = \frac{Gm^2\tau}{\hbar} \left| \frac{2}{d} - \frac{1}{r_+(\theta)} - \frac{1}{r_-(\theta)} \right|, \quad (29)$$

the natural one-parameter extension of Eqs. (11) and (15).

5.2 Multipole expansion and the $P_2(\cos\theta)$ structure

Writing $u \equiv \Delta/d$ and expanding the two inverse branch distances to fourth order in u (the odd-in- u terms cancel between r_+ and r_- because of the \pm displacement) yields

$$|\Delta\phi(\theta)| = \frac{Gm^2\tau}{\hbar d} \left[|1 - 3\cos^2\theta| u^2 + \frac{1}{4} |3 - 30\cos^2\theta + 35\cos^4\theta| u^4 + \mathcal{O}(u^6) \right]. \quad (30)$$

The angular factor in the leading u^2 term is $-2P_2(\cos\theta)$ where P_2 is the second Legendre polynomial: the BMV witness phase is a Newtonian dipole-dipole interaction at large d/Δ . Equation (30) reproduces $|\Delta\phi| = (2Gm^2\tau/\hbar d) u^2$ at $\theta = 0$ (parallel-axis leading-order, Eq. (12)) and $|\Delta\phi| = (Gm^2\tau/\hbar d) u^2$ at $\theta = \pi/2$ (perpendicular, Eq. (16)).

5.3 The magic angle: $\cos^2\theta_m = 1/3$

The leading u^2 coefficient $|1 - 3\cos^2\theta|$ in Eq. (30) vanishes at the unique angle

$$\cos^2\theta_m = \frac{1}{3} \iff \theta_m = \arccos(1/\sqrt{3}) \approx 54.7356^\circ. \quad (31)$$

At θ_m the dipole contribution to the BMV phase is identically zero and the next-order (quadrupole) term takes over. Its coefficient evaluates to

$$\frac{1}{4} |3 - 30/3 + 35/9| = \frac{1}{4} \cdot \frac{28}{9} = \frac{7}{9}, \quad (32)$$

so

$$|\Delta\phi(\theta_m)|_{\text{lead}} = \frac{7}{9} \frac{Gm^2\tau}{\hbar d} (\Delta/d)^4. \quad (33)$$

The witness criterion $|\Delta\phi| = \pi/2$ now fixes

$$\tau_{\text{BMV}}^{\text{magic,lead}} = \frac{9\pi\hbar d^5}{14 Gm^2 \Delta^4}, \quad (34)$$

which is parametrically longer than the cubic-geometry witness times of Eqs. (13) and (17) by a factor $\propto (d/\Delta)^2$.

Combining with the per-particle QGD rate $\Gamma = Gm^2/(\hbar\Delta)$ from Eq. (10) and the two-particle

exponent $2\Gamma\tau$, the visibility at the magic angle is

$$V_{\text{lead}}^{\text{magic}} = \exp\left[-\frac{9\pi}{7} (d/\Delta)^5\right], \quad (35)$$

a *quintic* suppression law, qualitatively distinct from the cubic $(\Delta/d)^3$ scaling of the two cardinal geometries. No CSL or Diósi–Penrose model can reproduce this scaling because in those models the decoherence rate depends only on the local mass distribution and is blind to the arm orientation; the $(d/\Delta)^5$ law is a direct fingerprint of the geometric structure of the Newtonian phase that QGD competes against.

The general one-parameter family follows from Eq. (30) at leading order:

$$V_{\text{lead}}(\theta) = \exp\left[-\frac{\pi (d/\Delta)^3}{|1 - 3 \cos^2 \theta|}\right], \quad (36)$$

which reproduces $V_{\text{lead}}^{\text{par}}$ at $\theta = 0$ (denominator $|1 - 3| = 2$), $V_{\text{lead}}^{\text{perp}}$ at $\theta = \pi/2$ (denominator $|1 - 0| = 1$), and diverges as $\theta \rightarrow \theta_m$ where the dipole vanishes and the leading-order approximation must be replaced by Eq. (35). The exponent of Eq. (36) is strictly monotone in θ on each of the two intervals $[0, \theta_m)$ and $(\theta_m, \pi/2]$: the magic angle is the worst-case orientation for residual visibility (best-case for QGD discrimination), with the cardinal geometries straddling it.

5.4 Exact-phase magic-angle visibility

Equation (35) retains only the leading u^4 term of the phase. The exact-phase calculation at θ_m proceeds without expansion. Defining the dimensionless function

$$g_m(u) \equiv \left| 2 - (1 + (2/\sqrt{3})u + u^2)^{-1/2} - (1 - (2/\sqrt{3})u + u^2)^{-1/2} \right|, \quad (37)$$

the exact magic-angle witness phase is $|\Delta\phi(\theta_m)| = (Gm^2\tau/\hbar d) g_m(u)$. Expanding $g_m(u)$ analytically gives the closed series (see Appendix B)

$$g_m(u) = \frac{7}{9} u^4 \left[1 - \frac{4}{7} u^2 + \mathcal{O}(u^4) \right], \quad (38)$$

in which the u^2 (dipole) coefficient is identically zero — the defining property of the magic angle — and the u^4 coefficient is exactly 7/9, recovering Eq. (33). The first non-trivial correction enters at u^6 with relative coefficient $-4/7$.

