The Past Hypothesis from Holographic Dark Energy

A Quantum-Geometric Resolution of Penrose's Weyl Curvature Asymmetry

Paper O of the Quantum-Geometric Duality Series

Marc Sperzel*

Independent Researcher
MSci Physics, King's College London

Abstract

Three apparently independent puzzles in fundamental physics—the Past Hypothesis (why was the early universe in a low-entropy state?), Penrose's Weyl Curvature Asymmetry (why does the Weyl tensor vanish at the Big Bang but diverge at black-hole singularities?), and the cosmological constant problem (why is $\rho_{\rm DE}/\rho_{\rm Pl} \sim 10^{-122}$?)—are shown, within the Quantum-Geometric Duality (QGD) framework, to be *a single mystery viewed through three lenses.* The connecting tissue is a single identity, exact in the de Sitter attractor of any FRW cosmology,

$$\rho_{\rm DE}^{\infty}\, S_{\rm dS}^{\infty} \;=\; \frac{3c^7}{8\,G^2\,\hbar},$$

relating the asymptotic dark-energy density to the asymptotic Bekenstein–Hawking entropy of the cosmic horizon. The identity itself is a tautology—both sides are functions of H_∞ alone, and the relation follows in three algebraic lines from the Friedmann equation and the Bekenstein–Hawking formula. Its content is not the algebra but the *coupling* of this relation to the Past Hypothesis and to Penrose's Weyl Curvature Hypothesis. As a consequence, the Past Hypothesis is implemented automatically by the holographic bound $S(t) \leq S_{\rm dS}(t)$ applied to a shrinking past horizon ($S_{\rm dS} \to 0$ as $t \to 0$); Penrose's 10^{123} is reinterpreted as the *capacity* of the de Sitter horizon rather than a probability of fine-tuning; and the smallness of $\rho_{\rm DE}$ in Planck units is the reciprocal of that capacity. The Weyl asymmetry is recovered as a consequence of the direction of cosmic decoherence: the Big Bang lies upstream of all branching ($\langle C\rangle = 0$, $\mathrm{Var}(C) = 0$) while black-hole singularities lie downstream (large $\langle C\rangle$, large $\mathrm{Var}(C)$); we state this as a heuristic operator-level conjecture motivated by three complementary readings of one underlying argument (holographic, kinematic-vacuum, no-environment). The framework introduces no new axioms and no new free parameters: the single number $\alpha = 3\Omega_{\rm DE}/(8\pi) \approx 0.082$ controlling Paper B's holographic dark energy density is identified, via the identity above, with the cosmic entropy budget in Planck units. We are explicit about the limits of the construction: α is fitted, not derived; saturation of the generalized second law is assumed and operates at the level of the holographic mode count, not the per-mode rate; the operator Weyl Curvature Hypothesis is heuristic, with several technical preconditions unaddressed; and the framework predicts no observable that discriminates against ΛCDM at current precision. What is gained is conceptual: one mystery dressed in three vocabularies.

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*ORCID: 0009-0000-6252-3155. Email: `me@marcsperzel.com`

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1 Introduction

Three of the most stubborn puzzles in modern fundamental physics carry different names, live in different sub-fields, and are usually attacked with different tools. They are, however, knotted together more tightly than the literature usually acknowledges.

The first puzzle is the *Past Hypothesis* [1, 2, 3]. Statistical mechanics is time-symmetric: nothing in the microscopic dynamics distinguishes the future from the past. Yet the macroscopic world is overwhelmingly time-asymmetric. The standard resolution is to add, by hand, an initial-condition postulate: at some early cosmic epoch the universe was in a microstate of extraordinarily low coarse-grained entropy. From that initial condition, the Second Law follows by ordinary statistical reasoning. The postulate is necessary but feels arbitrary: nothing in the dynamics demands it, and its plausibility is sometimes argued only on anthropic grounds.

The second puzzle is *Penrose's Weyl Curvature Asymmetry*, an especially sharp form of the Past Hypothesis cast in geometric language [3, 4]. The Weyl tensor $C_{\mu\nu\rho\sigma}$ encodes the tidal, gravitational-wave part of curvature—the part not algebraically determined by local matter. Penrose observed that at the Big Bang, the Weyl tensor was extraordinarily close to zero (no primordial gravitational radiation, no clumpiness), while at black-hole singularities the Weyl tensor diverges. Both endpoints are generic outcomes of gravitational dynamics; both are singularities; yet they differ qualitatively. Penrose's Weyl Curvature Hypothesis (WCH) elevates this empirical observation into a constraint: the boundary condition on cosmological initial data is $C_{\mu\nu\rho\sigma} \to 0$. He estimates the resulting fine-tuning at roughly $\exp(-10^{123})$.

The third puzzle is the *cosmological constant problem* [5]. Naive zero-point estimates of vacuum energy yield $\rho_{\rm vac} \sim \rho_{\rm Pl} \sim 10^{113}$ J/m^3, while observations [6, 7, 8, 9] require $\rho_{\rm DE} \sim 10^{-9}$ J/m^3. The ratio is $\sim 10^{-122}$. The numerical coincidence with Penrose's 10^{123} is striking and is not, on its face, a coincidence at all.

These three puzzles are usually presented separately. The Past Hypothesis is a problem in the foundations of statistical mechanics; the Weyl asymmetry is a problem in classical general relativity; the cosmological constant problem is a problem in quantum field theory. Different communities, different conferences, different review articles.

The thesis of this paper, developed within the framework of Quantum-Geometric Duality (QGD) [10, 11, 12, 13], is that these three puzzles are the same puzzle wearing three different sets of clothes. They are joined by a single identity, exact in the late-time de Sitter attractor of any FRW cosmology, that relates the asymptotic dark-energy density to the asymptotic horizon entropy:

$$\rho_{\rm DE}^{\infty}\, S_{\rm dS}^{\infty} \;=\; \frac{3c^7}{8\,G^2\,\hbar}. \tag{1}$$

The right-hand side is a Planck-scale constant of order unity in natural units. The left-hand side is the product of two quantities that, in any de Sitter universe, are inverses of each other up to that constant: dark-energy density and cosmic entropy capacity. Reducing the cosmological constant means raising the entropy capacity; raising the entropy capacity means lowering the dark-energy density. They are not independent.

We stress immediately what (1) is and what it is not. Both sides are functions of the single quantity H_∞; substituting the de Sitter Friedmann equation into the Bekenstein–Hawking formula gives the identity in three algebraic lines. The relation is therefore a re-labeling, not a reduction. The novelty of this paper is not the identity itself—versions of which appear in the holographic-dark-energy literature dating back to Banks [14] and the Fischler–Susskind N-bound proposal [15]—but the *coupling* of this trivial identity to the Past Hypothesis and to Penrose's Weyl Curvature Hypothesis within a single framework. The unification is conceptual: one mystery dressed in three vocabularies, not three puzzles reduced to a deeper principle.

From this identity, three corollaries follow immediately. (i) The cosmic entropy budget $S_{\mathrm{dS}}(t)$ is finite, computable, and vanishes at $t \to 0$. The Past Hypothesis becomes a kinematic consequence of the holographic bound: $S(t) \leq S_{\mathrm{dS}}(t)$ and the right-hand side goes to zero in the past. No fine-tuning. (ii) Penrose's 10^{123} is the value of S_{dS}^{∞} in nats; it is the *capacity* of the cosmic horizon, not a probability assigned to the actual Big Bang state. The Big Bang was low-entropy not because that was improbable but because there was no room. (iii) The smallness of $\rho_{\mathrm{DE}}/\rho_{\mathrm{Pl}}$ is structurally tied to the largeness of S_{dS}^{∞} via the identity (1); the cosmological constant problem and the Past Hypothesis are reciprocal facets of one mystery.

The Weyl Curvature Asymmetry then follows from the QGD account of cosmic decoherence [16, 17]. Within a single cosmic aeon, the Big Bang is upstream of all branching processes that produce semiclassical geometry; the Weyl tensor, as an operator, has neither a sharp expectation value nor a sharp variance at the past boundary. Black-hole singularities, by contrast, are downstream of decoherence; the Weyl operator has been measured into definite, high-curvature branches by interaction with the environment. The asymmetry within a single aeon is the direction of cosmic decoherence—no additional postulate beyond the existing QGD axioms.

We will be careful throughout to distinguish what is established from what is conjectured. The central identity (1) is an elementary consequence of the Friedmann equation and the Bekenstein–Hawking formula: it is exact. The holographic implementation of the Past Hypothesis is rigorous modulo the assumption of generalized-second-law (GSL) saturation, which QGD shares with Paper B [11]. The operator-level WCH is a heuristic conjecture supported by three complementary readings of one underlying argument (holographic, kinematic-vacuum, no-environment); we state it as such, flag the operator-theoretic well-definedness issues, and do not claim a proof. The cosmological-constant "solution" is, in honest framing, a unification rather than a derivation: $\alpha = 3\Omega_{\mathrm{DE}}/(8\pi) \approx 0.082$ remains fitted to observation, but it is now seen to control *simultaneously* the dark-energy density and the cosmic entropy capacity. One free parameter, not two. No new observable distinct from ΛCDM at current precision; the gain is conceptual.

The paper is organized as follows. Section 2 introduces the QGD holographic capacity and tabulates the cosmic entropy budget across cosmic history. Section 3 derives the central identity (1) and presents the numerical check. Section 4 develops the holographic implementation of the Past Hypothesis and shows why $\Lambda > 0$ is required by the framework's two-sided-budget formulation. Section 5 addresses Penrose's Weyl asymmetry as a consequence of the direction of cosmic decoherence and states the operator-level WCH conjecture. Section 6 surveys what is testable now, what may become testable, and the honest negative results (CMB non-Gaussianity is unobservable). Section 7 compares with alternative approaches—Penrose's conformal cyclic cosmology, Bousso's covariant entropy bound, Hartle–Hawking, multiverse/anthropic. Section 8 summarizes. Appendices collect numerical tables and a detailed walk through cosmic entropy history.

Conventions. We work in geometric and Planck units interchangeably, with conversions given explicitly where ambiguity may arise. The Hubble parameter $H(t) = \dot{a}/a$ is positive throughout the cosmic history we consider. The Bekenstein–Hawking entropy is $S = A/(4\ell_P^2)$

with $\ell_P = \sqrt{\hbar G/c^3}$ the Planck length. We use $S_{\mathrm{dS}}(t) \equiv \pi c^5/(H(t)^2 \hbar G)$ for the de Sitter horizon entropy at instantaneous Hubble rate $H(t)$. The asymptotic value $S_{\mathrm{dS}}^{\infty} \equiv S_{\mathrm{dS}}(H_\infty)$ is evaluated at the late-time Hubble rate $H_\infty = H_0\sqrt{\Omega_{\mathrm{DE}}}$.

2 Holographic Capacity in QGD

QGD takes as one of its six foundational axioms the holographic bound: the entropy of any region is bounded above by one quarter of its bounding area in Planck units [18, 19, 20]. We restate the bound in the cosmological setting and use it to define the central object of this paper, the cosmic entropy capacity $S_{\mathrm{dS}}(t)$.

Axiom 2.1 (Holographic Bound, QGD Axiom VI)**.** For any spacelike region $\mathcal{R}$ with bounding area $A(\mathcal{R})$, the von Neumann entropy of the quantum state supported on $\mathcal{R}$ satisfies

$$S(\mathcal{R}) \;\leq\; \frac{A(\mathcal{R})}{4\,\ell_P^2}, \qquad \ell_P \equiv \sqrt{\hbar G/c^3}. \tag{2}$$

Applied to the de Sitter horizon of a cosmological observer at instantaneous Hubble rate $H(t)$, the bound becomes a definite finite number [21]. The Hubble area is $A_H = 4\pi R_H^2$ with $R_H = c/H$; the Bekenstein–Hawking entropy is

$$S_{\mathrm{dS}}(t) \;=\; \frac{A_H(t)}{4\,\ell_P^2} \;=\; \frac{4\pi c^2/H(t)^2}{4\,\hbar G/c^3} \;=\; \frac{\pi c^5}{H(t)^2\,\hbar\,G}. \tag{3}$$

This is the *capacity*: the maximum entropy any subsystem in the observer's causal patch can carry, by Axiom 2.1. It is observer-dependent (each comoving observer sees their own horizon), but in a homogeneous FRW universe all comoving observers share the same value of $S_{\mathrm{dS}}(t)$.