Setting $|\Delta\phi(\theta_m)| = \pi/2$ and combining with $\Gamma = Gm^2/(\hbar\Delta)$ as before, the exact-phase visibility exponent is

$$-\ln V_{\text{exact}}^{\text{magic}} = \frac{\pi}{u g_m(u)}, \quad V_{\text{exact}}^{\text{magic}} = \exp\left[-\pi/(u g_m(u))\right]. \quad (39)$$

Substituting Eq. (38) yields the analytic expansion

$$-\ln V_{\text{exact}}^{\text{magic}} = \frac{9\pi}{7} (d/\Delta)^5 + \frac{36\pi}{49} (d/\Delta)^3 + \mathcal{O}(d/\Delta). \quad (40)$$

The leading term reproduces Eq. (35). The decisive qualitative feature is the *sign* of the

sub-leading correction: the $36\pi/49$ coefficient is positive, so

$$V_{\text{exact}}^{\text{magic}} < V_{\text{lead}}^{\text{magic}} \quad \text{for all } u > 0. \quad (41)$$

The exact phase makes the magic-angle suppression *stronger* than the leading quintic. This is the same direction as the perpendicular geometry (Eq. (24) versus Eq. (18)) and the opposite direction to the parallel-axis geometry (Eq. (21) versus Eq. (14)), completing the exact-vs-leading trilogy across the three computed geometries.

5.5 Numerical regime and discrimination window

Representative values of the magic-angle exact-phase exponent, mass-independent:

$$\begin{aligned} -\ln V_{\text{exact}}^{\text{magic}}(u = 0.5) &= 153.4 \quad (V \approx 2.4 \times 10^{-67}), \\ -\ln V_{\text{exact}}^{\text{magic}}(u = 1.0) &= 8.99 \quad (V \approx 1.24 \times 10^{-4}), \\ -\ln V_{\text{exact}}^{\text{magic}}(u = 2.0) &= 1.54 \quad (V \approx 0.21). \end{aligned} \quad (42)$$

The corresponding leading-order quintic exponents are 129.3, 4.04, and 0.126; the exact-versus-leading gap is 24.2, 4.95, and 1.41 respectively, with the same sign at all three points (exact more suppressive than leading). The two-term cubic correction $(36\pi/49)(d/\Delta)^3$ of Eq. (40) reproduces the gap qualitatively (18.5, 2.31, 0.29 at the same u) but underestimates it: higher-order corrections in d/Δ contribute at the $\sim 20\text{--}80\%$ level once $u \lesssim 1$. The closed-form Eq. (39) is therefore the only reliable evaluation in that regime.

The discrimination-feasible window $0.05 < V < 0.95$, computed numerically from Eq. (39), is

$$\text{magic-angle (exact):} \quad \Delta/d \in [1.465, 31.6]. \quad (43)$$

The exact-phase perpendicular window is the comparable $[1.322, 31.6]$, sharing the same upper edge ≈ 31.6 because both exponents scale as $\pi/(2u)$ for $u \rightarrow \infty$ and intersect the $V = 0.95$ threshold at the same point. The parallel exact-phase window $[0.664, 0.984]$ is roughly two orders of magnitude narrower than either of the other two (constrained from above by the formula's singularity at $\Delta = d$, where the parallel-axis arms touch the partner particle). The magic-angle and perpendicular geometries therefore both retain QGD-distinguishable residual visibility across a span of arm separations roughly twenty times wider than the parallel-axis layout permits.

5.6 Experimental and theoretical takeaways

Equations (36) and (39) extend the QGD prediction of Section 3 from a binary choice between two cardinal arm orientations to a continuous one-parameter family with three qualitatively distinct regimes:

- *Cubic regime*, $|\cos^2 \theta - 1/3|$ bounded away from zero: visibility scales as $\exp(-c(\theta)(d/\Delta)^3)$ with $c(\theta) = \pi/|1 - 3\cos^2 \theta|$, interpolating smoothly between $\pi/2$ (parallel) and π (perpendicular).
- *Quintic regime*, $\theta = \theta_m$: visibility scales as $\exp(-(9\pi/7)(d/\Delta)^5)$, with the dipole BMV phase contribution exactly cancelled by the geometric condition $\cos^2 \theta = 1/3$ and the

quadrupole taking over.

- *Transition region*, $|\cos^2 \theta - 1/3| \lesssim u^2$: the leading-order expansion breaks down and the exact formula $V = \exp(-\pi/(u g(u, \theta)))$ must be used; the discrimination window widens monotonically as $\theta \rightarrow \theta_m$.

For experimental design the implications are direct. An apparatus that can rotate its arms relative to the inter-particle axis can sweep three distinct QGD signatures in one platform. Demonstrating the $\cos^2 \theta_m = 1/3$ phase null is itself a non-trivial test of Newtonian gravity at mesoscopic separations: any theory whose phase response retains a non-zero dipole component at the magic angle (any modification that breaks P_2 structure) would manifest as residual BMV signal where QGD and Newtonian gravity predict the quintic-suppressed null. A combined measurement at $\theta = 0$, $\theta = \pi/2$, and $\theta = \theta_m$ tests three independent QGD predictions with the same hardware: cubic suppression with prefactor $\pi/2$, cubic suppression with prefactor π , and quintic suppression with prefactor $9\pi/7$.

Mass independence is preserved at every angle. At each θ the QGD rate $\Gamma \propto m^2$ and the witness time $\tau_{\text{BMV}}(\theta) \propto 1/m^2$ cancel exactly in the exponent, so $V(\theta)$ depends only on Δ/d and θ for any choice of arm orientation.

6 Numerical Predictions and Comparison with Competing Theories

6.1 Forecast for QGEM-class parameters

The QGEM proposal [3] considers two equal nanodiamond masses ($m \sim 10^{-17}$ kg each) in Stern–Gerlach interferometers, with arm separations $\Delta \sim 100\text{--}250 \mu\text{m}$ and inter-particle separations $d \sim 100\text{--}250 \mu\text{m}$. The QGD prediction depends only on the dimensionless ratio Δ/d and on the arm-to-axis angle, so we tabulate the visibility forecasts as a function of Δ/d in Table 1, separately for the three computed geometries. All entries are computed analytically from Eqs. (14), (18), (21), (24), (35), and (39); the same numbers are returned by the companion code referenced in Section 6.4.

The QGEM nominal design $\Delta \approx d$ is therefore in a regime where the parallel-axis QGD prediction breaks down formally and both the perpendicular and magic-angle predictions give strong but still observable suppression ($V_{\text{exact}}^{\text{perp}} \sim 5 \times 10^{-3}$, $V_{\text{exact}}^{\text{magic}} \sim 10^{-4}$). To work in the discrimination-feasible window the experiment can either:

- adopt the parallel-axis layout with $\Delta/d \in [0.85, 0.95]$ – a narrow regime within $\sim 15\%$ of the formula singularity, where QGD predicts $V_{\text{exact}}^{\text{par}} \in [0.49, 0.84]$ (i.e. 16–51% visibility loss);
- adopt the perpendicular-axis layout with $\Delta/d \in [1.32, 31.6]$ from Eq. (24), where QGD predicts $V_{\text{exact}}^{\text{perp}}$ varying smoothly between 0.05 and 0.95 with a clear cube-law signature in the suppression as Δ/d varies; or
- adopt the magic-angle layout with $\Delta/d \in [1.46, 31.6]$ from Eq. (39), where QGD predicts $V_{\text{exact}}^{\text{magic}}$ following the quintic law $\propto \exp(-(9\pi/7)(d/\Delta)^5 - (36\pi/49)(d/\Delta)^3)$ – a different functional form whose suppression decays more slowly with Δ/d .

Table 1: QGD-predicted residual visibility at the BMV witness criterion $\phi = \pi/2$, as a function of the geometric ratio Δ/d , in the three computed geometries. Values are mass-independent. Entries marked “–” lie outside the formula’s domain of validity (parallel-axis singularity at $\Delta = d$). “ $\theta = \theta_m$ ” is the magic angle $\arccos(1/\sqrt{3}) \approx 54.7^\circ$; “QG std.” is the standard quantum-gravity prediction (no decoherence: $V = 1$). Entries below $\sim 10^{-50}$ are effectively zero.

Δ/d	$\theta = 0$ (parallel)		$\theta = \pi/2$ (perp.)		$\theta = \theta_m$ (magic)		QG std.
	lead	exact	lead	exact	lead	exact	
0.30	5.4×10^{-26}	1.0×10^{-23}	2.9×10^{-51}	1.2×10^{-54}	~ 0	~ 0	1
0.50	3.5×10^{-6}	8.1×10^{-5}	1.2×10^{-11}	1.2×10^{-13}	7.3×10^{-57}	2.4×10^{-67}	1
0.70	1.0×10^{-2}	0.097	1.0×10^{-4}	4.1×10^{-6}	3.7×10^{-11}	8.2×10^{-16}	1
0.85	0.077	0.49	6.0×10^{-3}	4.3×10^{-4}	1.1×10^{-4}	1.1×10^{-7}	1
0.95	0.16	0.84	0.026	2.4×10^{-3}	5.4×10^{-3}	2.2×10^{-5}	1
1.00	0.21	–	0.043	4.7×10^{-3}	1.8×10^{-2}	1.2×10^{-4}	1
1.25	–	–	0.20	0.035	0.27	1.1×10^{-2}	1
1.50	–	–	0.39	0.10	0.59	5.9×10^{-2}	1
2.00	–	–	0.68	0.24	0.88	0.21	1
3.00	–	–	0.89	0.47	0.98	0.46	1

6.2 Comparison with competing theories

Five candidate predictions for the BMV/QGEM visibility, evaluated at the same parameters:

Table 2: Predictions for the BMV visibility from competing theoretical frameworks, at the QGEM nominal design point. “ V ” is the residual modulus of the off-diagonal density-matrix element at the witness criterion.

Framework	V at QGEM design	Distinctive scaling
Standard QG (no decoherence)	≈ 1	none
QGD (constrained-IF, this work)	$\exp(-c_g(\theta)(d/\Delta)^n)$	mass-indep.; $n = 3$ or 5 (magic)
Diósi–Penrose [5, 7]	$\exp(-\Gamma_{DP} \tau_{\text{BMV}})$	$\propto m^2$, smearing σ_0
CSL [8]	$\exp(-\lambda_{\text{CSL}} \tau_{\text{BMV}} N)$	free λ , density-dependent
Pikovski et al. [9]	$\exp(-\Gamma_{\text{PB}} \tau_{\text{BMV}})$	$\propto T \times$ (composition)
Classical (LOCC) gravity	1 but $C_{AB} = 0$	no entanglement at all

The QGD prediction is the only one in this list that is both *mass-independent* and *parameter-free*. The Diósi–Penrose rate is set by m^2 and depends on a regularisation length scale σ_0 associated with the mass smearing. CSL has the free collapse rate λ and the localisation length r_C . The Pikovski et al. time-dilation decoherence depends on the composition (number of internal degrees of freedom) and ambient temperature. Only QGD predicts a sharp $(d/\Delta)^3$ cubic law in the cardinal arm orientations and a $(d/\Delta)^5$ quintic law at the magic angle, with no fitted parameters and no mass dependence. None of the collapse models has any analogue of the magic-angle quintic law: the decoherence rate in CSL/D-P/Pikovski depends on the local matter distribution and is therefore blind to the orientation of the superposition arms relative to the inter-particle axis.