2.1 The cosmic entropy budget across history

Equation (3) can be evaluated at any cosmic epoch. The Hubble rate $H(t)$ tracks the dominant energy component: $H \propto a^{-2}$ in radiation, $H \propto a^{-3/2}$ in matter, $H \to H_\infty$ in the de Sitter attractor. The capacity therefore grows monotonically from its smallest value at the Planck epoch to a finite asymptotic value S_{dS}^{∞} set by the dark-energy density [22].

Table 1 shows S_{dS} at six representative epochs. The growth from approximately π nats at the Planck time to approximately 3×10^{122} nats in the asymptotic future is the structural fact that this paper hinges on.

Table 1. *Cosmic horizon entropy* $S_{\rm dS} = \pi c^5/(H^2\hbar G)$ *at representative epochs. The Hubble rates are standard estimates from Friedmann evolution with* Λ*CDM parameters; the asymptotic value uses* $H_\infty = H_0\sqrt{\Omega_{\rm DE}}$ *with the Planck 2018 values* $H_0 = 67.4$ *km/s/Mpc,* $\Omega_{\rm DE} = 0.689$.

Epoch	H (s^{-1})	$S_{\rm dS}$ (nats)	Comment
Planck time	$\sim 1/t_P \approx 1.85\times10^{43}$	$\sim \pi$	one Planck area
End of inflation	$\sim 10^{36}$	$\sim 10^{17}$	$H_{\rm inf} \sim 10^{14}$ GeV
Matter–radiation eq.	$\sim 10^{-12}$	$\sim 10^{88}$	$z_{\rm eq} \approx 3400$
Recombination	$\sim 5\times10^{-14}$	$\sim 10^{91}$	$z_{\rm LSS} \approx 1100$
Today	2.18×10^{-18}	2.27×10^{122}	$H_0 = 67.4$ km/s/Mpc (Planck 2018)
de Sitter asymptote	1.81×10^{-18}	3.29×10^{122}	$H_\infty = H_0\sqrt{\Omega_{\rm DE}}$

The numerical values span 122 orders of magnitude. We single out two features that will reappear repeatedly:

(i) $S_{\rm dS}(t) \to 0$ as $t \to 0$. This is forced by $H \to \infty$ in any FRW cosmology with a past singularity; it is the kinematic ingredient of the Past Hypothesis in Section 4.

(ii) $S_{\rm dS}(t) \to S_{\rm dS}^\infty$ as $t \to \infty$ with $S_{\rm dS}^\infty$ finite, provided $\Lambda > 0$. Without dark energy ($\Lambda \le 0$), the asymptotic capacity is either undefined (recollapse) or infinite (Minkowski limit). The finiteness of $S_{\rm dS}^\infty$ is exactly the structural role of dark energy in our story.

2.2 Saturation: from bound to budget

A bound is not the same as a budget. The holographic inequality (2) states only that $S(t) \le S_{\rm dS}(t)$; it does not require equality. A skeptic might ask why the actual cosmic entropy should track the bound rather than sitting comfortably below it.

QGD inherits this question from Paper B [11], where the generalized second law (GSL) saturation hypothesis is assumed in the same form: the cosmic-horizon entropy saturates in the de Sitter attractor, and the dark-energy density is set by that saturation. The same hypothesis appears here. We make it explicit:

Assumption 2.2 (GSL Saturation)**.** In any FRW cosmology with a de Sitter attractor, the actual entropy of the observer's causal patch tracks the holographic bound: $S(t) \sim S_{\rm dS}(t)$ at every epoch, with saturation $S(t)/S_{\rm dS}(t) \to 1$ in the asymptotic future.

This assumption is non-trivial. Within QGD it is motivated by the cosmic gravitational decoherence rate analyzed in Paper N [17] and the arrow-of-time analysis [16]: per-mode gravitational decoherence at rate $g(1)\,H/(4\pi)$ with $g(1) = 1 - (2/\pi)\,\mathrm{Si}(1) \approx 0.398$, when multiplied by the holographically-allowed number of horizon modes, gives a total cosmic entropy production rate that scales as H. The cosmic rate is derived in Paper N within a controlled approximation that assumes decoherent-histories branching, holographic finiteness of the mode count, and free/Gaussian factorization of mode contributions; see Paper N for the explicit list of working assumptions. We use Assumption 2.2 as a working hypothesis in what follows.

The per-mode rate alone does not saturate the bound. A cautionary clarification is needed at this point. The per-mode decoherence rate $g(1)\,H/(4\pi)$, integrated over the age of the universe ($N_{\text{efolds}} \sim 140$ in standard ΛCDM plus inflation), gives only $\sim N_{\text{efolds}}/(10\pi) \sim 4$ nats of entropy production *per mode*—far short of the asymptotic capacity $S_{\text{dS}}^{\infty} \sim 10^{122}$. The 122-order shortfall is closed not by the rate but by the *mode count*: each Hubble volume contains S_{dS} holographically distinguishable horizon modes, and summing the per-mode contribution over this count multiplies the result by precisely S_{dS} to give a total entropy production that tracks the capacity. The bound is saturated by the holographic mode count multiplying through, not by the per-mode rate alone. We state this explicitly because the per-mode rate by itself, integrated naively, does not saturate the bound; Assumption 2.2 requires the mode count to do the structural work. Within QGD this is consistent (the holographic bound supplies exactly the mode count needed), but the gap between the per-mode rate and the total budget should not be glossed over. Appendix B works the numerics explicitly.

2.3 Reading the budget

With Assumption 2.2 in place, the budget $S_{\text{dS}}(t)$ is not just an upper bound but the actual cosmic entropy at epoch t. Three readings will recur in the rest of the paper:

1. The Past Hypothesis (Section 4): $S_{\text{dS}}(t) \to 0$ in the past makes low initial entropy automatic.
2. Penrose's 10^{123} (Section 5): the asymptotic value $S_{\text{dS}}^{\infty} \approx 3 \times 10^{122}$ is the cosmic entropy *capacity*, not a probability.
3. The cosmological constant problem (Section 3): the inverse relation between ρ_{DE} and S_{dS}^{∞} is exact in the attractor.

All three readings are consequences of Axiom 2.1 applied to the cosmic horizon, plus Assumption 2.2. The next section makes the third reading explicit through the central identity.

3 The Dark-Energy / Horizon-Entropy Identity

We now derive the central identity of this paper. The argument is short: it is the conjunction of the holographic horizon entropy (3) with Paper B's holographic dark-energy formula [11] in the de Sitter attractor.

3.1 Paper B in one paragraph

Paper B of the QGD series derives, from Axiom 2.1 applied to the cosmic event horizon plus the GSL saturation hypothesis, the holographic dark-energy density

$$\rho_{\text{DE}} = \alpha \frac{c^2 H^2}{G}, \qquad \alpha = \frac{3\,\Omega_{\text{DE}}}{8\pi}. \tag{4}$$

The dimensionless coefficient α is fitted to the observed dark-energy fraction $\Omega_{\text{DE}} = 0.689 \pm 0.006$ [8], giving $\alpha = 0.082 \pm 0.001$. In the de Sitter attractor $\Omega_{\text{DE}} \to 1$ and $\alpha_{\infty} = 3/(8\pi)$. The framework does not derive the value of α; that is the residual mystery, equivalent to the magnitude of Λ.

In the late-time attractor, dark energy dominates the Friedmann equation:

$$H_\infty^2 = \frac{8\pi G}{3c^2}\,\rho_{\rm DE}^\infty, \tag{5}$$

which is the standard de Sitter expansion rate set by a cosmological constant $\Lambda \equiv 8\pi G\rho_{\rm DE}^\infty/c^4$.

3.2 The central identity

Substituting (5) into the horizon entropy formula (3) at $H = H_\infty$:

$$S_{\rm dS}^\infty = \frac{\pi c^5}{H_\infty^2\,\hbar G} = \frac{\pi c^5}{(8\pi G\,\rho_{\rm DE}^\infty/(3c^2))\,\hbar G} \tag{6}$$

$$= \frac{3\,c^7}{8\,G^2\,\hbar\,\rho_{\rm DE}^\infty}. \tag{7}$$

Rearranging gives the central identity:

Dark-energy/horizon-entropy identity. In the de Sitter attractor of any FRW cosmology,

$$\boxed{\rho_{\rm DE}^\infty\,S_{\rm dS}^\infty = \frac{3\,c^7}{8\,G^2\,\hbar}} \tag{8}$$

The right-hand side is a Planck-scale constant. Numerically,

$$\frac{3\,c^7}{8\,G^2\,\hbar} \approx 1.74\times 10^{113}\ \mathrm{J/m^3}, \tag{9}$$

which is the dark-energy density at one nat of cosmic entropy capacity.

The identity is a tautology—and that is fine. The derivation above occupies three algebraic lines: substitute the de Sitter Friedmann equation into the Bekenstein–Hawking formula. Both sides of (8) are functions of the single variable H_∞, so the identity is a re-labeling of one quantity, not a relation between two independent ones. We do not claim more for it than this. The substance of the paper lies not in the identity itself but in what it lets us do: *couple* the dark-energy density to the Past Hypothesis and to Penrose's Weyl Curvature Hypothesis within a single framework. Versions of (8) appear in the holographic-dark-energy literature dating to Banks [14], Fischler–Susskind [15], and Cohen–Kaplan–Nelson [23], and most explicitly in Li's holographic dark energy model [24]; we are not the first to write it down. We claim originality only for the conceptual coupling developed in Sections 4–5.

The identity has the structure of a conservation law in Planck units: dark-energy density and cosmic horizon entropy are inversely related through a constant of order unity in natural units. In terms of the Planck density $\rho_{\rm Pl} \equiv c^7/(\hbar G^2)$, the right-hand side of (8) is $(3/8)\,\rho_{\rm Pl}$, and the identity becomes

$$\frac{\rho_{\rm DE}^\infty}{\rho_{\rm Pl}} = \frac{3/8}{S_{\rm dS}^\infty}, \tag{10}$$

which we will use in Section 3.4 to reframe the cosmological constant problem.

3.3 Numerical check

Substituting today's measured values into the identity provides a non-trivial consistency check. We separate *today's* product from the *asymptotic* product because today is not yet at the de Sitter attractor.

Today. Using Planck 2018 values ($H_0 = 67.4$ km/s/Mpc, $\Omega_{\rm DE} = 0.689$, $\rho_{\rm crit}^{\rm today} = 7.67 \times 10^{-10}$ J/m^3), the observed dark-energy density is

$$\rho_{\rm DE}^{\rm today} = \Omega_{\rm DE}\, \rho_{\rm crit}^{\rm today} \approx 0.689 \times 7.67 \times 10^{-10} \text{ J/m}^3 \approx 5.28 \times 10^{-10} \text{ J/m}^3.$$

Today's horizon entropy is $S_{\rm dS}^{\rm today} \approx 2.27 \times 10^{122}$ (Table 1). The product is

$$\rho_{\rm DE}^{\rm today}\, S_{\rm dS}^{\rm today} \approx 1.20 \times 10^{113} \text{ J/m}^3.$$

This differs from the Planck-scale constant (9) by a factor $\Omega_{\rm DE} = 0.689$, as expected: the identity (8) holds only at the attractor where $\Omega_{\rm DE} \to 1$, while today is still in the matter-DE transition.

Asymptote. The dark-energy density does not dilute, so $\rho_{\rm DE}^{\infty} = \rho_{\rm DE}^{\rm today}$. The asymptotic horizon entropy is $S_{\rm dS}^{\infty} = S_{\rm dS}^{\rm today}/\Omega_{\rm DE} \approx 3.29 \times 10^{122}$. The product is

$$\rho_{\rm DE}^{\infty}\, S_{\rm dS}^{\infty} \approx 1.74 \times 10^{113} \text{ J/m}^3,$$

which matches the Planck-scale constant (9) to better than 1% (saturating exactly in the limit). The residual is at the level of present-day cosmological-parameter uncertainties.

This is a satisfying consistency check but not a new prediction: both sides of (8) are computed from the same set of cosmological parameters. The identity is a structural relation, not an empirical test.

3.4 The cosmological constant problem reframed

The cosmological constant problem [5] asks: why is $\rho_{\rm DE}/\rho_{\rm Pl} \sim 10^{-122}$ rather than $O(1)$ as quantum field theory naively predicts? The identity (10) provides an inversion of the question:

$$\frac{\rho_{\rm DE}^{\infty}}{\rho_{\rm Pl}} = \frac{3/8}{S_{\rm dS}^{\infty}} \sim \frac{1}{S_{\rm dS}^{\infty}}. \tag{11}$$

If $S_{\rm dS}^{\infty} \sim 10^{122}$, then $\rho_{\rm DE}^{\infty}/\rho_{\rm Pl} \sim 10^{-122}$ *automatically.* The hierarchy in the dark-energy density is the inverse of the hierarchy in the cosmic entropy capacity. The two are not separate numbers requiring separate explanations; they are reciprocal in Planck units.

This does not solve the cosmological constant problem. The residual question remains: why does the cosmic horizon hold 10^{122} nats and not 10^{60} or 10^{200}? But the question is now *one* question, not two. Asking "why is $\rho_{\rm DE}$ so small" is the same as asking "why is $S_{\rm dS}^{\infty}$ so large." Penrose's 10^{123} and the cosmological constant hierarchy are no longer independent puzzles to be explained separately; they are the same puzzle in two languages.

3.5 What we have shown and what we have not

The identity (8) is exact in any FRW cosmology that approaches a de Sitter attractor. It is an immediate consequence of the Friedmann equation and the Bekenstein–Hawking entropy formula. It involves no new physics beyond what is in Paper B and what was in the original de Sitter thermodynamics [21]. Its content is conceptual rather than empirical: it organizes two apparently independent dimensional puzzles into one.

What we have *not* shown is a microscopic derivation of either $\rho_{\mathrm{DE}}^{\infty}$ or S_{dS}^{∞} from first principles. The identity links them; it does not derive either separately. The mystery is preserved in dimensionless form: one number, $\alpha = 3\Omega_{\mathrm{DE}}/(8\pi)$ or equivalently $1/S_{\mathrm{dS}}^{\infty}$ in Planck units, is fitted to observation. Whether this number can be derived from a deeper principle is an open question that this paper does not answer.

4 Implementing the Past Hypothesis

The Past Hypothesis [1, 2] is normally stated as an external postulate: the early universe was in a microstate of extraordinarily low coarse-grained entropy, and this is the boundary condition that selects the thermodynamic arrow of time from the otherwise time-symmetric microscopic dynamics. Within QGD the postulate is unnecessary. The holographic bound (Axiom 2.1), applied to the shrinking cosmic horizon, implements the Past Hypothesis automatically.

4.1 The bound forces the initial condition

The argument is two lines. The holographic bound states

$$S(t) \;\le\; S_{\mathrm{dS}}(t) \;=\; \frac{\pi c^5}{H(t)^2\,\hbar G}. \tag{12}$$

In any FRW cosmology with a past singularity, $H(t) \to \infty$ as $t \to 0$. Therefore

$$S_{\mathrm{dS}}(t) \;\longrightarrow\; 0 \qquad \text{as } t \to 0, \tag{13}$$

and consequently

$$S(t) \;\longrightarrow\; 0 \qquad \text{as } t \to 0. \tag{14}$$

Low initial entropy is not fine-tuned; it is forced. There is no room for high entropy in the early universe because there is no horizon area to support it.

4.2 Why the bound is tracked: saturation revisited

A reader might object that (14) is only an upper bound. Even if $S_{\mathrm{dS}}(t) \to 0$, the actual entropy $S(t)$ could in principle be far below the bound at every epoch. Why does $S(t)$ track $S_{\mathrm{dS}}(t)$ rather than sitting comfortably below it?

The answer is Assumption 2.2 (GSL saturation) introduced in Section 2. Within QGD, saturation is motivated by the cosmic gravitational decoherence rate $\Gamma_{\mathrm{total}} = g(1)\,H/(4\pi) \approx H/(10\pi)$ derived in Paper N [17]: the universal cosmic decoherence rate scales as H, so cosmic entropy production keeps pace with cosmic capacity growth. Saturation is then *generic* rather than fine-tuned.

The Past Hypothesis as we state it in QGD is therefore not just "$S(0) \approx 0$" (which is trivial from the bound) but the stronger statement that $S(t)$ tracks $S_{\mathrm{dS}}(t)$ throughout cosmic history. Entropy production keeps pace with capacity growth. The arrow of time is the direction of this growth.

4.3 Why $\Lambda > 0$ is required by the framework's formulation

A sharper claim than "the Past Hypothesis is forced" is available within the same framework: the QGD reading of the Past Hypothesis, as a *finite, two-sided* statement about the cosmic entropy budget, requires dark energy to be positive ($\Lambda > 0$). This is a self-consistency condition on the particular formulation QGD has chosen, not a universal necessity. Other cosmologies admit their own Past Hypothesis formulations (we say which below); the present subsection establishes that the *specific* formulation we use here—bounded above by a finite asymptotic capacity, bounded below by a vanishing past capacity—closes only under $\Lambda > 0$. We consider the three possibilities in turn.

Case (i): $\Lambda < 0$ (AdS-like cosmology). A universe with negative cosmological constant eventually recollapses. There is no asymptotic cosmic horizon and no well-defined S_{dS}^{∞}. The Past Hypothesis "low initial entropy relative to the maximum" has no late boundary against which the "initial" value is measured. The cosmic entropy budget grows from zero in the past, but its maximum is reached only in the recollapse phase, which is not a stable asymptote but a turnaround.

Case (ii): $\Lambda = 0$ (Minkowski asymptote). A universe with vanishing cosmological constant has $H(t) \to 0$ as $t \to \infty$, so $S_{\mathrm{dS}}(t) \to \infty$. The holographic bound becomes vacuous in the future. The Past Hypothesis is one-sided: bounded below in the past ($S \to 0$), unbounded above in the future. There is no closed accounting of total cosmic entropy capacity.

Case (iii): $\Lambda > 0$ (de Sitter attractor; our case). The Hubble rate asymptotes to a finite $H_\infty > 0$, and the cosmic horizon entropy asymptotes to a finite S_{dS}^{∞}. The Past Hypothesis is two-sided: $S(t)$ grows from ~ 0 at the Planck epoch to S_{dS}^{∞} in the asymptotic future. Both endpoints are finite. The arrow of time has a beginning (where there is no room for entropy) and an asymptotic destination (where the capacity is saturated).

Only case (iii) gives the specific Past Hypothesis we have been developing: a closed, finite, two-sided budget. We turn the reasoning around:

> **Past-Hypothesis framework requirement on dark energy.** For the Past Hypothesis to have the closed two-sided implementation as a holographic-capacity statement developed in this paper, the universe must have $\Lambda > 0$. Dark energy is the structural condition that closes the QGD cosmic entropy budget under this formulation; without it, the Past Hypothesis would have to be re-stated differently (one-sided in the Minkowski case; turnaround-bounded in the AdS case).

Honest scope. This is a requirement of *our formulation*, not a universal theorem. Cosmologies with $\Lambda \leq 0$ are not incompatible with a Past Hypothesis as such; they admit alternative readings:

- In a $\Lambda < 0$ AdS-like recollapsing universe, the Past Hypothesis can be stated as "entropy near the Big Bang is small relative to the entropy at the Big Crunch." This is two-sided but bounded by a turnaround rather than an asymptote.
- In a $\Lambda = 0$ Minkowski-asymptote universe, the Past Hypothesis remains well-defined in its one-sided form (entropy grows monotonically from a low initial value), with the gradient defined by the time-derivative of an unbounded-above capacity. This is how the conventional non-cosmological Boltzmann/Penrose formulations operate.

QGD specifically chose the two-sided finite-budget reading because dark energy supplies a natural upper bound and because the holographic identity (8) pairs cleanly with it. Having made this choice, the framework then requires $\Lambda > 0$ for internal consistency. The argument selects the sign of Λ *conditional on the formulation*; it does not prove that any sensible Past Hypothesis demands $\Lambda > 0$. The magnitude $|\Lambda|$ remains fitted to observation in all three cases.

4.4 Status of the implementation

Let us be precise about what has and has not been shown.

Established. (i) The holographic bound (Axiom 2.1) plus the FRW cosmic expansion implies $S(t) \leq S_{\mathrm{dS}}(t)$ with $S_{\mathrm{dS}}(t) \to 0$ as $t \to 0$. This is rigorous given the axiom. (ii) In any cosmology with $\Lambda > 0$, S_{dS}^{∞} is finite. This is elementary. (iii) The QGD-specific two-sided, finite-budget formulation of the Past Hypothesis closes only under $\Lambda > 0$. This is the framework-requirement result above; it constrains the sign of Λ *conditional on the formulation*, not absolutely.

Assumed. (i) The GSL saturation hypothesis (Assumption 2.2): that $S(t)$ tracks $S_{\mathrm{dS}}(t)$ rather than remaining far below it. This is the QGD-internal motivation for saturation, supported by Paper N's per-mode decoherence analysis but not independently derived.

Not derived. (i) The value of Λ, or equivalently the value of S_{dS}^{∞}. We have explained why $\Lambda > 0$ is required by the framework's two-sided-budget formulation; we have not explained why Λ has the value it does, nor have we ruled out alternative cosmological frameworks (one-sided Minkowski, recollapsing AdS) that operate under different formulations of the Past Hypothesis. (ii) The full microphysics of how entropy is realized from capacity at each epoch. The QGD account is sketched via cosmic decoherence but not completed.

4.5 Comparison with the standard postulate

The conventional statement of the Past Hypothesis is essentially axiomatic: "the macrostate of the early universe had entropy far below the maximum compatible with its energy." QGD replaces this with a derived statement: "the cosmic entropy budget at early times was small because the holographic capacity was small." The first is postulated; the second follows from Axiom 2.1 plus saturation. QGD eliminates one postulate at the cost of accepting a different axiom (holographic bound) that it would have accepted anyway, since the bound is needed for black-hole thermodynamics, dark energy (Paper B), and dual-projection of QM and GR (Paper K).

This is the sort of axiomatic economy that has historically been associated with progress in fundamental physics. The Past Hypothesis is no longer a free postulate of cosmology; it is a consequence of the holographic bound applied to a shrinking past horizon. Whether this counts as a "solution" to the Past Hypothesis depends on whether one accepts the holographic bound as more fundamental than the Past Hypothesis itself. We do, and so does QGD.

5 The Weyl Curvature Asymmetry Resolved

Penrose's sharpest version of the Past Hypothesis is geometric. The Weyl tensor $C_{\mu\nu\rho\sigma}$—the tidal, gravitational-wave part of curvature—vanishes at the Big Bang and diverges at black-hole singularities [3, 4]. Both endpoints are singularities; both are generic outcomes of gravitational dynamics; yet they differ qualitatively. The asymmetry, Penrose's Weyl Curvature Hypothesis (WCH), is normally stated as an initial-condition postulate: the boundary condition on cosmological initial data is $C_{\mu\nu\rho\sigma} \to 0$.

This section recasts the asymmetry within QGD. The classical statement about a field becomes an operator-level conjecture about the cosmic state. The asymmetry is then explained, within a single aeon, by the direction of cosmic decoherence: the Big Bang is upstream of all branching processes, the black-hole singularity is downstream.

5.1 Classical statement and its problem

Classically, the WCH asserts

$$C_{\mu\nu\rho\sigma}(t \to 0) \;\to\; 0, \qquad C_{\mu\nu\rho\sigma}(\text{BH singularity}) \;\to\; \infty. \tag{15}$$

This is consistent with observation: the CMB is extraordinarily isotropic (no primordial gravitational radiation), while gravitational collapse generically produces BKL-like chaotic curvature divergences. As a phenomenological observation, (15) is hard to dispute.

As an explanation, however, (15) is unsatisfying. It is a boundary condition imposed on the classical field, not derived from any dynamical principle. Worse, it requires a distinction between past and future singularities that has no analogue in the time-symmetric Einstein equations. Penrose himself recognized this as the deepest puzzle of cosmology and devoted a substantial body of work to it [4].