6.3 Falsification routes

The experimental discrimination is binary at the QGEM design point:

- If QGEM observes the standard BMV entanglement amplitude ($V \approx 1$) at $\Delta/d \sim 1$ in any

of the three computed arm orientations, QGD's constrained-IF picture is *falsified*.

- If QGEM observes a cube-law suppression of the visibility, with the geometry-dependent prefactor (parallel: $\pi/2$, perpendicular: π) and no mass dependence across a parameter sweep, QGD is selected over Diósi–Penrose, CSL, and Pikovski-class models in a single experiment.
- If QGEM observes a quintic-law suppression at the magic angle $\theta_m = \arccos(1/\sqrt{3})$ with prefactor $9\pi/7$, this in addition rules out any decoherence mechanism whose rate is independent of the arm orientation relative to the inter-particle axis – a constraint that no collapse model can satisfy.

Failure modes that would not falsify QGD specifically: any visibility loss with explicit m^2 scaling (consistent with any collapse model), or any temperature dependence (consistent with Pikovski et al.). Both can be controlled by the standard experimental sweeps that QGEM is already planning.

6.4 Companion code

The numerical values in Tables 1 and 2 are computed by an open-source Python script (released alongside this paper) implementing the six QGD visibility formulae (cardinal-cubic in both arm orientations, exact-phase corrections in both, magic-angle quintic, exact magic-angle) and the standard BMV phase formula. The unit-test suite covers: the cube-law scaling, mass-independence, the leading-to-exact crossover in both cardinal geometries, the $P_2(\cos\theta)$ angular dependence, the magic-angle dipole-phase null, the quintic-law scaling with prefactor $9\pi/7$, the exact magic-angle closed form, and the analytic-series accuracy below the numerical seam $u = 10^{-2}$. All tests pass to machine precision.

7 Discussion and Conclusions

7.1 Summary of results

We have derived the QGD prediction for the BMV/QGEM gravitationally-induced-entanglement visibility, in the full one-parameter family of arm orientations parametrised by the angle θ between the superposition axis and the inter-particle axis. In the two cardinal geometries ($\theta = 0$ parallel, $\theta = \pi/2$ perpendicular) and at the magic angle ($\cos^2\theta_m = 1/3$), the visibility formulae are

$$\begin{aligned}
 V_{\text{lead}}^{\text{par}} &= \exp\left[-\frac{\pi}{2}(d/\Delta)^3\right], \\
 V_{\text{exact}}^{\text{par}} &= \exp\left[-\frac{\pi}{2}((d/\Delta)^3 - (d/\Delta))\right], \\
 V_{\text{lead}}^{\text{perp}} &= \exp\left[-\pi(d/\Delta)^3\right], \\
 V_{\text{exact}}^{\text{perp}} &= \exp\left[-\frac{\pi}{2}(1 + u^2 + \sqrt{1 + u^2})/u^3\right], \quad u \equiv \Delta/d, \\
 V_{\text{lead}}^{\text{magic}} &= \exp\left[-\frac{9\pi}{7}(d/\Delta)^5\right], \\
 V_{\text{exact}}^{\text{magic}} &= \exp\left[-\pi/(u g_m(u))\right], \quad g_m(u) \text{ from Eq. (37)},
 \end{aligned} \tag{44}$$

together with the leading-order angular interpolation $V_{\text{lead}}(\theta) = \exp[-\pi(d/\Delta)^3/|1 - 3\cos^2\theta|]$ between the cardinal geometries (Eq. (36)), all of them mass-independent and free of fitted

parameters. The leading prefactor doubles between the cardinal cubic geometries; the sign of the next-to-leading correction flips between them; and at the magic angle the dipole BMV phase vanishes identically and the suppression becomes quintic with prefactor $9\pi/7$, exact-phase strictly stronger than leading.

7.2 Origin of the cube law

Two physical scales control the BMV experiment. The phase scale is the gravitational interaction energy difference between branches, $\Delta E \sim Gm^2\Delta^2/d^3$ (for $\Delta \ll d$ in either geometry, up to the geometry-dependent prefactor). The decoherence scale is the per-particle dressed-state-overlap rate $\Gamma = Gm^2/(\hbar\Delta)$ derived from the constrained Feynman–Vernon influence functional [17, 16]. The witness time is $\tau \sim \hbar/\Delta E \sim \hbar d^3/(Gm^2\Delta^2)$. Their product

$$\Gamma \tau \sim \frac{Gm^2}{\hbar\Delta} \cdot \frac{\hbar d^3}{Gm^2\Delta^2} = (d/\Delta)^3 \quad (45)$$

gives the cube law and the cancellation of the mass. The cube power is therefore an algebraic consequence of the two timescales QGD identifies in the problem; it is not a free choice.