5.2 Operator-level reformulation

In QGD, the Weyl tensor is an operator $\hat{C}_{\mu\nu\rho\sigma}$ on the universal Hilbert space, with a well-defined expectation value $\langle \hat{C} \rangle$ and variance $\mathrm{Var}(\hat{C}) = \langle \hat{C}^2 \rangle - \langle \hat{C} \rangle^2$ in any state $|\Psi\rangle$. The classical statement (15) is incomplete at this level: a state with $\langle \hat{C} \rangle = 0$ but large $\mathrm{Var}(\hat{C})$ is a superposition of non-zero values, which is qualitatively different from a sharp zero.

We are led to the following stronger operator-level statement:

Conjecture 5.1 (QGD Weyl Curvature Hypothesis, Heuristic)**.** *Any solution $|\Psi\rangle$ of the Wheeler–DeWitt constraint $\hat{H}|\Psi\rangle = 0$ with no environment satisfies, at every past conformal boundary* *point of $|\Psi\rangle$'s support,*

$$\langle \hat{C}_{\mu\nu\rho\sigma} \rangle_{|\Psi\rangle} = 0, \tag{16}$$

$$\mathrm{Var}(\hat{C}_{\mu\nu\rho\sigma})_{|\Psi\rangle} = 0. \tag{17}$$

Condition (16) is the operator analogue of Penrose's classical statement. Condition (17) is the genuinely new content: it forbids the Weyl tensor from being in a superposition of non-zero values at the past boundary. The state must be in the kernel of the Weyl operator, not merely have zero mean.

Remark (Well-definedness caveats). Conjecture 5.1 is offered as a *heuristic* operator statement; a precise formulation requires three further technical specifications that lie beyond the scope of this paper:

1. **The operator $\hat{C}_{\mu\nu\rho\sigma}$ on which Hilbert space?** A constraint-quantization choice is needed. The simplest implementation is a minisuperspace truncation (homogeneous and isotropic backgrounds with small fluctuations); a fully covariant operator on the kinematic Hilbert space of canonical quantum gravity is not currently available. Throughout this section, references to the "Wheeler–DeWitt constraint" should be read as "the constraint at the minisuperspace level," which is a toy quantization and not a full quantum-gravitational construction.
2. **Which tensor components, in which frame?** The variance $\mathrm{Var}(\hat{C}_{\mu\nu\rho\sigma})$ depends on the choice of frame (tetrad) and on which scalar contractions of the Weyl tensor one is tracking; a careful gauge-invariant formulation (for example through the Weyl scalars $\Psi_0, \ldots, \Psi_4$ in a suitable Newman–Penrose frame, integrated over a small comoving volume) is needed.
3. **What is the "past conformal boundary of $|\Psi\rangle$'s support"?** This phrase presupposes that the cosmic wave function has a definite support structure with an identifiable past boundary, which itself requires a frame for "past" (a clock variable) and a notion of conformal compactification at the level of quantum geometry.

We do not address these in the present paper; we view the conjecture as motivating the operator-level reformulation of Penrose's WCH, not as a complete operator-theoretic statement. A full version would require constraint-quantization, frame, and boundary-support machinery that goes beyond the QGD framework as presently developed.

What we can offer in support of the heuristic conjecture is three motivating readings of one underlying argument, each of which makes (16)–(17) plausible from a different conceptual entry point.

5.3 Three readings of one argument

We present three motivating routes below, but we should be honest about what they are: *three complementary readings of a single underlying argument*, not three logically independent derivations. All three rely on the same core step—the past conformal boundary has vanishing bounding area, $A_{\text{boundary}} \to 0$—and differ only in which downstream consequence they emphasize. Listing them as three is a pedagogical convenience rather than independent evidence.

Route A: Holographic. At a past conformal boundary, the bounding area $A_{\text{boundary}} \to 0$, so the holographic bound $S \leq A/(4\ell_P^2) \to 0$. A state with $\text{Var}(\hat{C}) > 0$ carries non-trivial gravitational entropy through the realized branching of the metric structure; the bound forces this contribution to vanish at the boundary. We caveat: this is a bound on what *could* carry entropy, not a derivation of which state *is* present.

Route B: Kinematic vacuum. The Weyl tensor is a semiclassical observable. Its sharp values require a branching of $|\Psi\rangle$ through environment-induced decoherence. At a past conformal boundary no decoherence has yet occurred (no clock, no observer, no environment in the QGD sense), so $\hat{C}$ has neither a definite eigenvalue nor a definite superposition of non-zero eigenvalues. We resolve the ambiguity by assigning the boundary state to the vacuum sector of $\hat{C}$, where both (16) and (17) hold trivially. Note that "no decoherence yet" is, on closer inspection, equivalent to Route A's "no holographic room for branching," since branching is decoherence and entropy is the count of distinguishable branches. We caveat: "the vacuum sector has $\hat{C} = 0$" requires an explicit construction.

Route C: No-environment plus Axiom I. Globally $|\Psi\rangle$ is pure (the universe-as-a-whole has no exterior), so the global von Neumann entropy is zero. At any moment, the entropy carried by matter is bounded by the available gravitational entropy capacity, which in turn is bounded by the boundary area in Planck units. At a past conformal boundary the latter goes to zero, so the matter entropy goes to zero, and consequently the realized branching of the geometry (which is what gives non-trivial $\text{Var}(\hat{C})$) goes to zero. This is once again the same core step as Route A; the difference is that Route C makes the global purity of $|\Psi\rangle$ explicit. We caveat: this assumes the past conformal boundary has $A \to 0$, which is the standard cosmological assumption but should be derived from the quantum constraint.

Summary. The three routes share a single mechanism: $A_{\text{boundary}} \to 0 \Rightarrow S_{\max} \to 0 \Rightarrow \langle\hat{C}\rangle = \text{Var}(\hat{C}) = 0$ at the boundary. Route A emphasizes the holographic-bound step, Route B the absence-of-decoherence reading, Route C the no-environment global purity. They are not three independent proofs; they are three vocabularies for one heuristic. We retain the labeling because the different vocabularies illuminate different aspects, but the cumulative weight is one motivating argument, not three.

5.4 The asymmetry within a single aeon

Conjecture 5.1 is symmetric in past and future conformal boundaries: it states that $\hat{C}$ has $\langle\hat{C}\rangle = \text{Var}(\hat{C}) = 0$ at *any* conformal boundary. It does not, by itself, distinguish the Big Bang from the future heat death.

The asymmetry between Big Bang and black-hole singularity is not at a past versus future conformal boundary of the global $|\Psi\rangle$. Both are local singularities within a single cosmic aeon. The asymmetry is at the level of *decoherence direction*, which is a property of the cosmic state within the aeon, not of its global boundary.

Within a single aeon:

- The *Big Bang* is upstream of all decoherence processes. No semiclassical clock yet exists, no environment has measured the Weyl operator into definite branches, no branching of

$|\Psi\rangle$ has occurred. The Weyl operator is in its kinematic vacuum: $\langle\hat{C}\rangle = 0$, $\mathrm{Var}(\hat{C}) = 0$. This matches the classical WCH statement (15) at the Big Bang.

- The *black-hole singularity* is downstream of decoherence. Substantial gravitational entanglement has accumulated between the collapsing matter and the rest of the universe; the Weyl operator has been measured many times into definite branches; the BKL-like chaotic behavior is the classical description of a superposition of high-variance branches each with large $\hat{C}$ eigenvalue. The expectation $\langle\hat{C}\rangle$ on the relevant branch is large; the variance $\mathrm{Var}(\hat{C})$ is also large, reflecting the chaotic structure. This matches the classical WCH statement at the BH singularity.

The asymmetry follows from a *single* fact—the direction of emergent time as the direction of cosmic decoherence—applied to both endpoints. The Big Bang is upstream of the arrow; the BH singularity is downstream. There is no separate postulate of past-future asymmetry; the asymmetry is the arrow of time itself, applied at two different stages of cosmic evolution.

The cleanest formulation of Penrose's WCH that we have been able to construct is therefore this: *the Weyl tensor is in its kinematic vacuum at the upstream end of decoherence; it is in a high-variance, large-mean branch at the downstream end.* The mechanism is the direction of cosmic gravitational decoherence at per-mode rate $g(1)\,H/(4\pi)$ [17, 16].

5.5 What is and is not established

Established. (i) The classical statement of the WCH is recoverable as the semiclassical limit of Conjecture 5.1. (ii) Within a single aeon, the Big-Bang/BH asymmetry is explained as upstream/downstream relative to cosmic decoherence, with no additional postulate beyond the QGD arrow-of-time framework already in place.

Conjectured. Conjecture 5.1 itself. We have given one heuristic argument in three vocabularies (holographic, kinematic-vacuum, no-environment); none of the three readings constitutes a proof, and they are not logically independent of each other. A full derivation would require (i) a precise operator construction for $\hat{C}_{\mu\nu\rho\sigma}$ on the kinematic Hilbert space of a sufficiently complete canonical quantization (likely beyond minisuperspace), (ii) a derivation of the boundary-area-vanishing condition $A_{\text{boundary}} \to 0$ from the Wheeler–DeWitt constraint rather than as an external geometric assumption, and (iii) either an explicit Hartle–Hawking-like construction of $|\Psi\rangle$ in the vicinity of the past boundary or a proof that the no-environment condition forces the operator-vacuum form there. All three remain open problems.

Not addressed. Cyclicity. Penrose's Conformal Cyclic Cosmology (CCC) attempts to identify the future conformal boundary of one aeon with the past conformal boundary of the next, generating a cyclic sequence [4]. Within QGD this identification fails on technical grounds: the would-be Thermofield-Double pairing between aeons requires a shared Hamiltonian and a shared inverse temperature, neither of which is provided by the global Wheeler–DeWitt constraint [25]. The QGD reading of CCC is therefore not cyclic but block-static: any conformal boundary of $|\Psi\rangle$ has the same kinematic-vacuum form, but "aeons" are not chronologically connected in the way Penrose proposed. We address this comparison in Section 7.

6 Observational Status

A theoretical framework that resolves three foundational puzzles is interesting; a framework that also makes novel testable predictions is what discriminates fundamental physics from natural philosophy. This section examines, with explicit honesty, what the present framework predicts at the level of current and near-future observations. The verdict is mixed: there are predictions, but most are either already-known (and consistent with ΛCDM at current precision) or in-principle-testable but parametrically out of reach.

6.1 What is testable now

Laboratory gravitational decoherence (BMV-class experiments). The QGD decoherence rate

$$\Gamma_{\text{dec}} = \frac{GM^2}{\hbar\, d} \equiv \frac{E_G}{\hbar} \tag{18}$$

applies to two coherent mass distributions of mass M separated by distance d.[1] The Bose–Marletto–Vedral (BMV) class of experiments [27, 28] is designed to test whether two masses can become gravitationally entangled—which, if observed, demonstrates the quantum character of gravity and incidentally tests the QGD rate (18). The setup is detailed in Paper M of this series [26]: for $M \sim 10^{-9}$ kg and $d \sim 10^{-4}$ m, $\Gamma_{\text{dec}}^{-1} \sim 10^{-9}$ s, which is the relevant decoherence timescale for a BMV experiment. This is the gold-standard test of the QGD framework as a whole; if confirmed at the predicted rate, it provides empirical support for the gravitational-decoherence mechanism that underlies the cosmic-decoherence argument of Section 2 and Paper N. The framework presented in this paper inherits its testability through this connection.

ISW suppression from dynamical dark energy. Paper B's $\rho_{\text{DE}} = \alpha c^2 H^2/G$ predicts an effective equation of state $w_{\text{eff}}(z)$ that differs slightly from $w = -1$ at $z \sim 0.5$–1, and a corresponding modification of the late-time integrated Sachs–Wolfe (ISW) effect at large angular scales ($\ell < 30$). The predicted suppression is at the 1–2% level—below current Planck cosmic variance [8] but potentially accessible to a combined CMB-S4 + LSS analysis later this decade. This is an in-principle test of Paper B's framework, which our identity (8) inherits.

6.2 What is testable in principle but not at current precision

Time variation of $w(z)$ and the holographic relation. DESI 2024 [9] reports mild hints of $w(z) \neq -1$, with statistical significance below 3σ at the time of writing. If real, $w(z)$ variation would test the holographic relation: the QGD framework predicts that any time variation of $\rho_{\text{DE}}(z)$ is correlated with a corresponding time variation of $S_{\text{dS}}(z)$, with the product saturating the Planck-scale constant in the de Sitter attractor (Section 3). Standard quintessence has no such constraint. A precise $w(z)$ reconstruction could distinguish the two in principle. In practice, the precision required to discriminate is well beyond current DESI and probably beyond Stage-IV LSS at $z \lesssim 1$. This is a future-decade prediction at best.

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[1] The expression $\Gamma_{\text{dec}} = E_G/\hbar$ is the QGD prediction; the energy scale $E_G = GM^2/d$ is established as the classical gravitational potential energy of the two configurations and is independent of the rate identification, but the assertion that the decoherence rate equals $E_G/\hbar$ follows from the QGD saturation principle / Hamiltonian-constraint argument developed in Papers K and M [13, 26]. The energy scale and the rate identification are logically distinct steps; only the former is rigorous in the classical sense.

Cosmic horizon entropy as a derived prediction. The identity (8) predicts $S_{\rm dS}^{\infty}$ from $\rho_{\rm DE}^{\infty}$. A precise measurement of $H_\infty = H_0\sqrt{\Omega_{\rm DE}}$ from large-scale structure pins down $S_{\rm dS}^{\infty}$ to better than 1%. This is the cosmic-information capacity of the universe-as-causal-patch. While not a novel prediction (the Bekenstein–Hawking entropy of the cosmic horizon is standard), the QGD framing gives this number deep meaning: it is the entropy budget of the universe, the source of the Past Hypothesis, and the dual of dark energy.

6.3 What is *not* testable: the honest negative result

A theoretical hope at the start of this program was that QGD's per-mode cosmic decoherence at rate $g(1)\,H/(4\pi)$ [17] would imprint a measurable signature on the CMB through modified non-Gaussianity. We have completed this calculation [29] and the verdict is negative.

The per-mode accumulated decoherence functional from horizon exit during inflation to last scattering, with the de Sitter form factor $g(x) = 1 - (2/\pi)\,\mathrm{Si}(x)$ properly applied [30], is

$$D_k(\eta_{\rm LSS}) \;=\; \frac{1}{4\pi}\int_{\ln a_*(k)}^{\ln a_{\rm LSS}} g\!\left(\frac{aH}{ck}\right) d\ln a \;\approx\; 0.35\text{–}0.51 \quad \text{per mode,} \tag{19}$$

where the dominant contribution comes from the deep sub-horizon period during inflation, not from horizon crossing or the super-horizon phase. The induced non-Gaussianity is a multiplicative suppression of the standard inflationary value:

$$f_{\rm NL}^{\rm QGD} \;=\; e^{-D_k(\eta_{\rm LSS})}\, f_{\rm NL}^{\rm standard}, \qquad |\Delta f_{\rm NL}^{\rm QGD}| \;\sim\; 5\times10^{-3}\text{–}1.5\times10^{-2}. \tag{20}$$

The current Planck 2018 bound is $f_{\rm NL}^{\rm local} = -0.9 \pm 5.1$ [31]; the CMB-S4 target sensitivity is $\sigma(f_{\rm NL}) \sim 1$ [32]. The QGD prediction is roughly 60–200 times below CMB-S4 sensitivity. *QGD-induced non-Gaussianity is not observable at any planned CMB experiment.*

A more striking diagnosis appears when QGD is compared to the standard squeezing-induced classicality mechanism of Kiefer, Polarski, and Starobinsky [33]. The squeezing parameter $r_k(\eta) = \ln(a/a_*)$ reaches $r \sim 60$ for CMB pivot modes by the end of inflation, producing phase-space elongation by a factor $e^{2r} \sim 10^{52}$. This classicalizes inflationary perturbations *without* genuine decoherence. The squeezing rate $r_{\rm dot} = H$ is roughly 30 times faster than the QGD decoherence rate $g(1)\,H/(4\pi) \approx 0.032\,H$:

$$\frac{\Gamma_{\rm QGD}}{r_{\rm dot}} \;=\; \frac{g(1)}{4\pi} \;\approx\; 0.032. \tag{21}$$

Squeezing-induced classicality therefore dominates the apparent quantum-to-classical transition of inflationary perturbations. QGD's contribution is real but observationally redundant: it certifies a conclusion that squeezing already produces.

We list this as an honest negative result. The QGD framework is consistent with all current and foreseeable CMB data; it does not predict a discriminating signature in the CMB sector. The discriminating tests are elsewhere—in laboratory BMV experiments at micron scale and microgram mass, in the MOND-like behavior of low-acceleration galaxy dynamics at $z > 2$ [34], and (in principle) in precise reconstruction of $w(z)$ as DESI and successor surveys mature.

6.4 Summary of observational status

Table 2 collects the predictions and their observational status.

Table 2. *Observational status of QGD predictions relevant to the framework of this paper. "Status" is graded from "testable now" to "not testable in the foreseeable future." Below-sensitivity predictions are listed for completeness; they do not constitute observational signatures of QGD against ΛCDM.*

Prediction	QGD value	Status
BMV-class lab decoherence rate	$\Gamma = GM^2/(\hbar d)$	Testable this decade; Paper M setup [26].
ISW large-scale suppression	1–2% at $\ell < 30$	Below Planck cosmic variance; possibly CMB-S4 + LSS.
Identity $\rho_{\rm DE}^\infty S_{\rm dS}^\infty = 3c^7/(8G^2\hbar)$	Exact in attractor	Structural consistency relation, not a novel test.
$S_{\rm dS}^\infty \approx 3 \times 10^{122}$	Predicted from H_∞	Standard; informational interpretation new.
$\Lambda > 0$ requirement (framework formulation)	Sign-determination	Consistent with observation; not a discriminating test.
CMB non-Gaussianity $\lvert\Delta f_{\rm NL}\rvert$	$\sim 5 \times 10^{-3}$–10^{-2}	60–200× below CMB-S4 sensitivity; not testable.
$a_0(z) = cH(z)/(2\pi)$ MOND scale	Galaxy dynamics at $z > 2$	JWST/Euclid/Rubin, 5–10 yr horizon [34].

The framework of this paper makes no novel discriminating prediction beyond ΛCDM at current precision. The contribution is conceptual, not observational. We acknowledge this explicitly: a theoretical framework that "solves" three puzzles without producing a new testable signature is, by definition, an organizational improvement rather than an empirical advance. Whether the conceptual gain is worth the cost is a judgement we leave to the reader.

7 Discussion

The framework developed in this paper sits in a landscape populated by other proposed resolutions of the Past Hypothesis, the Weyl Curvature Asymmetry, and the cosmological constant problem. We organize the discussion by direct comparison with the most influential alternatives.

7.1 Penrose's Conformal Cyclic Cosmology

The most direct competitor is Penrose's Conformal Cyclic Cosmology (CCC) [4]. CCC takes the future conformal boundary of one aeon and identifies it, via a conformal rescaling, with the past conformal boundary of the next. The Big Bang then becomes the conformal future infinity of a previous aeon, and the WCH boundary condition is automatically satisfied at every aeon-aeon junction because the conformal rescaling smooths the Weyl tensor at the boundary.

QGD endorses several of Penrose's intuitions and rejects others.

Endorsed: mass-as-clock and the role of conformal symmetry. The Page–Wootters argument that mass is necessary for a clock with a Lorentz-invariant rate is sound [35]. The deep version of this—mass as the minimal Weyl-symmetry-breaking parameter in the Lagrangian—is the foundation on which both CCC and QGD's block-static reading rest. Massless fields alone do not provide a clock with a fixed rate; mass is the canonical conformal-frame-breaking object.

Rejected: cyclicity via TFD. Within QGD, the would-be Thermofield-Double identification between aeons fails on three independent technical grounds [25]: (i) no shared Hamiltonian (the Wheeler–DeWitt constraint is $\hat{H}|\Psi\rangle = 0$ globally, leaving no global energy spectrum); (ii) no shared inverse temperature (the de Sitter temperature $T_{\text{dS}} = H/(2\pi)$ requires a static patch with a timelike Killing vector, which the spacelike past conformal boundary does not provide); (iii) no canonical mode pairing (Bunch–Davies modes on the two sides of a putative aeon-aeon junction live in different conformal frames). The eternal-AdS-black-hole TFD construction works because the bifurcation horizon has a Killing symmetry that forces the structure; de Sitter conformal infinity has no such symmetry. The TFD identification is rhetoric, not derivation, in the de Sitter context.

Rejected: ontological mass-shedding. CCC requires the late universe to have no scales, which Penrose addresses by postulating that all massive particles eventually decay—black holes via Hawking evaporation, protons via GUT mechanisms, and electrons via hypothetical "erebons." The erebon hypothesis is ad hoc; the Standard Model has stable electrons. QGD avoids the need for erebons by replacing ontological mass-shedding with operational horizon dilution [25]: massive particles persist ontologically, but they are diluted past the holographic bound of any observer's de Sitter horizon, so mass is operationally inaccessible in the far future. This is the weaker but more defensible position.

What QGD takes from CCC. The reframing of the 10^{123} as a geometric quantity (Section 5); the use of conformal boundaries as the natural setting for the WCH; the recognition that the Past Hypothesis and the cosmological constant problem are intertwined. What QGD does not take: cyclicity, TFD structure, Hawking-point predictions, erebons.

The picture that survives is not "CCC inside QGD" but "the holographic bound forces low entropy at every conformal boundary, including the Big Bang." Penrose's intuitions are partly endorsed, partly rejected, and the surviving content is the holographic-capacity reading of the Past Hypothesis developed in Section 4.

7.2 Bousso's Covariant Entropy Bound

Bousso's covariant entropy bound [19, 20] is the most general formulation of the holographic principle in arbitrary spacetimes. The bound applies to lightsheets emanating from arbitrary spacelike surfaces and yields, for the de Sitter horizon, exactly the $S_{\text{dS}} = A/(4\ell_P^2)$ entropy we use throughout this paper.

The relation between our framework and the Bousso bound is straightforward: the Bousso bound is the most general statement; QGD specializes it to the cosmic horizon and adds the dynamical-dark-energy link (8). The Bousso bound by itself does not relate horizon entropy to dark-energy density; our identity does so by combining the bound with Paper B's holographic dark energy formula and the de Sitter Friedmann equation.

QGD does not contradict Bousso; it elaborates the bound into a dynamical statement about dark energy.

7.3 Prior work on holographic dark energy and the N-bound

A substantial body of prior work has linked the cosmological constant to horizon entropy via essentially the same algebraic identity we use. We are not the first to write down a relation of the form (8), and we should be explicit about where the present paper stands relative to that literature.

N-bound and Banks. Banks [14] and Fischler–Susskind [15] proposed in 1998–2000 that the cosmological constant is fundamentally an entropy quantity: the de Sitter horizon carries a finite number $N = S_{\rm dS}^{\infty}$ of degrees of freedom, the cosmological constant is whatever value makes this number what it observably is, and the smallness of Λ in Planck units *is* the largeness of N. The argument was rediscovered, with variations, in several later works. The structural content of our identity (8) is, in this sense, twenty-five years old; we are recovering and extending it within the QGD framework, not introducing it.

Holographic foundations. The holographic-principle literature on which all of this rests includes 't Hooft's original dimensional-reduction proposal [36] and Susskind's "world as a hologram" formulation [37], both of which propose that the information content of a region is bounded by the area of its boundary in Planck units. Our Axiom 2.1 is a specialization of this principle to cosmological horizons.

Cohen–Kaplan–Nelson and Li. Cohen, Kaplan and Nelson [23] proposed an effective-field-theory upper bound on the vacuum energy that uses a horizon IR cutoff and a UV cutoff at the same scale, yielding $\rho_{\rm vac} \sim H^2 M_P^2$—essentially our holographic dark-energy formula in another vocabulary. Li's holographic dark energy model [24] sharpened this into the dynamical relation $\rho_{\rm DE} \propto c^2 H^2/G$ that our Paper B inherits as its starting point.

Padmanabhan. The closest single body of prior work to the present synthesis is Padmanabhan's thermodynamic-aspects-of-gravity program [38], which has, over more than a decade, explicitly related dark energy to horizon information and argued that the gravitational field equations are statistical-mechanical identities. The structural overlap with our framework is substantial; the differences lie in the specific axiomatic basis (QGD's six axioms plus the duality correspondence) and in the coupling to the Past Hypothesis and the operator-level Weyl Curvature Hypothesis developed in Sections 4–5.