7.3 What is, and is not, derived

The cube-law and quintic-law forms of the visibility, the leading and exact-phase prefactors, the magic-angle identification, and the $P_2(\cos\theta)$ angular interpolation are derived from QGD’s constrained Feynman–Vernon influence functional plus the standard Newtonian phase formula expanded in the Legendre generating function for the branch distances (Appendix B). The dressed-coherent-state overlap structure of the gravitational field, the saturation of the overlap exponent at $\xi \ln(d/\ell_P)$, and the post-saturation linear-rate $\Gamma = Gm^2/(\hbar\Delta)$ are inputs from the companion paper [17] on the constrained influence functional.

Two results are not derived but conjectured:

- *Validity beyond linearised gravity.* The derivation uses the Newtonian limit of linearised general relativity throughout. At QGEM-class parameters this is well within the regime of validity (graviton wavelength \gg particle separation), but extension to higher mass scales would require post-Newtonian corrections to both the BMV phase formula and the constrained-IF dressed-state overlap.
- *Effect of imperfect coherent-state alignment.* We have assumed that the gravitational field for each matter configuration is exactly in the constraint-enforced dressed coherent state. Departures from exact alignment (due to, e.g., decoherence of the gravitational field by external sources) would weaken the QGD prediction; the magnitude of this effect has not been estimated.

7.4 Limitations and caveats

Saturation regime: the linear-rate formula $\Gamma \propto 1/\Delta$ applies for $\tau > d/c$ (the gravitational sound-crossing time). At QGEM scales $d/c \sim 10^{-12}$ s while $\tau_{\text{BMV}} \sim 1$ s, so the saturated regime dominates by twelve orders of magnitude. For interferometers with $\tau \lesssim d/c$ the rate is suppressed and the prediction does not apply.

Visibility versus concurrence: we have computed the visibility of the off-diagonal element of the two-particle reduced density matrix, not the full two-qubit concurrence. For the BMV witness criterion, the off-diagonal modulus dominates the concurrence monotonically; the cube law therefore translates directly to a concurrence suppression. A full treatment of the four-by-four matter density matrix is straightforward and would refine the numerical prefactors by $\mathcal{O}(1)$ but does not alter the cube law or the mass-independence.

Environmental decoherence: any non-gravitational decoherence (residual gas, blackbody radiation, internal degrees of freedom of the nanodiamond) [14] acts in parallel and adds to the QGD suppression. The QGEM background-decoherence budget [3] of $\Gamma_{\text{env}} \tau \lesssim 1$ is comparable to the QGD suppression at $\Delta/d \sim 1$ in the parallel-axis layout, so distinguishing the gravitational signature from the background requires the parameter sweep across Δ/d that the cube law predicts.

7.5 Relation to other master-equation derivations

Several master equations for gravitationally induced decoherence have been derived from quantising linearised gravity perturbatively. Anastopoulos and Hu [10] and Blencowe [11] obtain a Lindblad equation by tracing over the transverse-traceless graviton bath; the resulting rate is $\mathcal{O}(G^2)$ in the matter–graviton coupling and depends on the temperature of the graviton bath. The QGD rate (10) is $\mathcal{O}(G)$ in the same coupling because the constrained influence functional retains the Hamiltonian-constraint sector of the gravitational field, which the perturbative trace integrates out [17, 16]. At QGEM-class parameters the QGD rate exceeds the Anastopoulos–Hu/Blencowe rate by many orders of magnitude (the BMV cube-law prediction would be invisible in either of the perturbative treatments), so an observation of cube-law suppression at the QGEM design point would also discriminate between QGD and these perturbative master equations.

7.6 Place among other near-term tests of QGD

This prediction sits alongside two other QGD-specific experimental signatures, summarised in Table 3.

Table 3: *QGD-specific experimental discriminators among near-term proposals.*

Test	Observable	Discriminates QGD from
Single-mass decoherence [16]	$\Gamma_{\text{dec}} = GM^2/(\hbar d)$	no decoherence
Single-mass entanglement decay [18]	$C_{AB}(t) = C_{AB}(0)e^{-GM^2t/\hbar d}$	no decoherence
<i>This work (BMV/QGEM)</i>	$V \propto \exp(-(d/\Delta)^3)$	no decoherence, CSL, Diósi–Penrose, Pikovski

The single-mass tests [16, 18] discriminate QGD from theories that predict no gravitational decoherence at all; they cannot distinguish QGD from collapse models that also predict mass-dependent decoherence. The BMV test is the only near-term proposal whose mass-independent cube law is incompatible with all extant collapse models simultaneously.

7.7 Conclusions

QGD predicts that gravitational decoherence of the source masses suppresses the BMV/QGEM entanglement visibility by a mass-independent factor whose form depends on the arm-to-axis angle θ . In the two cardinal cubic geometries the suppression is $V \sim \exp(-(d/\Delta)^3)$ with prefactor $\pi/2$ (parallel) or π (perpendicular); at the magic angle $\cos^2 \theta_m = 1/3$ the dipole BMV phase vanishes identically and the suppression becomes quintic $V \sim \exp(-(9\pi/7)(d/\Delta)^5)$. Including the exact phase, the parallel-axis formula is more permissive than its leading-order limit while the perpendicular and magic-angle formulae are more suppressive; the magic-angle exact-phase exponent acquires a positive $(36\pi/49)(d/\Delta)^3$ cubic correction, completing the exact-versus-leading trilogy across the three computed geometries. At the QGEM design point $\Delta = d/2$ all three predictions are essentially zero residual visibility; the discrimination-feasible regime requires either parallel-axis $\Delta \sim 0.85d-0.95d$, perpendicular $\Delta \in [1.3d, 31.6d]$, or magic-angle $\Delta \in [1.5d, 31.6d]$. Observation of standard BMV entanglement at QGEM-class parameters falsifies QGD's constrained-IF picture; observation of mass-independent cubic suppression in the cardinal geometries together with quintic suppression at the magic angle selects QGD over all extant collapse models in a single experiment, since no collapse mechanism depends on the orientation of the superposition arms relative to the inter-particle axis.

Appendix

A Algebra of the Exact-Phase Visibility Formulae

This appendix records the algebraic steps from the exact branch energies of Section 2 to the exact-phase visibility formulae of Section 4. Throughout, write $u \equiv \Delta/d$.

A.1 Parallel-axis arms: from energy to visibility

The four-branch interaction energies for the parallel-axis layout are

$$E_{LL} = E_{RR} = -\frac{Gm^2}{d}, \quad E_{LR} = -\frac{Gm^2}{d-\Delta}, \quad E_{RL} = -\frac{Gm^2}{d+\Delta}. \quad (46)$$

The witness phase difference at time τ is

$$\Delta\phi_{\text{par}}(\tau) = \frac{\tau}{\hbar}(E_{LL} + E_{RR} - E_{LR} - E_{RL}) = -\frac{Gm^2\tau}{\hbar} \left[\frac{2}{d} - \frac{1}{d-\Delta} - \frac{1}{d+\Delta} \right]. \quad (47)$$

Algebraically,

$$\frac{2}{d} - \frac{1}{d-\Delta} - \frac{1}{d+\Delta} = \frac{2(d^2 - \Delta^2) - d(d+\Delta) - d(d-\Delta)}{d(d^2 - \Delta^2)} = -\frac{2\Delta^2}{d(d^2 - \Delta^2)}, \quad (48)$$

so

$$|\Delta\phi_{\text{par}}(\tau)| = \frac{2Gm^2\Delta^2\tau}{\hbar d(d^2 - \Delta^2)}. \quad (49)$$

Setting this equal to $\pi/2$ (the BMV witness criterion) gives

$$\tau_{\text{BMV}}^{\text{par,exact}} = \frac{\pi\hbar d(d^2 - \Delta^2)}{4Gm^2\Delta^2} = \frac{\pi\hbar}{4Gm^2} \frac{d^3}{\Delta^2} (1 - u^2). \quad (50)$$

The leading-order limit ($u \rightarrow 0$) gives $\tau_{\text{BMV}}^{\text{par,lead}} = \pi\hbar d^3/(4Gm^2\Delta^2)$ as in Eq. (13).

The QGD visibility uses the per-particle rate $\Gamma = Gm^2/(\hbar\Delta)$ and the two-particle exponent $2\Gamma\tau$:

$$\begin{aligned} -\ln V_{\text{exact}}^{\text{par}} &= 2\Gamma\tau_{\text{BMV}}^{\text{par,exact}} = 2 \frac{Gm^2}{\hbar\Delta} \cdot \frac{\pi\hbar d(d^2 - \Delta^2)}{4Gm^2\Delta^2} \\ &= \frac{\pi}{2} \frac{d(d^2 - \Delta^2)}{\Delta^3} = \frac{\pi}{2} \left[(d/\Delta)^3 - (d/\Delta) \right], \end{aligned} \quad (51)$$

which is Eq. (21). The leading cube-law term recovers Eq. (14); the sub-leading $-(d/\Delta)$ term carries a positive sign in the visibility (i.e. less suppression than the leading-order formula), because the exact phase reaches the witness threshold faster than its leading-order approximation predicts.

A.2 Perpendicular-arm geometry: from energy to visibility

The four-branch interaction energies for the perpendicular-arm layout are

$$E_{LL} = E_{RR} = -\frac{Gm^2}{d}, \quad E_{LR} = E_{RL} = -\frac{Gm^2}{\sqrt{d^2 + \Delta^2}}. \quad (52)$$

The witness phase difference is

$$|\Delta\phi_{\text{perp}}(\tau)| = \frac{2Gm^2\tau}{\hbar} \left[\frac{1}{d} - \frac{1}{\sqrt{d^2 + \Delta^2}} \right] = \frac{2Gm^2\tau}{\hbar d} \left[1 - \frac{1}{\sqrt{1+u^2}} \right]. \quad (53)$$

Setting this equal to $\pi/2$ gives

$$\tau_{\text{BMV}}^{\text{perp,exact}} = \frac{\pi\hbar d}{4Gm^2} \frac{1}{1 - 1/\sqrt{1+u^2}} = \frac{\pi\hbar d}{4Gm^2} \frac{\sqrt{1+u^2}}{\sqrt{1+u^2} - 1}. \quad (54)$$

Rationalising the denominator,

$$\frac{\sqrt{1+u^2}}{\sqrt{1+u^2} - 1} = \frac{\sqrt{1+u^2}(\sqrt{1+u^2} + 1)}{(1+u^2) - 1} = \frac{1+u^2 + \sqrt{1+u^2}}{u^2}. \quad (55)$$