Where the novelty actually sits. The identity (8) is therefore not original to this paper, and we have updated the framing in Section 3 accordingly. What is novel here is the *coupling*: we connect this prior-art identity to (a) the Past Hypothesis (via the shrinking past horizon making $S_{\rm dS}(t \to 0) \to 0$ automatic) and (b) Penrose's Weyl Curvature Hypothesis (via the upstream-downstream-of-cosmic-decoherence reading). Neither of these connections appears, to our knowledge, in the Banks/Fischler–Susskind/Cohen–Kaplan–Nelson/Li/Padmanabhan threads. The conceptual unification is the contribution; the identity is a tool that already existed.

7.4 Standard Λ-driven heat death

In the most conservative cosmological framework—standard ΛCDM, with Λ a fundamental constant and no further explanation sought—the Past Hypothesis is an external postulate and the cosmological constant problem is an open puzzle of quantum field theory. The framework is empirically successful at present precision but admits no explanation for the magnitude of Λ or for the low initial entropy.

Our framework makes the Past Hypothesis a consequence rather than a postulate, while preserving full consistency with ΛCDM phenomenology. The cosmological constant is reframed as one face of a two-faced object whose other face is the cosmic entropy capacity. The magnitude Λ remains fitted to observation; we have not derived it from first principles.

Against standard ΛCDM, our framework offers conceptual economy (one mystery viewed through three lenses rather than three distinct mysteries) without offering empirical discrimination. The choice between them at present is a matter of theoretical taste, not data.

7.5 Hartle–Hawking and the wave function of the universe

The Hartle–Hawking "no-boundary" proposal [39] provides a candidate boundary condition for the cosmic wave function: $\Psi_{\mathrm{HH}}[h_{ij}]$ is computed as a path integral over compact Euclidean geometries with a single boundary at the given three-metric. The proposal directly addresses the Past Hypothesis by selecting a specific quantum state of the universe at small scale factor.

Our framework is complementary to Hartle–Hawking, not competing with it. Hartle–Hawking proposes a specific boundary condition; QGD proposes a capacity argument for any state with the holographic bound. The two are mutually consistent: a Hartle–Hawking initial state automatically satisfies the QGD capacity bound at small scale factor, since both predict $S \to 0$ as $A \to 0$. The QGD argument is more general (it requires only the holographic bound, not the specific HH path-integral construction) but less specific (it does not pick out a single wave function).

A complete account would presumably combine HH (for the specific state) with QGD's capacity argument (for the general bound). We leave this synthesis to future work.

7.6 Multiverse and anthropic explanations

Anthropic resolutions of the cosmological constant problem [40] appeal to a vast landscape of possible universes with different values of Λ and select our value by the requirement that observers exist. The mechanism is statistical rather than dynamical. Similarly, anthropic resolutions of the Past Hypothesis appeal to the fact that observers can only exist in regions of the multiverse where the local Past Hypothesis holds, irrespective of whether it is generic.

Our framework provides an alternative that does not invoke a multiverse. The Past Hypothesis is forced by the holographic bound applied to a shrinking past horizon (Section 4); the cosmological constant has the value it does by virtue of being reciprocal to the cosmic entropy capacity (Section 3); the magnitude $\alpha = 3\Omega_{\mathrm{DE}}/(8\pi)$ is the residual mystery, fitted to observation but constrained to lie in a narrow window by self-consistency requirements.

This is not a logical refutation of the multiverse—the multiverse and QGD can both be true. But the QGD framework is more economical: it explains the structure of the puzzle from a single number α, without requiring an unobservable ensemble of other universes.

7.7 Where does this leave us

The honest summary is that our framework offers conceptual unification at the cost of conjectural commitments (the operator-level WCH; saturation of the GSL; the holographic bound applied cosmologically) and without offering novel discriminating observations against ΛCDM. Whether this trade is worth making depends on theoretical priorities: those who value parsimony in the foundational structure will find the unification attractive; those who require empirical discrimination will find it premature.

We close this discussion with a remark on what the framework does *not* replace. It does not replace the need for a microscopic theory of quantum gravity. The holographic bound, the GSL, and the dark-energy formula (4) are all phenomenological inputs in our framework—each is well-motivated within QGD but none is derived from a quantum-gravity Lagrangian. The framework is best understood as the most economical organization of these inputs that the present state of theoretical knowledge permits, not as a final theory.

8 Conclusions

We have argued that three apparently independent puzzles in fundamental physics—the Past Hypothesis, Penrose's Weyl Curvature Asymmetry, and the cosmological constant problem—are one mystery viewed through three lenses. The connecting tissue is the identity

$$\rho_{\mathrm{DE}}^{\infty}\, S_{\mathrm{dS}}^{\infty} \;=\; \frac{3\,c^7}{8\,G^2\,\hbar}, \tag{22}$$

which is exact in the de Sitter attractor of any FRW cosmology with $\Lambda > 0$. The right-hand side is a Planck-scale constant of order unity (equal to $(3/8)\,\rho_{\mathrm{Pl}}$, where $\rho_{\mathrm{Pl}} = c^7/(\hbar G^2)$ is the Planck density). The left-hand side is the product of two quantities, each parametrically large or small in Planck units, that are reciprocal to each other up to that constant: the asymptotic dark-energy density and the asymptotic Bekenstein–Hawking entropy of the cosmic horizon. We have stressed throughout that the identity is a tautology (both sides are functions of H_∞ alone, and the derivation is three algebraic lines from Friedmann plus Bekenstein–Hawking); the substance of the paper is the conceptual coupling of this prior-art identity [14, 15, 24, 38] to the Past Hypothesis and Penrose's Weyl Curvature Hypothesis, not the identity itself.

8.1 What this gives us

The Past Hypothesis becomes a corollary, not a postulate. The holographic bound $S(t) \leq S_{\mathrm{dS}}(t)$ applied to a shrinking past horizon gives $S(t) \to 0$ as $t \to 0$ automatically. No fine-tuning of initial entropy is required: the entropy is small because the capacity is small. The strong form (entropy tracking capacity rather than sitting below it) follows from the QGD GSL-saturation assumption motivated by Paper N's cosmic decoherence analysis.

Penrose's 10^{123} is a capacity, not a probability. The number is the value of S_{dS}^{∞} in nats. It quantifies the maximum information any observer can ever encode in their causal patch. It is large because the universe is large; it is not a measure of fine-tuning of initial conditions but the size of the cosmic address book.

The cosmological constant problem is the inverse of the cosmic entropy budget. The hierarchy $\rho_{\rm DE}/\rho_{\rm Pl} \sim 10^{-122}$ is reciprocal to $S_{\rm dS}^{\infty} \sim 10^{122}$. The two are not independent puzzles: $\alpha = 0.082$ controls both simultaneously. One mystery in dimensionless form remains; two have been merged.

The Weyl Curvature Asymmetry is the direction of cosmic decoherence. Within a single aeon, the Big Bang is upstream of all branching processes (kinematic vacuum: $\langle \hat{C} \rangle = \mathrm{Var}(\hat{C}) = 0$); black-hole singularities are downstream (large mean and variance). The asymmetry is not a separate postulate but a consequence of the cosmic arrow of time as the direction of QGD gravitational decoherence.

Dark energy is required by the framework's formulation. $\Lambda > 0$ is the structural condition that closes the QGD cosmic entropy budget under the specific two-sided, finite-budget formulation developed in this paper. Cosmologies with $\Lambda \leq 0$ admit alternative Past Hypothesis formulations (one-sided in the Minkowski limit, turnaround-bounded in the AdS case); QGD specifically chose the formulation that pairs cleanly with the holographic identity, and that choice then selects $\Lambda > 0$ for internal consistency. The sign of Λ is determined *conditional on the formulation*, not absolutely; the magnitude $|\Lambda|$ remains fitted to observation.

8.2 What we have not done

We have been careful throughout to flag the limits of the construction. They bear repeating in summary:

- **α is not derived from first principles.** The coefficient $\alpha = 3\Omega_{\rm DE}/(8\pi) \approx 0.082$ controlling the dark-energy density is fitted to observations. We have shown that it equivalently sets $1/S_{\rm dS}^{\infty}$; we have not derived its value. The residual cosmological constant problem (why α has this magnitude) persists.
- **GSL saturation is assumed.** The holographic bound is an upper bound; that the actual cosmic entropy saturates it is an additional assumption (Assumption 2.2). Within QGD this is motivated by the per-mode decoherence rate of Paper N, but it is not independently derived.
- **The operator Weyl Curvature Hypothesis is conjectured.** Conjecture 5.1 states that $\langle \hat{C} \rangle = \mathrm{Var}(\hat{C}) = 0$ at past conformal boundaries. Three motivating routes are given (holographic, kinematic-vacuum, no-environment); none is a proof.
- **No novel observable distinct from ΛCDM at current precision.** The framework predicts the same expansion history, the same CMB power spectrum, and CMB non-Gaussianity that is too small to be measured. The discriminating tests of QGD are elsewhere (BMV lab experiments, MOND-like dynamics at high z), not in the framework of this paper.

8.3 The unified picture

The unification we have argued for has the following logical flow. The QGD holographic bound (Axiom 2.1) applied to the cosmic horizon supplies two ingredients in parallel: Paper B's holographic dark-energy formula $\rho_{\rm DE} = \alpha c^2 H^2/G$, and the cosmic entropy capacity $S_{\rm dS}(t) \leq \pi c^5/(H^2 \hbar G)$ (Section 2). In the de Sitter attractor, substituting the Friedmann equation into the Bekenstein–Hawking formula gives the central identity $\rho_{\rm DE}^{\infty} S_{\rm dS}^{\infty} = 3c^7/(8G^2\hbar)$. From this single relation, three corollaries follow: (i) the Past Hypothesis, as the cosmic-capacity statement $S(t \to 0) \to 0$; (ii) Penrose's Weyl asymmetry, as the upstream-versus-downstream-of-decoherence reading of the Big Bang and black-hole singularities; (iii) the cosmological-constant hierarchy $\rho_{\rm DE}/\rho_{\rm Pl} \sim 1/S_{\rm dS}^{\infty}$ as a reciprocal relation, not two independent numbers.

The same axiom, applied to the same cosmic horizon, gives three faces of the same physics. Dark energy makes the asymptotic entropy capacity finite; the finite capacity implements the Past Hypothesis; the Past Hypothesis defines the arrow of time; the arrow of time produces the Weyl asymmetry as upstream-versus-downstream of decoherence; and the smallness of the dark-energy density in Planck units is the reciprocal of the cosmic entropy capacity. One axiom, one identity, one parameter (α), three puzzles dressed in three vocabularies.

8.4 Honest assessment

What this paper offers is a single conceptual rearrangement: three apparently independent foundational puzzles are seen, within QGD, to be three readings of one mystery. The identity (22) that licenses this rearrangement is a tautology, well-known in the holographic-dark-energy literature [14, 15, 24, 38]; the novelty is the coupling of this identity to the Past Hypothesis and to Penrose's Weyl Curvature Hypothesis. The framework introduces no new axioms beyond the six already in QGD, no new free parameters beyond the single α already in Paper B, and—honestly—no new observable that discriminates against ΛCDM at current precision.

The strength of the contribution is therefore conceptual rather than empirical. We have not solved any of the three puzzles in the strong sense of deriving them from a deeper principle; we have argued that explaining the value of $\alpha \approx 0.082$ would simultaneously explain the Past Hypothesis, the Weyl asymmetry, and the smallness of the cosmological constant in Planck units. This is a rearrangement of the unknown, not its elimination. Whether the rearrangement is worth making—whether seeing one mystery in three vocabularies is more illuminating than seeing three independent mysteries—is a judgement we leave to the reader.

The framework is best read as a step, not a destination. The destination would be a first-principles derivation of α from a microscopic theory of quantum gravity. We do not have that theory. What we have is one organization of the present puzzle landscape—one in which the holographic bound, applied to the cosmic horizon and coupled to the dark-energy density via the de Sitter attractor, makes the Past Hypothesis a corollary of capacity, recasts Penrose's 10^{123} as a capacity rather than a probability, and ties the smallness of $\rho_{\rm DE}/\rho_{\rm Pl}$ to the largeness of $S_{\rm dS}^{\infty}$. The conjectural commitments (operator-level WCH with its well-definedness caveats; GSL saturation requiring the holographic mode count to do the structural work; the saturation gap between per-mode and total rates noted in Section 2) are real and we have flagged them throughout. Whether they hold up under deeper scrutiny is the question that will decide whether this is the right organization of the puzzle landscape or merely one organization among several.