Hence

$$\tau_{\text{BMV}}^{\text{perp,exact}} = \frac{\pi\hbar d}{4Gm^2} \frac{1+u^2 + \sqrt{1+u^2}}{u^2}. \quad (56)$$

The two-particle visibility exponent is then

$$\begin{aligned} -\ln V_{\text{exact}}^{\text{perp}} &= 2\Gamma \tau_{\text{BMV}}^{\text{perp,exact}} = 2 \frac{Gm^2}{\hbar\Delta} \cdot \frac{\pi\hbar d}{4Gm^2} \frac{1+u^2 + \sqrt{1+u^2}}{u^2} \\ &= \frac{\pi}{2} \frac{d}{\Delta} \frac{1+u^2 + \sqrt{1+u^2}}{u^2} = \frac{\pi}{2} \frac{1+u^2 + \sqrt{1+u^2}}{u^3}, \end{aligned} \quad (57)$$

which is Eq. (24). In the wide-arm limit $u \rightarrow 0$, $1+u^2 + \sqrt{1+u^2} \rightarrow 2 + \mathcal{O}(u^2)$, so $-\ln V_{\text{exact}}^{\text{perp}} \rightarrow \pi/u^3$, recovering the leading cube law $V_{\text{lead}}^{\text{perp}} = \exp(-\pi(d/\Delta)^3)$ of Eq. (18).

A.3 Sub-leading expansion

Expanding both exact-phase exponents in u :

$$\begin{aligned} -\ln V_{\text{exact}}^{\text{par}} &= \frac{\pi}{2} [u^{-3} - u^{-1}] = \frac{\pi}{2} u^{-3} - \frac{\pi}{2} u^{-1}, \\ -\ln V_{\text{exact}}^{\text{perp}} &= \frac{\pi}{2} \frac{1+u^2 + \sqrt{1+u^2}}{u^3} \\ &= \frac{\pi}{2} u^{-3} (2 + \frac{3}{2}u^2 + \mathcal{O}(u^4)) = \pi u^{-3} + \frac{3\pi}{4} u^{-1} + \mathcal{O}(u). \end{aligned} \quad (58)$$

The leading-order coefficients are $\pi/2$ (parallel) and π (perpendicular), giving the factor-of-two prefactor difference of Eq. (19). The sub-leading u^{-1} coefficients are $-\pi/2$ (parallel) and $+3\pi/4$ (perpendicular), with opposite signs: the parallel-axis exact formula is more permissive than its leading-order limit by $\exp(\pi u/2)$, the perpendicular-axis exact formula is more suppressive by $\exp(3\pi u/4)$.

B Algebra of the Oblique-Arm and Magic-Angle Formulae

This appendix records the algebraic steps from the angular branch distances of Eq. (28) to the multipole expansion of Eq. (30), and from there to the exact magic-angle series (38). Throughout, $u \equiv \Delta/d$ and the displacement vector is $\hat{n} = (\cos\theta, \sin\theta, 0)$.

B.1 Legendre generating function for the branch distances

The two non-trivial branch distances at general θ are $r_{\pm}(\theta) = d\sqrt{1 \pm 2u \cos \theta + u^2}$. Their reciprocals admit the Legendre generating function expansion

$$\frac{1}{r_{\pm}(\theta)} = \frac{1}{d} \sum_{\ell \geq 0} (\mp 1)^{\ell} P_{\ell}(\cos \theta) u^{\ell}, \quad (59)$$

valid for $u < 1$. Summing the two branches, the odd- ℓ terms cancel,

$$\frac{1}{r_{+}(\theta)} + \frac{1}{r_{-}(\theta)} = \frac{2}{d} \sum_{\ell \geq 0} P_{2\ell}(\cos \theta) u^{2\ell}. \quad (60)$$

The witness phase combination of Eq. (29) is therefore

$$\frac{2}{d} - \frac{1}{r_{+}} - \frac{1}{r_{-}} = -\frac{2}{d} \sum_{\ell \geq 1} P_{2\ell}(\cos \theta) u^{2\ell}. \quad (61)$$

Inserting the standard Legendre values

$$P_2(\cos \theta) = \frac{1}{2}(3 \cos^2 \theta - 1), \quad P_4(\cos \theta) = \frac{1}{8}(35 \cos^4 \theta - 30 \cos^2 \theta + 3), \quad (62)$$

and taking the modulus gives, with $|\Delta\phi| = (Gm^2\tau/\hbar)|2/d - 1/r_{+} - 1/r_{-}|$,

$$\begin{aligned} |\Delta\phi(\theta)| &= \frac{Gm^2\tau}{\hbar d} \left[2|P_2|u^2 + 2|P_4|u^4 + \mathcal{O}(u^6) \right] \\ &= \frac{Gm^2\tau}{\hbar d} \left[|1 - 3 \cos^2 \theta|u^2 + \frac{1}{4}|3 - 30 \cos^2 \theta + 35 \cos^4 \theta|u^4 + \mathcal{O}(u^6) \right], \end{aligned} \quad (63)$$

recovering Eq. (30). The leading u^2 angular factor is exactly $-2P_2(\cos \theta)$ in modulus; its zero $\cos^2 \theta_m = 1/3$ defines the magic angle of Eq. (31).

B.2 Magic-angle phase function $g_m(u)$

At $\theta = \theta_m$, $\cos \theta_m = 1/\sqrt{3}$, so $2 \cos \theta_m = 2/\sqrt{3}$. Define

$$x_{\pm}(u) \equiv (1 \pm \frac{2}{\sqrt{3}}u + u^2)^{-1/2} = \sum_{\ell \geq 0} (\mp 1)^{\ell} P_{\ell}(1/\sqrt{3}) u^{\ell}, \quad (64)$$

so that $g_m(u) = |2 - x_{+}(u) - x_{-}(u)|$ as in Eq. (37). From the parity-summed series,

$$g_m(u) = \left| -2 \sum_{\ell \geq 1} P_{2\ell}(1/\sqrt{3}) u^{2\ell} \right|. \quad (65)$$

The relevant Legendre values are

$$\begin{aligned} P_2(1/\sqrt{3}) &= \frac{1}{2} \left(3 \cdot \frac{1}{3} - 1 \right) = 0, \\ P_4(1/\sqrt{3}) &= \frac{1}{8} \left(\frac{35}{9} - \frac{30}{3} + 3 \right) = \frac{1}{8} \cdot \frac{35-90+27}{9} = -\frac{7}{18}, \\ P_6(1/\sqrt{3}) &= \frac{1}{16} \left(\frac{231}{27} - \frac{315}{9} + \frac{105}{3} - 5 \right) = \frac{1}{16} \cdot \frac{231-945+945-135}{27} = \frac{2}{9}. \end{aligned} \quad (66)$$

The u^2 coefficient of g_m vanishes (this is the magic-angle defining property $P_2(1/\sqrt{3}) = 0$); the u^4 coefficient is $-2P_4(1/\sqrt{3}) = 7/9$; the u^6 coefficient is $-2P_6(1/\sqrt{3}) = -4/9$. Hence

$$g_m(u) = \frac{7}{9}u^4 - \frac{4}{9}u^6 + \mathcal{O}(u^8) = \frac{7}{9}u^4 \left[1 - \frac{4}{7}u^2 + \mathcal{O}(u^4) \right], \quad (67)$$

which is Eq. (38). The leading coefficient is exactly 7/9, reproducing Eq. (33) and hence $V_{\text{lead}}^{\text{magic}} = \exp[-(9\pi/7)(d/\Delta)^5]$ of Eq. (35). The first correction enters at relative order u^2 with negative coefficient $-4/7$.

The choice of absolute value in Eq. (37) requires the two-term series to be sign-checked for the range $u \in [0, 1]$. Equation (67) is positive in this range ($1 - (4/7)u^2 > 1 - 4/7 = 3/7 > 0$), and direct numerical evaluation of $|2 - x_+(u) - x_-(u)|$ confirms positivity at the few percent level for all $u \in [0, 1]$ checked. Numerically at $u = 0.5$: the two-term truncation gives $g_m \approx (7/9)(0.0625)(1 - 1/7) = 0.0417$; the exact closed form Eq. (37) gives $g_m(0.5) = 0.04097$, agreeing to better than 2×10^{-2} relative accuracy. At $u = 0.1$ the agreement is sub-permille; at $u = 0.01$ it is sub-microsecond — the series is the appropriate evaluation below the seam $u_{\text{seam}} = 10^{-2}$ where the closed form Eq. (37) loses thirteen significant digits to catastrophic cancellation ($x_+ + x_- \rightarrow 2$).

B.3 Exact-phase visibility expansion

Substituting Eq. (67) into Eq. (39),

$$-\ln V_{\text{exact}}^{\text{magic}} = \frac{\pi}{u g_m(u)} = \frac{\pi}{u} \cdot \frac{9}{7u^4} \cdot \left[1 - \frac{4}{7}u^2 \right]^{-1}. \quad (68)$$

Expanding the bracket as a geometric series in $(4/7)u^2$ and keeping the first two terms,

$$-\ln V_{\text{exact}}^{\text{magic}} = \frac{9\pi}{7u^5} \left[1 + \frac{4}{7}u^2 + \mathcal{O}(u^4) \right] = \frac{9\pi}{7} (d/\Delta)^5 + \frac{36\pi}{49} (d/\Delta)^3 + \mathcal{O}(d/\Delta), \quad (69)$$

which is Eq. (40). The sub-leading cubic correction has the *positive* sign ($+36\pi/49$) and therefore strengthens the suppression relative to the pure-quintic leading term, as stated in Eq. (41).

B.4 Cardinal-geometry consistency check

The Legendre generating-function form (61) reproduces the parallel-axis and perpendicular results of Sections 3 and 4 at $\theta = 0$ and $\theta = \pi/2$ respectively. At $\theta = 0$, $\cos \theta = 1$, and $1/r_{\pm}(0) = (1/d)(1 \mp u)^{-1} = (1/d) \sum_{\ell} (\mp 1)^{\ell} u^{\ell}$, recovering Eq. (11). At $\theta = \pi/2$, $\cos \theta = 0$ and only the $u^{2\ell}$ terms with $P_{2\ell}(0)$ non-zero contribute. Eq. (59) gives $\sum_{\ell} P_{2\ell}(0) u^{2\ell} = (1 + u^2)^{-1/2}$ (the standard Taylor series at $x = 0$), so

$$\frac{1}{r_+(\pi/2)} + \frac{1}{r_-(\pi/2)} = \frac{2}{d} (1 + u^2)^{-1/2}, \quad (70)$$

recovering Eq. (15). Both cardinal geometries are therefore reproduced from the same Legendre identity, with the magic angle representing the unique zero of the leading multipole.

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