Appendices

A Numerical Tables

This appendix collects the numerical values underlying the calculations of the main text. All quantities use the CODATA 2018 values of fundamental constants and the Planck 2018 cosmological parameters.

A.1 Fundamental constants

Table 3. *Fundamental constants used throughout.*

Quantity	Symbol	Value
Gravitational constant	G	6.67430×10^{-11} m^3/(kg s^2)
Reduced Planck constant	$\hbar$	$1.054571817 \times 10^{-34}$ J s
Speed of light	c	2.99792458×10^{8} m/s
Boltzmann constant	k_B	1.380649×10^{-23} J/K
Planck length	ℓ_P	$1.6162550 \times 10^{-35}$ m
Planck time	t_P	$5.3912464 \times 10^{-44}$ s
Planck mass	m_P	2.1764340×10^{-8} kg
Planck density	$\rho_{\mathrm{Pl}} = c^7/(\hbar G^2)$	5.155×10^{96} kg/m^3

A.2 Cosmological parameters

Table 4. *Cosmological parameters from Planck 2018 [8] (TT,TE,EE+lowE+lensing).*

Quantity	Symbol	Value
Hubble constant	H_0	(67.4 ± 0.5) km/s/Mpc
Hubble rate today (SI)	H_0	$(2.184 \pm 0.016) \times 10^{-18}$ s^{-1}
Dark energy fraction	Ω_{DE}	0.689 ± 0.006
Matter fraction	Ω_m	0.311 ± 0.006
Critical density today	$\rho_{\mathrm{crit}} = 3c^2H_0^2/(8\pi G)$	$(7.67 \pm 0.11) \times 10^{-10}$ J/m^3
Dark energy density today	$\rho_{\mathrm{DE}}^{\mathrm{today}} = \Omega_{\mathrm{DE}}\,\rho_{\mathrm{crit}}$	$(5.28 \pm 0.10) \times 10^{-10}$ J/m^3
Asymptotic Hubble rate	$H_\infty = H_0\sqrt{\Omega_{\mathrm{DE}}}$	1.81×10^{-18} s^{-1}
Holographic dark energy coefficient	$\alpha = 3\Omega_{\mathrm{DE}}/(8\pi)$	0.0823 ± 0.0007

The critical density and dark-energy density are derived from H_0 and Ω_{DE} via $\rho_{\mathrm{crit}} = 3c^2H_0^2/(8\pi G)$, ensuring internal consistency with the body-text Planck 2018 value $H_0 = 67.4$ km/s/Mpc.

A.3 Cosmic entropy capacity at representative epochs

Table 5. *The cosmic horizon entropy* $S_{\mathrm{dS}} = \pi c^5/(H^2\hbar G)$ *at representative epochs of cosmic history. Hubble rates are standard* Λ*CDM estimates with the parameters above.*

Epoch	Redshift z	H (s^{-1})	S_{dS} (nats)
Planck epoch	$\sim 10^{32}$	$\sim 1.85 \times 10^{43}$	$\pi \approx 3.14$
End of inflation	$\sim 10^{30}$	$\sim 10^{36}$	$\sim 1 \times 10^{17}$
QCD transition	$\sim 10^{12}$	$\sim 10^{12}$	$\sim 1 \times 10^{65}$
Matter–radiation eq.	3.4×10^3	$\sim 10^{-12}$	$\sim 1 \times 10^{88}$
Recombination	1.1×10^3	5×10^{-14}	$\sim 4 \times 10^{91}$
$z = 1$ (cosmic noon)	1	5.0×10^{-18}	4.1×10^{121}
Today	0	2.18×10^{-18}	2.27×10^{122}
de Sitter asymptote	$\to -1$ (eqn-of-state)	1.81×10^{-18}	3.29×10^{122}

The total range spans 122 orders of magnitude. The cosmic entropy capacity has grown from approximately π nats at the Planck epoch to approximately 3.3×10^{122} nats in the de Sitter asymptote, an increase of 122 orders of magnitude over the full history of the universe.

A.4 Numerical check of the central identity

We tabulate $\rho_{\mathrm{DE}}\, S_{\mathrm{dS}}$ and the right-hand side of the identity (8) at three reference epochs.

Table 6. *Numerical check of the dark-energy / horizon-entropy identity* (8)*. The product* $\rho_{\mathrm{DE}}\, S_{\mathrm{dS}}$ *should equal the Planck-scale constant* $3c^7/(8G^2\hbar)$ *in the de Sitter attractor only. "Ratio" is the product divided by the constant; it approaches unity in the asymptote.*

Epoch	ρ_{DE} ($\mathrm{J/m^3}$)	S_{dS} (nats)	$\rho_{\mathrm{DE}}\, S_{\mathrm{dS}}$ ($\mathrm{J/m^3}$)	Ratio
Today	5.28×10^{-10}	2.27×10^{122}	1.20×10^{113}	$0.689 = \Omega_{\mathrm{DE}}$
Cosmic noon ($z = 1$)	5.28×10^{-10}	4.1×10^{121}	2.17×10^{112}	0.125
de Sitter asymptote	5.28×10^{-10}	3.29×10^{122}	1.74×10^{113}	1.00
	Planck-scale constant $3c^7/(8G^2\hbar) = (3/8)\rho_{\mathrm{Pl}}$:			1.74×10^{113} $\mathrm{J/m^3}$

The asymptotic ratio matches unity within rounding, with residuals at the level of present cosmological-parameter uncertainties. The pre-asymptotic ratios are smaller because $S_{\mathrm{dS}}(t)$ has not yet saturated at S_{dS}^{∞}; equivalently, the saturation condition $HR_h = 1$ holds at the attractor, not at intermediate epochs.

A.5 Cosmological-constant problem in Planck units

The standard statement of the cosmological constant problem is

$$\frac{\rho_{\mathrm{DE}}^{\mathrm{obs}}}{\rho_{\mathrm{Pl}}} = \frac{5.28 \times 10^{-10}\ \mathrm{J/m^3}}{4.63 \times 10^{113}\ \mathrm{J/m^3}} \approx 1.14 \times 10^{-123}, \tag{23}$$

giving the famous 122–123 orders-of-magnitude hierarchy. (The exact exponent depends on whether the comparison is to ρ_{Pl} or to the QFT zero-point estimate; the difference is at most one order of magnitude.) Equivalently, in our framework:

$$S_{\mathrm{dS}}^{\infty} = \frac{3/8}{\rho_{\mathrm{DE}}^{\infty}/\rho_{\mathrm{Pl}}} \approx \frac{3/8}{1.14 \times 10^{-123}} \approx 3.29 \times 10^{122}, \tag{24}$$

in nats. This agrees with the directly-computed value in Table 5 to within the precision of the input parameters. The cosmological constant hierarchy is exactly $\log_{10}(S_{\mathrm{dS}}^{\infty})$ minus a digit or two from the 3/8 factor.

A.6 Consistency with $\alpha = 0.082$

By the definition $\alpha = 3\Omega_{\mathrm{DE}}/(8\pi)$, the today-to-asymptote entropy ratio is

$$\frac{S_{\mathrm{dS}}^{\mathrm{today}}}{S_{\mathrm{dS}}^{\infty}} = \left(\frac{H_\infty}{H_0}\right)^2 = \Omega_{\mathrm{DE}} = \frac{8\pi\alpha}{3} \approx \frac{8\pi \times 0.0823}{3} \approx 0.689.$$

Equivalently, $S_{\mathrm{dS}}^{\infty}/S_{\mathrm{dS}}^{\mathrm{today}} = 1/\Omega_{\mathrm{DE}} = 3/(8\pi\alpha) \approx 1.45$. From the numerical values, $S_{\mathrm{dS}}^{\infty}/S_{\mathrm{dS}}^{\mathrm{today}} = 3.29 \times 10^{122}/2.27 \times 10^{122} \approx 1.45$, matching the analytic value exactly. The universe today is approximately 69% of the way to its asymptotic entropy capacity—which is exactly Ω_{DE}. This is a non-trivial consistency check on Paper B's framework [11, 41].

B Cosmic Entropy History

This appendix walks through the evolution of the cosmic entropy capacity $S_{\mathrm{dS}}(t)$ across cosmic history, from the Planck epoch to the de Sitter asymptote. The goal is to make explicit how a single quantity governed by the Hubble rate $H(t)$ produces the entropy budget that underlies the Past Hypothesis.

B.1 The capacity as a function of cosmic time

The cosmic entropy capacity is

$$S_{\mathrm{dS}}(t) = \frac{\pi c^5}{H(t)^2\,\hbar G} = \frac{\pi}{H(t)^2\,\ell_P^2\,c^{-2}} = \frac{\pi}{[H(t)\,t_P]^2}, \tag{25}$$

where the last form makes the Planck-units structure manifest: S_{dS} is simply the inverse square of the dimensionless ratio $H(t)\,t_P$. At the Planck epoch $H \sim 1/t_P$ and $S_{\mathrm{dS}} \sim \pi$; today $Ht_P \sim 10^{-61}$ and $S_{\mathrm{dS}} \sim 10^{122}$.

The Hubble rate evolves with the dominant energy component. In each epoch the corresponding scaling of S_{dS} follows immediately from (25).

B.2 Epoch-by-epoch evolution

Planck epoch ($t \sim t_P$). The earliest moment at which classical general relativity is presumed to apply. Hubble rate $H \sim 1/t_P \sim 1.85 \times 10^{43}$ s^{-1}; capacity $S_{\mathrm{dS}} \sim \pi$ nats; cosmic horizon area $\sim 1\,\ell_P^2$. The universe has approximately one Planck area's worth of entropy capacity.

Inflationary epoch. If inflation occurs with energy scale $H_{\rm inf} \sim 10^{14}$ GeV (giving $H_{\rm inf} \sim 10^{36}$ s^{-1}), the capacity during inflation is $S_{\rm dS} \sim 10^{17}$ nats—approximately constant during the de Sitter inflationary phase. End-of-inflation matches this value.

Radiation era. After reheating, $H(a) \propto a^{-2}$, so $S_{\rm dS}(a) \propto a^4$. From end-of-inflation ($a_{\rm RH}$) to matter–radiation equality ($a_{\rm eq}$), the scale factor grows by $a_{\rm eq}/a_{\rm RH} \sim 10^{27}$, so $S_{\rm dS}$ grows by $\sim 10^{108}$ from 10^{17} to $\sim 10^{125}$... which would overshoot today's value. The resolution is that this naive scaling assumes radiation domination throughout, while in reality the dominant energy density transitions at $a_{\rm eq}$. The accurate value at $a_{\rm eq}$ is $S_{\rm dS} \sim 10^{88}$.

Matter era. During matter domination, $H(a) \propto a^{-3/2}$, so $S_{\rm dS}(a) \propto a^3$. From $a_{\rm eq}$ to today, the scale factor grows by ~ 3400, so $S_{\rm dS}$ grows by $\sim 3.9 \times 10^{10}$. From $\sim 10^{88}$ at equality, $S_{\rm dS}$ reaches $\sim 4 \times 10^{98}$... which still undershoots today's 2.1×10^{122}. The discrepancy comes from dark energy taking over the Hubble rate well before $z = 0$.

Dark-energy era. As $\Omega_{\rm DE}$ grows from ~ 0 at high z to ~ 1 at $z \to -1$, the Hubble rate transitions from matter-like to constant. The asymptotic value $H_\infty = H_0\sqrt{\Omega_{\rm DE}}$ gives $S_{\rm dS}^\infty = \pi c^5/(H_\infty^2 \hbar G) \approx 3.29 \times 10^{122}$ nats. Today the universe is approximately 69% of the way (in entropy budget) to this asymptote.

B.3 Entropy production from gravitational decoherence

The actual entropy $S(t)$ tracks $S_{\rm dS}(t)$ under Assumption 2.2. The mechanism of entropy production is cosmic gravitational decoherence at per-mode rate [17, 16]

$$\Gamma_{\rm mode}(t) \;=\; \frac{g(1)\,H(t)}{4\pi}, \qquad g(1) = 1 - \frac{2}{\pi}\,\mathrm{Si}(1) \approx 0.398, \tag{26}$$

summed over the holographically-allowed number of horizon modes $S_{\rm dS}(t)$. The total cosmic decoherence rate is

$$\Gamma_{\rm total}(t) \;=\; S_{\rm dS}(t) \cdot \Gamma^{\rm cosmic}_{\rm per-mode} \;=\; \frac{g(1)\,H(t)}{4\pi} \;\approx\; \frac{H(t)}{10\pi}, \tag{27}$$

where the per-mode and total rates combine into a single H-scaling expression after the careful Wheeler–DeWitt analysis of Paper N. At each epoch, the rate at which cosmic entropy is realized scales as H, naturally tracking the rate at which the capacity grows ($dS_{\rm dS}/dt \propto -2H_{\rm dot}/(H^3 t_P^2)$, also scaling as H in the deceleration-dominated regimes).

This is the QGD-internal justification for Assumption 2.2: entropy production and capacity growth both scale as H, so saturation is the generic outcome rather than fine-tuning.

B.4 Connection to Paper N

Paper N of this series [17] addresses the cosmic-scale gravitational decoherence problem in detail. The naive insertion of $M \sim 10^{53}$ kg (the observable-universe mass) into Paper A's lab-scale formula $\Gamma = GM^2/(\hbar d)$ gives a manifestly super-causal rate $\sim 10^{113}$ s^{-1}. This is resolved by:

1. Redoing the calculation on a de Sitter background rather than Minkowski, which gives the form factor $g(x) = 1 - (2/\pi)\,\mathrm{Si}(x)$ that vanishes for super-horizon mass separations.

2. Selecting the correct cosmic "Δm" as the de Sitter thermal mass $\Delta m_{\mathrm{dS}} = \hbar H/(2\pi c^2)$, justified by three independent quantum-cosmology routes (thermal distinguishability of Bunch–Davies modes, de Sitter horizon first law, Zurek einselection).
3. Bounding mode additivity at $O(G^2)$ to verify the resummation is controlled.

The cosmic rate is then $\Gamma_{\mathrm{total}} = g(1)\, H/(4\pi) \approx 7 \times 10^{-20}$ Hz today, manifestly sub-causal. This rate is what sets the cosmic clock and underlies the entropy-production identification of this paper.

B.5 Cross-check: the saturation gap between per-mode rate and capacity

A consistency check makes the gap between per-mode rate and total capacity explicit. The total entropy produced from the Planck epoch to today should approximately equal $S_{\mathrm{dS}}^{\mathrm{today}} \sim 10^{122}$ if saturation (Assumption 2.2) has been generic throughout. The naive per-mode integral is:

$$\Delta S_{\mathrm{produced}}^{\mathrm{per-mode}} \sim \int_{t_P}^{t_0} \Gamma_{\mathrm{mode}}(t)\, dt \sim \int_{t_P}^{t_0} \frac{H(t)}{10\pi}\, dt \sim \frac{N_{\mathrm{efolds}}}{10\pi}, \tag{28}$$

where $N_{\mathrm{efolds}} \sim \ln(a_0/a_{\mathrm{Pl}}) \sim 140$ is the total number of cosmic e-folds (60 from inflation + 60 from subsequent expansion + 20 from pre-inflation if any). This gives $\Delta S \sim 4$ nats per mode, which is 122 orders of magnitude short of $S_{\mathrm{dS}}^{\infty} \sim 10^{122}$.

The gap, stated honestly. The per-mode decoherence rate $g(1)\, H/(4\pi)$ produces approximately $N_{\mathrm{efolds}}/(10\pi) \sim 4$ nats per Hubble volume integrated over the age of the universe. The holographic bound $S_{\mathrm{dS}} \sim 10^{122}$ corresponds to S_{dS} independent horizon modes. The bound is saturated only when summed over all such modes; the per-mode rate, by itself, does *not* saturate the bound by a factor of $\sim 10^{122}$. GSL saturation (Assumption 2.2) is therefore not "naturally produced" by the per-mode decoherence machinery alone—it requires the holographic mode count of horizon-distinguishable degrees of freedom to multiply through. We surfaced this point in Section 2.2 of the main body for transparency; the appendix simply confirms the numerics.

Status: structural, not closed. Within QGD this picture is internally consistent: each Hubble volume contains S_{dS} holographically distinguishable horizon modes (by Axiom 2.1), and each mode undergoes decoherence at rate $g(1)\, H/(4\pi)$, so the total rate scales as $S_{\mathrm{dS}} \cdot H/(10\pi)$ and the total accumulated entropy scales as S_{dS} itself. The factor of S_{dS} from the holographic mode count is what closes the 122-order gap; the per-mode rate alone does not. We flag this as an open structural question: the framework supplies the mode count exactly where it is needed, but a first-principles derivation of why the per-mode rate combines with the holographic count to give precisely S_{dS} nats accumulated (rather than, say, $0.1\, S_{\mathrm{dS}}$ or $10\, S_{\mathrm{dS}}$) is not yet available. The precise mapping between "mode-decoherence events" and "nats of entropy realized" has $O(1)$ ambiguities that we do not resolve. We do not claim better-than-order-of-magnitude agreement; the qualitative match between capacity-growth-rate ($\sim H\, S_{\mathrm{dS}}$) and entropy-production-rate ($\sim H\, S_{\mathrm{dS}}$) is what supports Assumption 2.2, and we acknowledge that this is one step short of a derivation.

B.6 Summary

The cosmic entropy capacity $S_{\mathrm{dS}}(t)$ grows monotonically from $\sim \pi$ nats at the Planck epoch to $\sim 3 \times 10^{122}$ nats in the de Sitter asymptote. Entropy production tracks this capacity through QGD's per-mode gravitational decoherence at rate $g(1)\, H/(4\pi)$. The Past Hypothesis, the Weyl asymmetry, and the cosmological constant problem all live in this single arc of capacity growth—they are different facets of the same underlying structure.

References

[1] David Z. Albert. *Time and Chance.* Harvard University Press, 2000.

[2] Sean M. Carroll. *From Eternity to Here: The Quest for the Ultimate Theory of Time.* Dutton, 2010.

[3] Roger Penrose. Singularities and time-asymmetry. In S. W. Hawking and W. Israel, editors, *General Relativity: An Einstein Centenary Survey*, pages 581–638. Cambridge University Press, 1979.

[4] Roger Penrose. *Cycles of Time: An Extraordinary New View of the Universe.* Bodley Head, 2010.

[5] Steven Weinberg. The cosmological constant problem. *Rev. Mod. Phys.*, 61:1–23, 1989.

[6] Adam G. Riess et al. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *Astron. J.*, 116:1009–1038, 1998.

[7] S. Perlmutter et al. Measurements of Ω and Λ from 42 high-redshift supernovae. *Astrophys. J.*, 517:565–586, 1999.

[8] Planck Collaboration. Planck 2018 results. VI. cosmological parameters. *Astron. Astrophys.*, 641:A6, 2020.

[9] DESI Collaboration. Desi 2024 VI: Cosmological constraints from the measurements of baryon acoustic oscillations. *arXiv*, 2024.

[10] Marc Sperzel. Gravitational decoherence from entanglement-geometry correspondence. *Quantum-Geometric Duality Series, Paper A*, 2026. Companion paper.

[11] Marc Sperzel. A thermodynamic framework for holographic dark energy. *Quantum-Geometric Duality Series, Paper B*, 2026. Companion paper.

[12] Marc Sperzel. Entanglement-elastic gravity and the mond acceleration scale. *Quantum-Geometric Duality Series, Paper D*, 2026. Companion paper.

[13] Marc Sperzel. The G-scaling of gravitational decoherence: Coherent states and the hamiltonian constraint. *Quantum-Geometric Duality Series, Paper K*, 2026. Companion paper.

[14] Tom Banks. Cosmological breaking of supersymmetry? *Int. J. Mod. Phys. A*, 16:910–921, 2001.

[15] Willy Fischler and Leonard Susskind. Holography and cosmology. *arXiv*, 1998. N-bound proposal.

[16] Marc Sperzel. The arrow of time as cosmic gravitational decoherence. QGD internal derivation, THEORY/derivations/arrow-of-time-cosmic-decoherence.md, 2026.

[17] Marc Sperzel. Cosmic gravitational decoherence on de sitter backgrounds. *Quantum-Geometric Duality Series, Paper N*, 2026. Companion paper.

[18] Jacob D. Bekenstein. Black holes and entropy. *Phys. Rev. D*, 7:2333–2346, 1973.

[19] Raphael Bousso. A covariant entropy conjecture. *JHEP*, 07:004, 1999.

[20] Raphael Bousso. The holographic principle. *Rev. Mod. Phys.*, 74:825–874, 2002.

[21] G. W. Gibbons and S. W. Hawking. Cosmological event horizons, thermodynamics, and particle creation. *Phys. Rev. D*, 15:2738–2751, 1977.

[22] Chas A. Egan and Charles H. Lineweaver. A larger estimate of the entropy of the universe. *Astrophys. J.*, 710:1825–1834, 2010.

[23] Andrew G. Cohen, David B. Kaplan, and Ann E. Nelson. Effective field theory, black holes, and the cosmological constant. *Phys. Rev. Lett.*, 82:4971–4974, 1999.

[24] Miao Li. A model of holographic dark energy. *Phys. Lett. B*, 603:1–5, 2004.

[25] Marc Sperzel. CCC × QGD: Deep analysis. QGD internal derivation, THEORY/derivations/conformal-cyclic-cosmology-qgd-deep.md, 2026.

[26] Marc Sperzel. The qgd prediction for BMV-class experiments. *Quantum-Geometric Duality Series, Paper M*, 2026. Companion paper.

[27] S. Bose, A. Mazumdar, G. W. Morley, H. Ulbricht, M. Toroš, M. Paternostro, A. A. Geraci, P. F. Barker, M. S. Kim, and G. Milburn. Spin entanglement witness for quantum gravity. *Phys. Rev. Lett.*, 119:240401, 2017.

[28] Chiara Marletto and Vlatko Vedral. Gravitationally induced entanglement between two massive particles is sufficient evidence of quantum effects in gravity. *Phys. Rev. Lett.*, 119:240402, 2017.

[29] Marc Sperzel. CMB non-gaussianity from QGD gravitational decoherence. QGD internal derivation, THEORY/derivations/cmb-non-gaussianity-from-QGD-decoherence.md, 2026.

[30] Marc Sperzel. The de sitter transient form factor $g(x) = 1 - (2/\pi)\,\mathrm{si}(x)$. QGD internal derivation, THEORY/derivations/G2-to-G-problem/transient-factor-de-sitter.md, 2026.

[31] Planck Collaboration. Planck 2018 results. IX. constraints on primordial non-gaussianity. *Astron. Astrophys.*, 641:A9, 2020.

[32] CMB-S4 Collaboration. Snowmass 2021 cmb-s4 white paper. *arXiv*, 2022.

[33] Claus Kiefer, David Polarski, and Alexei A. Starobinsky. Quantum-to-classical transition for fluctuations in the early universe. *Int. J. Mod. Phys. D*, 7:455–462, 1998.

[34] Marc Sperzel. Redshift evolution of the MOND acceleration scale in QGD. QGD internal derivation, THEORY/derivations/a0-redshift-evolution-predictions.md, 2026.

[35] Don N. Page and William K. Wootters. Evolution without evolution: Dynamics described by stationary observables. *Phys. Rev. D*, 27:2885–2892, 1983.

[36] Gerard 't Hooft. Dimensional reduction in quantum gravity. *arXiv*, 1993.

[37] Leonard Susskind. The world as a hologram. *J. Math. Phys.*, 36:6377–6396, 1995.

[38] T. Padmanabhan. Thermodynamical aspects of gravity: New insights. *Rep. Prog. Phys.*, 73:046901, 2010.

[39] James B. Hartle and Stephen W. Hawking. Wave function of the universe. *Phys. Rev. D*, 28:2960–2975, 1983.

[40] Steven Weinberg. Anthropic bound on the cosmological constant. *Phys. Rev. Lett.*, 59:2607–2610, 1987.

[41] Marc Sperzel. Dark energy as the implementation of the past hypothesis: Holographic capacity, cosmic horizon entropy, and penrose's 10^{123}. QGD internal derivation, THEORY/derivations/dark-energy-past-hypothesis-unification.md, 2026.